Multiplicity of ground states for the scalar curvature equation

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We study the existence and multiplicity of radial ground states for the scalar curvature equation

$$\Delta u + K(|x|)\, u^{\frac{n+2}{n-2}} = 0, \quad x \in \mathbb{R}^n, \quad n \geq 3$$

where $K : \mathbb{R}^+ \to \mathbb{R}^+$ is a C^1 reciprocally symmetric, bounded function,

$$0 < \underline{K} \leq K(r) \leq \overline{K} \quad \forall\, r > 0,$$

which decreases in $(0, 1)$ and increases in $(1, \infty)$. Chen and Lin in [1] had shown the existence of a large number of fast decay solutions, i.e. solutions which decay at infinity like $|x|^{2-n}$, if K is a sufficiently small perturbation of a positive constant. Our main purpose is to improve and extend the result in [1] to a non-perturbative situation: by developing a constructive argument based on some elementary tools of phase plane analysis, we are able to prove multiplicity results whenever the ratio $\overline{K}/\underline{K}$ is smaller than some computable (and not so small) values.

This is a joint work with Matteo Franca and Andrea Sfecci.

References

[1] C.C. Chen, C.S. Lin, Blowing up with infinite energy of conformal metrics on S^n, *Comm. Partial Differential Equations* **24** (1999), 785–799.