Existence of one non-zero solution for a two point boundary value problem involving a fourth-order equation

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The following fourth-order problem will be discussed

$$\begin{cases} u^{(iv)}(x) = \lambda f(x, u(x)) \text{ in } [0,1] \\ u(0) = u'(0) = 0 \\ u''(1) = 0 \quad u'''(1) = \mu g(u(1)) \end{cases} \qquad (P_{\lambda,\mu})$$

where $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is an L^1- Carathéodory function, $g : \mathbb{R} \to \mathbb{R}$ is a continuous function and λ, μ are positive parameters. The problem $(P_{\lambda,\mu})$ describes the static equilibrium of a flexible elastic beam of length 1, clamped at its left end $x = 0$ and resting on an elastic device at its right end $x = 1$ (given by g), when, along its length, a load f is added to cause deformation.
Object of this talk, will be the existence of one non-zero solution for the problem $(P_{\lambda,\mu})$. Precisely, using a variational approach, under conditions involving the antiderivatives of f and g, we will obtain two precise intervals of the parameters λ and μ for which the problem $(P_{\lambda,\mu})$ admits at least one non-zero classical solution.

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