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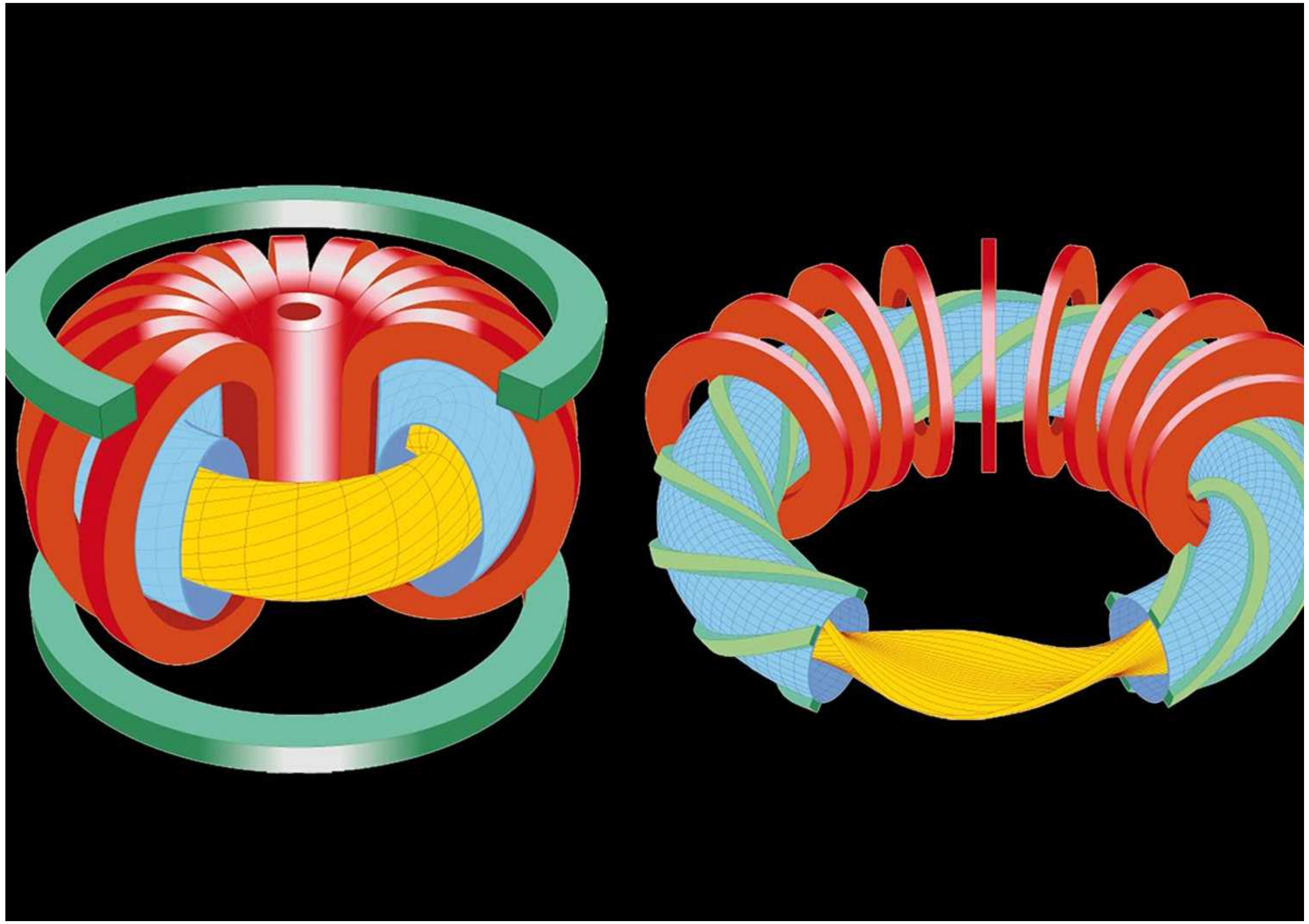
GEDO 2018, Ancona

Complicated dynamics in a model of charged particles



Cambon and co-workers, *Chaotic motion of charged particles in toroidal magnetic configurations*, Chaos 24, 2014

$$H_{\text{eff}} := \frac{\dot{r}^2}{2} + \frac{A^2}{2r^2} + \frac{(B_0)^2}{8} r^2 + F^2(r) = E_c + V_0(r) + F^2(r),$$

for

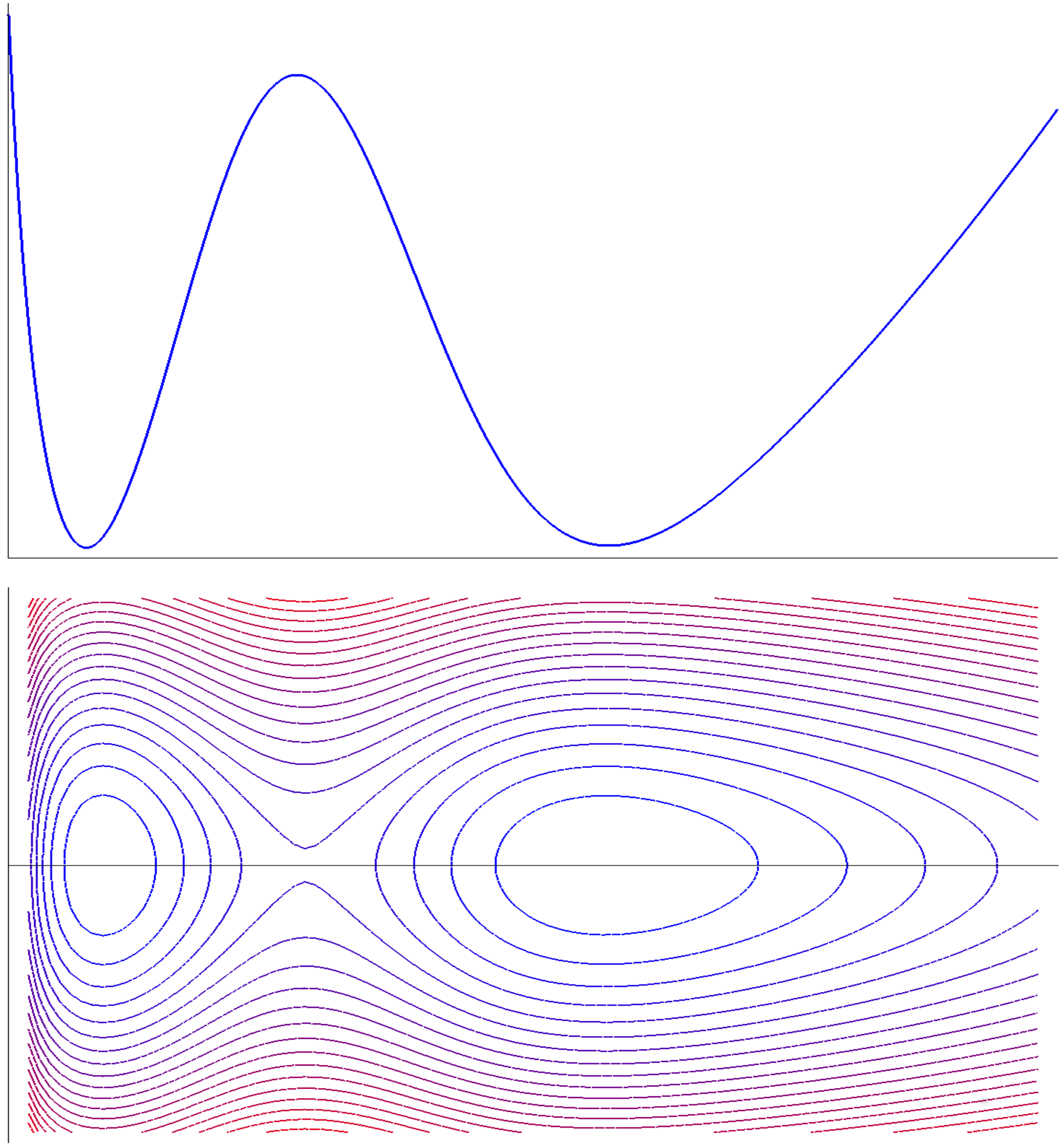
$$F(r) := ar^2 \exp\left(-\frac{r^2}{c^2}\right),$$

where A, a, c **are positive constants and** B_0 **is the magnetic field.**

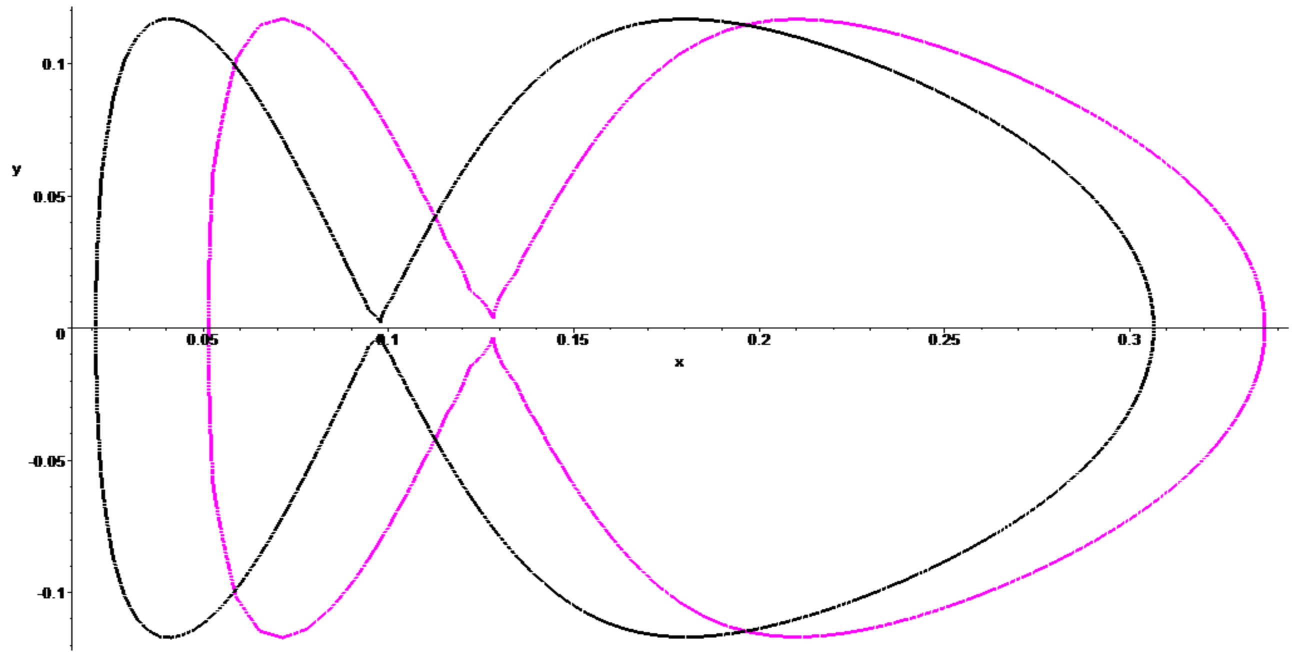
$$\begin{cases} \dot{x} = y \\ \dot{y} = -g(x), \end{cases}$$

for $x := r > 0,\ y = \dot{r}$ **and**

$$g(x) := \frac{d}{dx}\left(V_0(x) + F(x)^2\right) = -\frac{A^2}{x^3} + \frac{{B_0}^2}{4} x + 2F(x)F'(x),$$

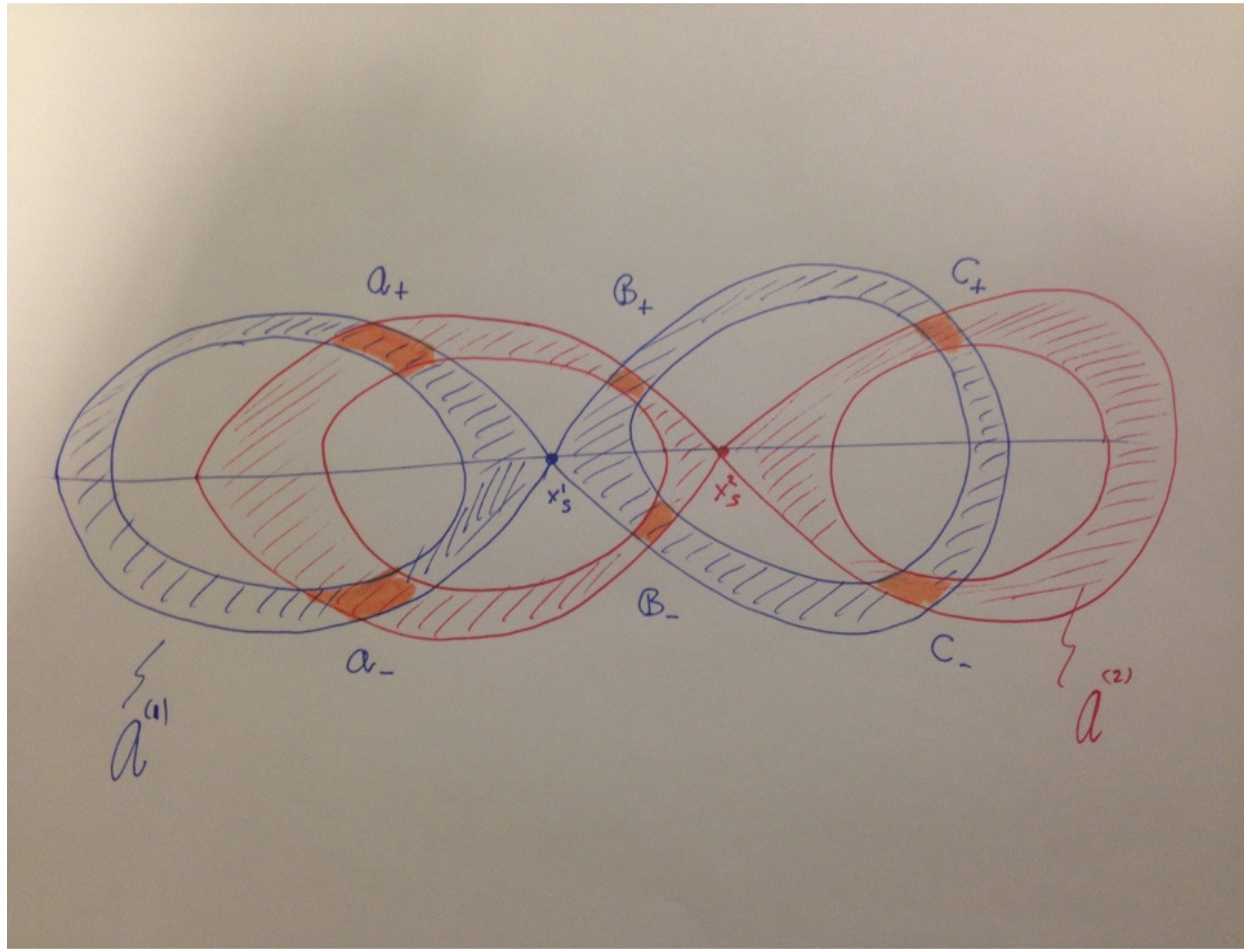


Perturbation



$$B(t) := \begin{cases} B_1(t), & \textbf{for}\ \ 0 \le t < T_1 \\ B_2(t), & \textbf{for}\ \ T_1 \le t < T_1 + T_2 = T, \end{cases}$$

Rectangular regions



Crossing condition: $\boldsymbol{a}^{(1)} \cap \boldsymbol{a}^{(2)} = \{\mathfrak{a}_{\pm}\ \mathfrak{b}_{\pm}, \mathfrak{a}_{\pm}\}$

Poincare maps:

Let Φ_i be the Poincaré map on the time-interval $[0, T_i]$ associated with the system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -g_i(t, x) \end{cases}$$

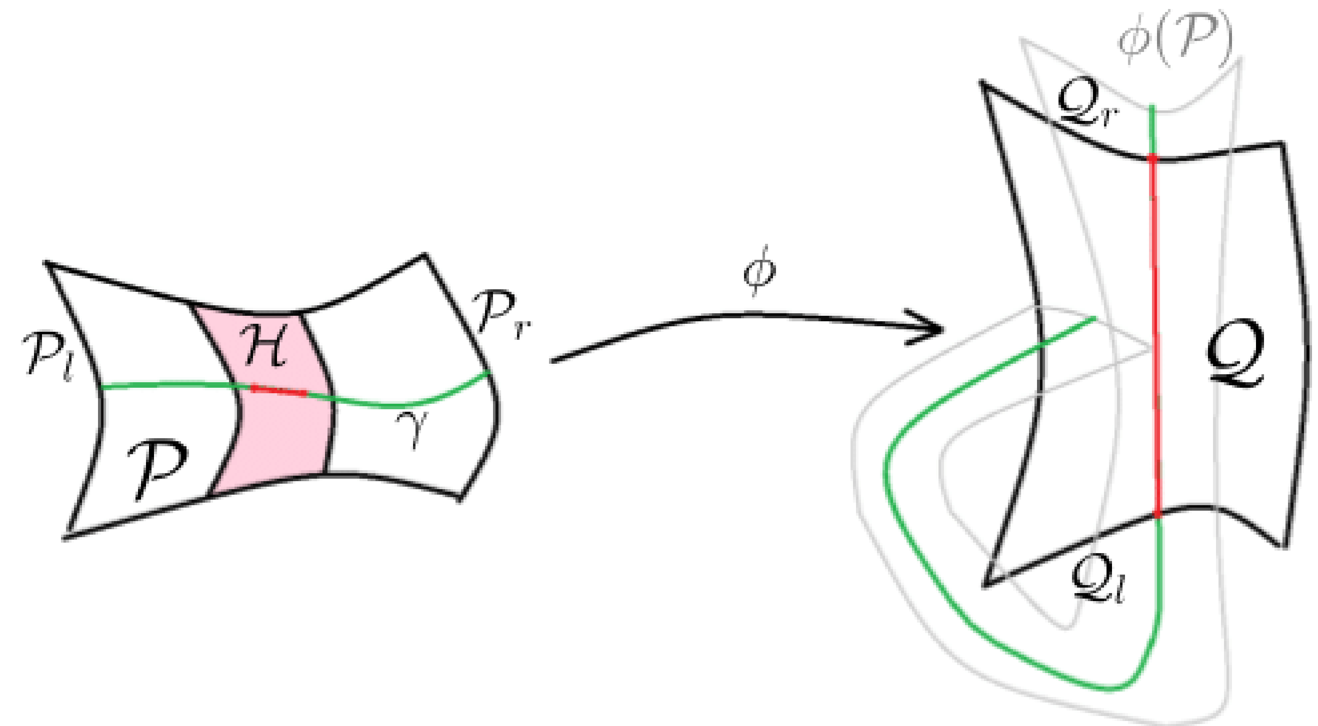
for $i = 1, 2$.
Then also

$$\Phi = \Phi_2 \circ \Phi_1$$

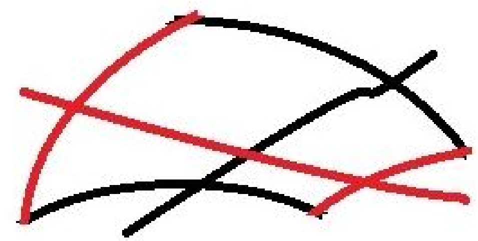
is a Poincaré map associated to the planar conservative system.

Objective: Φ induces chaotic dynamics on m-symbols.

SAP methods:


$\mathcal{P}_l$
$\mathcal{H}$
$\mathcal{P}_r$
$\mathcal{P}$
γ
ϕ
$\phi(\mathcal{P})$
$\mathcal{Q}_r$
$\mathcal{Q}$
$\mathcal{Q}_l$

Convention used:



The paths in black means $\overset{\smile}{[\,]}_{\pm}$ otherwise $\overset{\frown}{[\,]}_{\pm}$.

Main result:

Lemma 0.0.1 ***Given any positive integer*** l_1, ***it holds that***

$$\Phi_1 : \breve{\mathfrak{a}}_+ \stackrel{\Leftrightarrow}{\longrightarrow}^{l_1} \widehat{\mathfrak{a}}_-,$$

provided that $T_1 > l_1 \tau_l^{(1)}$.

Main result:

Lemma 0.0.2 ***There exist times*** τ_1^* ***and*** τ_2^*, ***such that, for any positive integer*** l_1, l_2, ***it holds that:***

$$\Phi_1 : \overset{\smile}{\mathfrak{a}}_{\pm} \Longrightarrow^{l_1} \overset{\frown}{\mathfrak{a}}_{\pm}, \qquad \overset{\smile}{\mathfrak{b}}_{\pm} \Longrightarrow^{l_1} \overset{\frown}{\mathfrak{b}}_{\pm}, \ \overset{\frown}{\mathfrak{c}}_{\pm}, \qquad \overset{\smile}{\mathfrak{c}}_{\pm} \Longrightarrow^{l_1} \overset{\frown}{\mathfrak{b}}_{\pm}, \ \overset{\frown}{\mathfrak{c}}_{\pm},$$

provided that $T_1 > l_1 \tau_1^*$.

$$\Phi_2 : \overset{\frown}{\mathfrak{a}}_{\pm} \Longrightarrow^{l_2} \overset{\smile}{\mathfrak{a}}_{\pm}, \ \overset{\smile}{\mathfrak{b}}_{\pm}, \qquad \overset{\frown}{\mathfrak{b}}_{\pm} \Longrightarrow^{l_2} \overset{\smile}{\mathfrak{a}}_{\pm}, \ \overset{\smile}{\mathfrak{b}}_{\pm}, \qquad \overset{\frown}{\mathfrak{c}}_{\pm} \Longrightarrow^{l_2} \overset{\smile}{\mathfrak{c}}_{\pm},$$

provided that $T_2 > l_2 \tau_2^*$.

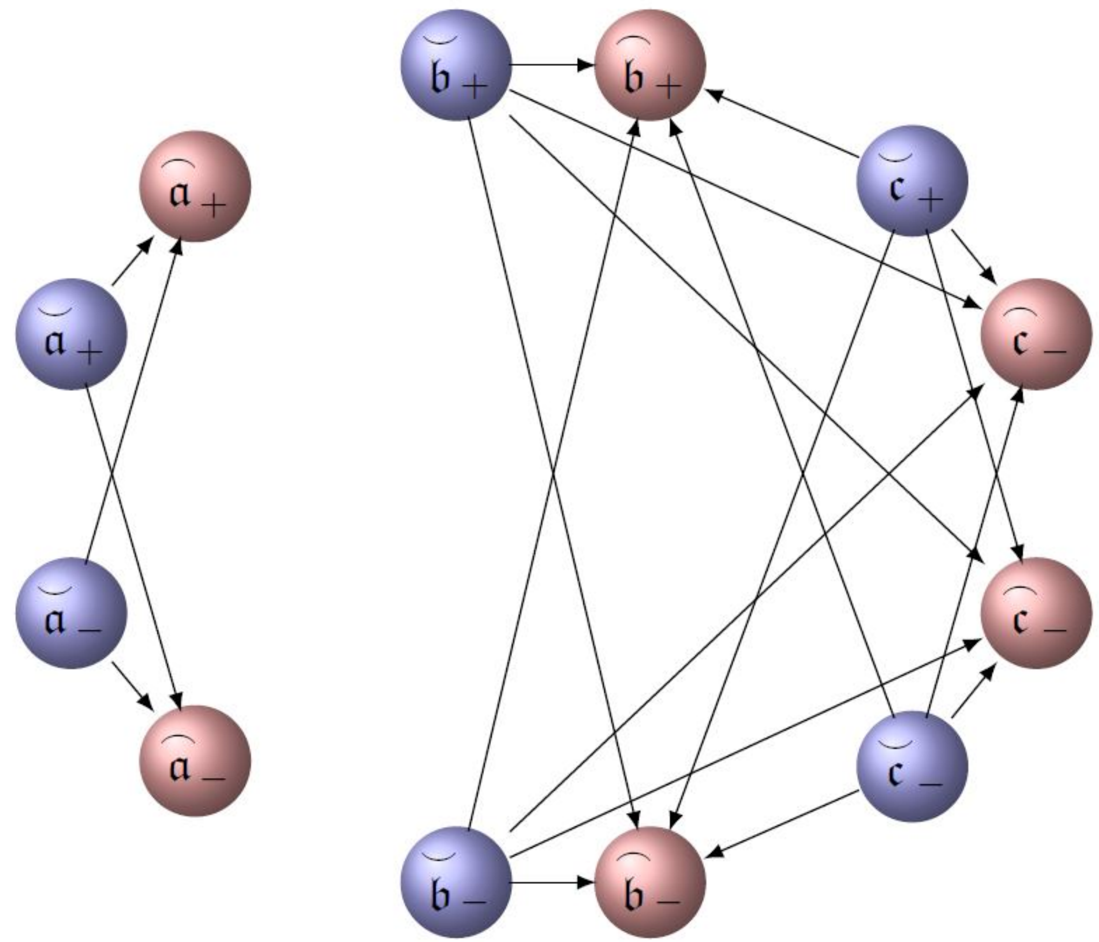


FIGURE 1: This graph represents all the possible connections by the partial Poincaré map Φ_1. The arrows correspond to the $\rightarrow$ symbol. The integer ℓ_1 is not indicated but it can be arbitrarily chosen provided that $T_1 > \ell_1 \tau_1^*$.

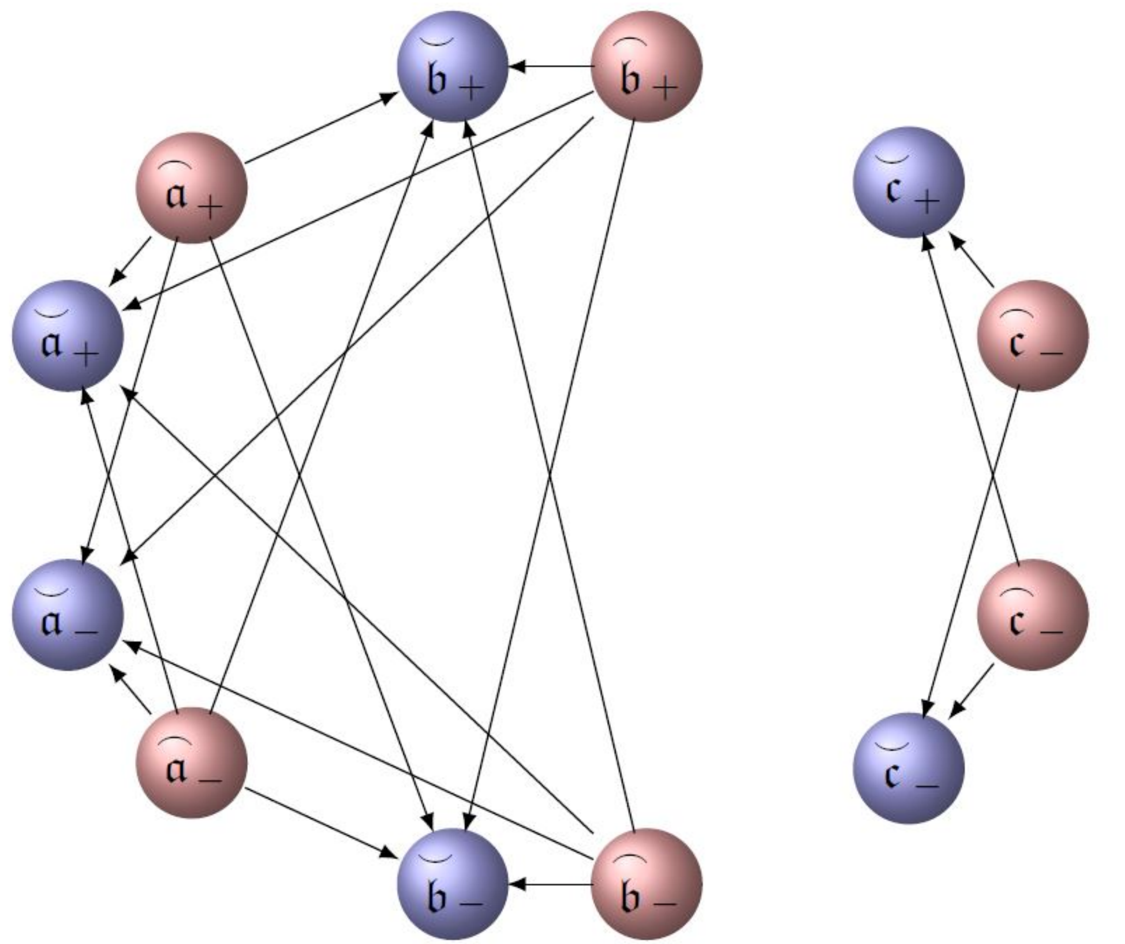


FIGURE 2: This graph represents all the possible connections by the partial Poincaré map Φ_2. The arrows correspond to the $\overset{\bumpeq}{\longrightarrow}$ symbol. The integer ℓ_2 is not indicated but it can be arbitrarily chosen provided that $T_2 > \ell_2 \tau_2^*$.

Applaying:

Lemma 0.0.3 ***Let*** $\widehat{\mathcal{R}} := (\mathcal{R}, \mathcal{R}^-)$ ***be an oriented rectangle and*** $\psi : \mathcal{R} \to \mathbb{R}^2$ ***be a continuous and one-to-one map. Suppose that***

$$\psi : \widehat{\mathcal{R}} \Longleftrightarrow\!\!\!\!\!\!\longrightarrow^m \widehat{\mathcal{R}},$$

for some $m \geq 2$. ***Then*** ψ ***induces chaotic dynamics on*** m ***symbols on the set*** $\mathcal{R}$.

Considering:

$\mathcal{R} \in \{\mathfrak{a}_\pm\, \mathfrak{b}_\pm, \mathfrak{a}_\pm\}$

$\Phi = \Phi_2 \circ \Phi_1$

$\widehat{\mathcal{R}} \in \{\overset{\smile}{\mathfrak{a}}_\pm, \overset{\smile}{\mathfrak{b}}_\pm, \overset{\smile}{\mathfrak{c}}_\pm\}$ **and** $m = \ell_1 \times \ell_2 \geq 2$

than the following connections are available:

$$\overset{\smile}{\mathfrak{a}}_+ \longrightarrow \overset{\frown}{\mathfrak{a}}_+ \longrightarrow \overset{\smile}{\mathfrak{a}}_+ , \quad \overset{\smile}{\mathfrak{a}}_+ \longrightarrow \overset{\frown}{\mathfrak{a}}_- \longrightarrow \overset{\smile}{\mathfrak{a}}_+$$

$$\overset{\smile}{\mathfrak{a}}_- \longrightarrow \overset{\frown}{\mathfrak{a}}_- \longrightarrow \overset{\smile}{\mathfrak{a}}_- , \quad \overset{\smile}{\mathfrak{a}}_- \longrightarrow \overset{\frown}{\mathfrak{a}}_+ \longrightarrow \overset{\smile}{\mathfrak{a}}_-$$

and therefore, we find that

$$\Phi : \overset{\smile}{\mathfrak{a}}_\pm \longrightarrow^2 \overset{\smile}{\mathfrak{a}}_\pm ,$$

In the last formula we use the convention that $[]_\pm \longrightarrow []_\pm$ **means that only the two possibilities** $[]_+ \longrightarrow []_+$ **and** $[]_- \longrightarrow []_-$ **are available.**

Thank you for your attention !