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Facoltà di Scienze
Corso di Laurea in Scienze Biologiche

Corso di recupero per l'assolvimento degli obblighi formativi (OFA) dell'insegnamento di Matematica

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Disequazioni esponenziali e logaritmiche, identità goniometriche

(i) Risolvere le seguenti disequazioni esponenziali:

(1) $e^{4x} > 4,$ $\qquad \left(\log\sqrt{2}, +\infty\right)$

(2) $2e^{x+3} > 5,$ $\qquad \left(-3+\log\frac{5}{2}, +\infty\right)$

(3) $5^{2x} - 2\cdot 5^x - 3 \geq 0,$ $\qquad [\log_5 3, +\infty)$

(4) $(e^x-2)(-e^{-x}-1) \geq 0,$ $\qquad (-\infty, \log 2]$

(5) $2\left(\frac{1}{9}\right)^{\frac{x+1}{2}} - \left(\frac{1}{9}\right)^x + \frac{5}{3} \geq 0,$ $\qquad \left[2\log_{1/9}\frac{5}{3}, +\infty\right)$

(6) $|3^{2x} - 3^x| < 2,$ $\qquad (-\infty, \log_3 2)$

(7) $\dfrac{3\cdot 2^{2x+2}-12}{2^x} \leq 2^x + 7\cdot 2^{2x} - 7 - 2^{3x},$ $\qquad (-\infty, 0] \cup [\log_2 3, 2]$

(8) $2\cdot e^{3x} - 9\cdot e^{2x} + e^x + 12 \leq 0,$ $\qquad \left[\log\frac{3}{2}, \log 4\right]$

(9) $\dfrac{3\cdot e^{2x}}{4-e^x} \geq 1,$ $\qquad [0, \log 4)$

(10) $\left(2^{x^2} - \frac{1}{3}\right)(5^{3x} - 6\cdot 5^{2x} + 3\cdot 5^x + 10) \leq 0,$ $\qquad [\log_5 2, 1]$

(11) $3^{\sqrt{x}+1} - 9^{\sqrt{x}} + 4 \geq 0,$ $\qquad \left[0, \log_3^2 4\right]$

(12) $\dfrac{2-5^x}{2\cdot 5^x - 2} + \dfrac{2}{25^x - 5^x} \leq \dfrac{3-5^x}{5^x-1},$ $\qquad (-\infty, 0) \cup \{\log_5 2\}$

(13) $$\left(e^{\sqrt{2x+3}} - e^x\right)(e^{2x} - e^x - 2) \le 0. \qquad \left[-\frac{3}{2}, \log 2\right] \cup [3, +\infty)$$

(ii) Applicando le proprietà dei logaritmi, trasformare le seguenti espressioni:

(1) $$\log_a x^2 y^3, \quad \log_a \frac{xy}{z}, \quad \log_a \sqrt{\frac{xy}{z}}, \quad \log_a x^2 \sqrt[3]{y}, \quad \log_a \frac{\sqrt[3]{xy^2}}{z^3}, \quad \log_a \sqrt[3]{\frac{x^2\sqrt{yz^3}}{t^4}};$$

$$\Big[2\log_a x + 3\log_a y,\ \log_a x + \log_a y - \log_a z,\ \frac{1}{2}\left(\log_a x + \log_a y - \log_a z\right),$$
$$2\log_a x + \frac{1}{3}\log_a y,\ \frac{1}{3}(\log_a x + 2\log_a y) - 3\log_a z,$$
$$\frac{1}{3}\left(2\log_a x + \frac{1}{2}(\log_a y + 3\log_a z) - 4\log_a t\right)\Big]$$

(2) $$3\log_a x + \frac{1}{2}\log_a y - \frac{1}{4}\log_a z, \quad \log_a 2 + \log_a x - \frac{1}{2}\log_a y - \frac{1}{2}\log_a z;$$

$$\left[\log_a \frac{x^3\sqrt{y}}{\sqrt[4]{z}},\ \log_a \frac{2x}{\sqrt{yz}}\right]$$

(iii) Applicando le proprietà dei logaritmi, verificare le seguenti uguaglianze:

(1) $$\log_a b = -\log_a \frac{1}{b}, \quad \log_a b = -\log_{1/a} b, \quad \log_a b = \frac{1}{\log_b a}.$$

(iv) Risolvere le seguenti disequazioni logaritmiche:

(1)	$\log_{\sqrt[3]{5}} x < 6,$	$(0, 25)$
(2)	$\log_{\sqrt{2}} x > 4,$	$(4, +\infty)$
(3)	$\log_{1/2} x < 4,$	$\left(\frac{1}{16}, +\infty\right)$
(4)	$\log_{1/10} x > \frac{1}{2},$	$\left(0, \frac{\sqrt{10}}{10}\right)$
(5)	$\log_{1/3}(x^2 - 3) > 0,$	$\left(-2, -\sqrt{3}\right) \cup \left(\sqrt{3}, 2\right)$
(6)	$\log_2 \left(x^2 - \frac{3}{4}\right) < -2,$	$\left(-1, \frac{\sqrt{3}}{2}\right) \cup \left(\frac{\sqrt{3}}{2}, 1\right)$
(7)	$\log_{1/2}(x^2 - x) > \log_{1/2} 6,$	$(-2, 0) \cup (1, 3)$
(8)	$\log_{1/2}(3x - 5) < \log_{1/4}(2x - 1),$	$\left(\frac{16 + \sqrt{22}}{9}, +\infty\right)$
(9)	$\log_5 \frac{1 + \lvert x\rvert}{1 - \lvert x\rvert} > \log_5 2,$	$\left(-1, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, 1\right)$
(10)	$\log(7 - x) > 2\log(x + 3) - \log(12 - x),$	$(-3, 3)$
(11)	$1 \geq \log 2x - 2\log x,$	$\left[\frac{2}{e}, +\infty\right)$
(12)	$\log \left(x - \frac{3}{2}\right) < -\log x,$	$\left(\frac{3}{2}, 2\right)$
(13)	$\log^2 x - 3\log x - 4 < 0,$	$\left(e^{-1}, e^4\right)$
(14)	$\log^4_{1/2} x - \log^3_{1/2} x \geq 0,$	$\left(0, \frac{1}{2}\right] \cup [1, +\infty)$
(15)	$(\log_2 \lvert x\rvert)^2 + 2\log_2 \lvert x\rvert - 3 < 0,$	$\left(-2, -\frac{1}{8}\right) \cup \left(\frac{1}{8}, 2\right)$
(16)	$\log_2 \frac{x + \sqrt{x^2 + 9}}{2x} > 1,$	$\left(0, \frac{3}{2\sqrt{2}}\right)$
(17)	$\frac{\log_2^2 x - 5\log_2 x + 6}{1 - \log_2^2 x} \geq 0,$	$\left(\frac{1}{2}, 2\right) \cup [4, 8]$

(v) Verificare le seguenti identità goniometriche:

(1) $$\frac{1-\cos^2\alpha}{\sin\alpha\cos\alpha}+\frac{\cos^4\alpha-\sin^4\alpha}{1-\sin^2\alpha}=1+\operatorname{tg}\alpha(1-\operatorname{tg}\alpha),$$

(2) $$\frac{\sin^3\alpha-\sin\alpha}{\cos\alpha}+\frac{1}{\operatorname{ctg}\alpha}=\operatorname{tg}\alpha-\frac{\operatorname{ctg}\alpha}{\operatorname{ctg}^2\alpha+1},$$

(3) $$(\sin\alpha+\cos\alpha)^3=3(\sin\alpha+\cos\alpha)-2(\sin^3\alpha+\cos^3\alpha),$$

(4) $$\frac{1-\cos^4\alpha}{\sin^2\alpha+\operatorname{tg}^2\alpha}=\cos^2\alpha,$$

(5) $$\sin^2\alpha\operatorname{tg}\alpha+\cos^2\alpha\operatorname{ctg}\alpha+2\sin\alpha\cos\alpha=\operatorname{tg}\alpha+\operatorname{ctg}\alpha,$$

(6) $$\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha,$$

(7) $$4\sin\alpha\sin\left(\frac{2}{3}\pi-\alpha\right)\sin\left(\frac{\pi}{3}-\alpha\right)=\sin3\alpha,$$

(8) $$\frac{\cos2\alpha}{\sin\left(\frac{\pi}{4}-\alpha\right)}-\frac{\sin2\alpha}{\sin\left(\frac{\pi}{4}+\alpha\right)}=\frac{1}{\sin\left(\frac{3}{4}\pi-\alpha\right)},$$

(9) $$\frac{\cos3\alpha+\sin3\alpha}{\cos\alpha-\sin\alpha}-\sin2\alpha=1+2\sin\alpha\cos\alpha,$$

(10) $$\frac{1+\sin2\alpha}{1+\cos2\alpha}=\frac{1}{2}(1+\operatorname{tg}\alpha)^2,$$

(11) $$\operatorname{tg}2\alpha-\operatorname{tg}\left(\alpha+\frac{\pi}{4}\right)=-\frac{1}{\cos2\alpha},$$

(12) $$\cos^4\alpha+\sin^2\alpha-\cos^2 2\alpha=\frac{3}{4}\sin^2 2\alpha,$$

(13) $$\frac{1+\sin\alpha}{\cos\alpha}=\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}},$$

(14) $$2\left(\sin\frac{\alpha}{2}-\sin^3\frac{\alpha}{2}\right)=\left(2\cos^3\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)\operatorname{tg}\alpha,$$

(15) $$\frac{1-\cos2\alpha}{1-\cos\alpha}=4\cos^2\frac{\alpha}{2},$$

(16) $$\frac{\operatorname{ctg}\frac{\alpha}{2}-\operatorname{tg}\frac{\alpha}{2}}{\operatorname{ctg}\frac{\alpha}{2}+\operatorname{tg}\frac{\alpha}{2}}=\cos\alpha,$$

(17) $$\frac{\cos\alpha}{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}-\frac{\sin\alpha}{\cos\left(\frac{\alpha}{2}+\frac{\pi}{4}\right)}-\frac{1}{\sin\left(\frac{5}{4}\pi-\frac{\alpha}{2}\right)}=2\sqrt{2}\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right),$$

(18) $$\operatorname{tg}\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)-\operatorname{tg}\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)=2\operatorname{tg}\alpha,$$

(19) $$\sin\alpha\sin2\alpha+\sin\alpha\sin4\alpha+\sin\alpha\sin6\alpha=\sin3\alpha\sin4\alpha,$$

(20) $$\sin^2 4\alpha-\sin^2 2\alpha=\sin6\alpha\sin2\alpha,$$

(21) $$\frac{\cos3\alpha+\cos4\alpha+\cos5\alpha}{\sin3\alpha+\sin4\alpha+\sin5\alpha}=\operatorname{ctg}4\alpha,$$

(22) $$\cos^4\alpha+\sin^2\alpha-\cos^2 2\alpha=\frac{3}{4}\sin^2 2\alpha,$$

(23) $$\frac{\sin5\alpha+\sin3\alpha}{\sin5\alpha-\sin3\alpha}=\operatorname{tg}4\alpha\operatorname{ctg}\alpha.$$