Some existence results for boundary value problems associated with singular equations

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We prove some existence results for boundary value problems associated with (possibly degenerate) non-linear differential equations of the following type

$$\Big(\Phi\big(a(t,x(t))\,x'(t)\big)\Big)' = f(t,x(t),x'(t)) \quad \text{a.e. on } [0,T].$$

Here, $\Phi : \mathbb{R} \to \mathbb{R}$ is a strictly increasing homeomorphism, $a : [0,T] \times \mathbb{R} \to \mathbb{R}$ is continuous and $f : [0,T] \times \mathbb{R}^2 \to \mathbb{R}$ is a Carathéodory function.

Following an approach similar to that exploited in [1, 2], our existence results rely on a suitable combination of fixed-point techniques (applied to an auxiliary abstract equation) with the well-known method of lower/upper solutions. Such an approach is powerful enough to allows us to consider also the case when a vanishes on a set with zero Lebesgue measure, provided that f satisfies a weak form of a Wintner-Nagumo growth condition.

References

[1] A. Calamai, C. Marcelli, F. Papalini: *Boundary Value Problems for singular second order equations*, preprint (2018)

[2] C. Marcelli, F. Papalini: *Boundary value problems for strongly nonlinear equations under a Wintner-Nagumo growth condition*, Bound. Value Probl. **2017**, 1–15 (2017).