Multiple solutions for an elliptic problem involving the $p-$Laplacian

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Consider the following parameter-dependent Dirichlet boundary value problem

$$(\mathrm{D}_\lambda): \begin{cases} -\Delta_p u = \lambda f(u), & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where,

- $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth enough boundary $\partial\Omega$,
- $f : \mathbb{R} \to \mathbb{R}$ continuous, $\lambda > 0$, $p > N$,

$-\Delta_p u$ is the p-Laplacian operator

$(-\Delta_p u, v) := \int_\Omega |\nabla u(x)|^{p-2} \nabla u(x) \nabla v(x) dx \;\forall\; v$ in $W_0^{1,p}(\Omega)$.

A basic model

Weak solution:

$$u \in W_0^{1,p}(\Omega),\ \int_\Omega |\nabla u(x)|^{p-2}\nabla u(x)\nabla v(x)\ dx = \lambda \int_\Omega f(u(x))v(x)dx,$$

$$\forall\ v\ \in\ W_0^{1,p}(\Omega).$$

Subsolution (Supersolution):

$$u \in W^{1,p}(\Omega),\ u_{\partial\Omega} \le 0,\ \int_\Omega |\nabla u(x)|^{p-2}\nabla u(x)\nabla v(x)dx \le \lambda \int_\Omega f(u(x))v(x)dx \quad (\ge)$$

$$\forall v \in\ W_0^{1,p}(\Omega) \cap L_+^p(\Omega).$$

The goal

The aim of this talk is to give an overview on a novel variational approach to investigate multiple solutions for problem (D_λ), jointly developed with S. Carl and R. Livrea.

We describe open intervals of parameters $\Lambda_k \subset \mathbb{R}_+$, $k = 1, 2$, such that

for any $\lambda \in \Lambda_1$, problem (D_λ) has at least two constant-sign solutions,

while for $\lambda \in \Lambda_2 \subset \Lambda_1$, there exist a third sign-changing solution.

Moreover, for such solutions, we give a priori estimate on the sup-norm $\|\cdot\|$, uniformly with respect to the parameters.

The goal is achieved combining classical variational methods with sub-super solutions arguments.

The novelty introduced in our approach to study problem (D_λ) can be summarize in two steps:

- We apply an abstract result due to Bonanno + C. (2010) to obtain the existence of constant-sign solutions to problem (D_λ);
- We arrange the technique developed by Carl and Motreanu to find a third sign-changing solution.

References

G. Bonanno + P. C., *Nonlinear difference equations through varitional methods.* In: D.Y. GAO, D. MOTREANU. Handbook of Nonconvex Analysis and Applications. Iternational Press of Boston, 2010.

G. Bonanno + P. Candito, *Non-differentiable functionals and applications to elliptic problems with discontinuous nonlinearities*, J. Differential Equations,(2008)

G. Bonanno and S. A. Marano, *On the structure of the critical set of non-differentiable functions with a weak compactness condition*, Appl. Anal., (2010)

G. Bonanno + G. Molica Bisci, *Infinitely many solutions for a boundary value problem with discontinuous nonlinearities*, Bound, Value Probl., (2009).

To prove the existence of a sign-changing solution we adopted the technique developed in

S. Carl and D. Motreanu, *Constant-sign and sign-changing solutions for nonlinear eigenvalue problems*, Nonlinear Anal. (2008).

where the Authors deals with this type of problem

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u - g(x,u), & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where, $1 < p < +\infty$, $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function such that

(i) $\lim_{|s|\to+\infty} \frac{g(x,s)}{|s|^{p-2}s} = +\infty$, (ii) $\lim_{|s|\to 0} \frac{g(x,s)}{|s|^{p-2}s} = 0$, uniformly in Ω.

(iii) g is bounded on bounded sets.

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- S. Liu, *Multiple solutions for coercive p-Laplacian equations*, J. Math. Anal. Appl. (2006) (D_1, $1 < p < +\infty$, F is $p-$sublinear at infinity)