Global components of positive bounded variation solutions of a one-dimensional indefinite quasilinear Neumann problem

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We investigate the topological structure of the set of the positive solutions of the one-dimensional quasilinear indefinite Neumann problem

$$-\left(u'/\sqrt{1+u'^2}\right)' = \lambda a(x) f(u) \text{ in } (0,1), \quad u'(0) = 0,\ u'(1) = 0,$$

where $\lambda \in \mathbb{R}$ is a parameter, $a \in L^\infty(0,1)$ changes sign, and $f \in C^1(\mathbb{R})$ is positive in $(0,+\infty)$.

Our attention is focused on the case $f(0) = 0$ and $f'(0) = 1$, where we can prove a bifurcation result for this problem in the space of bounded variation functions. Namely, we establish the existence of global connected components of the set of the positive solutions, emanating from the line of the trivial solutions at the two principal eigenvalues of the linearized problem around 0. The solutions in these components are regular, as long as they are small, while they may develop jump singularities at the nodes of the weight function a, as they become larger, thus showing the possible coexistence along the same component of regular and singular solutions.

This is joint work with Julian López-Gómez.