Optimization Days

an international workshop on Calculus of Variations

Ancona, Università Politecnica delle Marche, June 6-8, 2011

Abstracts

- **Gabriele Bonanno** (Università di Messina): *Some remarks on the variational methods*

  Some remarks on the classical Ambrosetti-Rabinowitz theorem are presented. In particular, it is observed that the geometry of the mountain pass, if the function is bounded from below, is equivalent to the existence of at least two local minima. On the other hand, local minimum theorems for a suitable class of functions are established. Hence, by starting from the above remarks, multiple critical points results are presented and some applications to nonlinear differential problems are obtained.

- **Pierre Bousquet** (Université de Provence Aix-Marseille 1): *Continuity of solutions for a problem in the Calculus of Variations*

  We consider the following problem in the Calculus of Variations :

  $$\text{To minimize } u \mapsto \int_\Omega F(\nabla u(x)) + G(x, u(x))\, dx \quad , \; u \in W^{1,1}(\Omega),$$

  under a Dirichlet boundary condition : $u_{|\partial\Omega} = \phi$. Here, Ω is a bounded open set in $\mathbb{R}^n$, $F : \mathbb{R}^n \to \mathbb{R}$ is convex, $G : \Omega \times \mathbb{R} \to \mathbb{R}$ is smooth and $\phi : \partial\Omega \to \mathbb{R}$ is continuous. We do not assume any growth assumption from above on F.

  We address the question of the continuity of a solution u when Ω satisfies further geometric assumptions.

- **Alessio Brancolini** (Politecnico di Bari): *Regularity results on optimal patterns for the branched transportation problem*

  In many natural and artificial systems branched structures arise. In order to model this kind of structures, several new functionals were considered. In all of them, the transportation cost between the source and the destination (both measures) is given by lenght times m^a, with $0 < a < 1$. The subadditivity of the a-power function forces the mass to move together as much as possible and the branched structure arises.

In the framework of Maddalena-Morel-Solimini's model, we consider two regularity problems. The first is the Holder regularity of the landscape function (a natural mathematical object which gives the transportation cost of a sigle mass particle on the network), which we prove under general assumptions on the regularity of the destination measure. The second is the "fractal regularity" of the network. Given a portion of a branch of the optimal network, we give estimates on the number of branches that bifurcate from it. (Coauthor: Sergio Solimini)

- **Giuseppe Buttazzo** (Università di Pisa): *Spectral optimization problems*

  In the lecture I shall present some classes of shape optimization problems of the form

  $$\min\big\{F(\Omega)\ :\ \Omega\in\mathcal{A}\big\}$$

  where the unknown Ω varies in a suitable family $\mathcal{A}$ of domains and the cost function $F(\Omega)$ depends on Ω through the solution u_Ω of an elliptic PDE in Ω. Two main cases fall in this scheme: the case of integral functionals

  $$F(\Omega)=\int_{\mathbb{R}^d} j\big(x,u_\Omega(x)\big)\,dx$$

  and the case of functions of the spectrum of an elliptic operator

  $$F(\Omega)=\Phi\big(\lambda(\Omega)\big).$$

  One of the main issues is concerned with the existence of optimal domains; while in general this cannot be expected, as some simple counterexamples show, in several interesting cases optimal domains actually exist.

  In the lecture several open problems and new direction of research will be addressed.

- **Pasquale Candito** (Università Mediterranea di Reggio Calabria): *Multiple solutions for an elliptic problem involving the $p-$Laplacian*

  The aim of this talk is to give an overview on a novel variational approach, jointly developed with S. Carl and R. Livrea, to investigate multiple solutions for a quasilinear elliptic problem. The goal is achieved combining classical variational methods with sub-supersolution arguments.

- **Arrigo Cellina** (Università di Milano-Bicocca): *On the non-occurrence of the Lavrentiev phenomenon*

  We show that the Lavrentiev phenomenon does not occurr for a class of variational functionals on a multi-dimensional domain. Joint work with G. Bonfanti.

- **Bernard Dacorogna** (École Polytechnique Fédérale de Lausanne): *On the equation* $\det \nabla u = f$

  We discuss existence of a solution u of the Dirichlet problem

  $$\begin{cases} \det \nabla u(x) = f(x) & x \in \Omega \\ u(x) = x & x \in \partial\Omega \end{cases}$$

  where Ω is a bounded smooth domain and f satisfies

  $$\int_\Omega f = \operatorname{meas} \Omega$$

  with no sign hypothesis on f.

  This is a joint work with G. Cupini and O. Kneuss.

- **Mikil Foss** (University of Nebraska-Lincoln): *Partial Continuity for Vectorial Problems*

  In this talk, I will describe some recent work on the development of a low-order regularity theory for elliptic and parabolic problems in the vectorial setting. To be more precise, consider the elliptic system

  $$\operatorname{div}[a(x, u, Du)] = 0 \quad \text{in } \Omega,$$

  where $\Omega \subset \mathbb{R}^n$ is a bounded domain. The vector field $a : \Omega \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to \mathbb{R}^N$ satisfies natural p-growth and ellipticity assumptions with $p \geq 2$. I will present a result establishing that there is an open set of full measure in Ω on which the solution is Hölder continuous. The key assumption for the problem being considered is that the vector field a is only assumed to be continuous with respect to the arguments x and u. This distinguishes the result from others which provide regularity for the gradient of the solution while requiring at least Hölder continuity of the field a with respect to x and u. The analogous results for variational problems and parabolic systems will also be discussed. The work to be presented was done in collaboration with V. Bögelein (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany)and G. Mingione (University of Parma, Italy).

- **Francesca Gladiali** (Università di Sassari): *Global bifurcation results for semilinear elliptic equations*

  We consider radial positive solutions of semilinear elliptic problems with power nonlinearities either in an annulus or in the exterior of a ball. We study the

spectrum of the linearized operator and we characterize the degeneracy points in dependence either of the annulus or the exponent of the nonlinear term. This allow us prove that there are infinitely many points of nonradial bifurcation along the curve of radial positive solutions. This bifurcation is indeed global and obeys at the so called Rabinowitz alternative. These results apply also to the supercritical case. (Coauthors: M. Grossi, F. Pacella, P.N. Srikanth).

- **Bernd Kawohl** (Universität Köln): *Variations on the p-Laplacian*

  In this talk I address several issues involving Dirichlet problems for the classical p–Laplacian operator $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ for $p \in (1,\infty)$. First I look at p harmonic functions as $p \to \infty$ and $p \to 1$. Then I compare the p–Laplacian with its normalized version $\Delta_p^N u := \frac{1}{p}|\nabla u|^{2-p}\Delta_p u$ and study equations like $-\Delta_p u = 1$ or $-\Delta_p^N u = 1$. Finally I present results and open problems on the eigenvalue problem $-\Delta_p u = \lambda |u|^{p-2}u$.

- **Francesco Leonetti** (Università di l'Aquila): *Integrability for solutions of some quasilinear elliptic systems*

  Elliptic systems may have singular solutions as De Giorgi's counterexample shows. We consider quasilinear elliptic systems whose off-diagonal coefficients are small when the solution is large and we show higher integrability.

- **Giovanni Leoni** (Carnegie mellon University): *New characterizations of Sobolev functions and of functions of bounded variation*

  Using nonlocal functionals we present some new characterizations of Sobolev functions and of functions of bounded variation in arbitrary open sets. This is joint work with D. Spector.

- **Salvatore Angelo Marano** (Università di Catania) *On a Dirichlet problem with p-Laplacian and set-valued nonlinearity*

  The existence of solutions to a homogeneous Dirichlet problem for a p-Laplacian differential inclusion is studied via a fixed-point type theorem concerning operator inclusions in Banach spaces. Some meaningful special cases are then worked out.

- **Elvira Mascolo** (Università di Firenze): *Regularity of solutions to quasilinear elliptic systems*

  The study of the regularity of weak solutions to nonlinear elliptic systems has been strongly motivated, and at the same time conditioned, by the De Giorgi's example of existence of unbounded weak solutions.

In a joint paper with G. Cupini and P. Marcellini we prove the local boundedness of weak solutions to the following class of quasilinear elliptic systems, including the case of general elliptic equations:

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(\sum_{j=1}^{n} a_{ij}\left(x,u,Du\right)\, u_{x_j}^{\alpha} + b_i^{\alpha}\left(x,u,Du\right) \right) = f^{\alpha}\left(x,u,Du\right)$$

$\forall\, \alpha = 1,2,\ldots,m$. The regularity is proved assuming the anisotropic and p,q-growth of the leading part a_{ij}, the pertubation term b_i^{α} and the data f^{α} under sharp conditions on the exponents.

- **Giuseppe Mingione** (Università di Parma): *Nonlinear aspects of Calderón-Zygmund theory*

  CZ theory is classical in the linear case, where via the analysis of singular integrals allows to infer in an optimal way regularity properties of solutions to linear elliptic and parabolic equations from that of the data. I will try to outline a number of results that, put together, seem to form a nonlinear CZ theory.

- **Giovanni Molica Bisci** (Università Mediterranea di Reggio Calabria): *On some variational problems in Riemannian and Fractal Geometry*

  We present some resent results, obtained in collaboration with G. Bonanno and V. Rădulescu on some variational problems arising from Geometry. This study is motivated by the Emden-Fowler equation that appears in mathematical physics. In the second part, under an appropriate oscillating behavior either at zero or at infinity of the nonlinear term, the existence of a sequence of weak solutions for an eigenvalue Dirichlet problem, on a fractal domain, is proved. Our approach is based on variational methods. The abstract results are contained in [G. Bonanno, G. Molica Bisci and V. Rădulescu, Multiple solutions of generalized Yamabe equations on Riemannian manifolds and applications to Emden–Fowler problems, Nonlinear Anal. Real Word and Applications, 2011, 1–10, DOI 10.1016/jonrwa.2011.03.012] and [G. Bonanno, G. Molica Bisci and V. Rădulescu, Variational analysis for a nonlinear elliptic problem on the Sierpiński gasket, submitted for publication].

- **Piero Montecchiari** (Università Politecnica delle Marche) *Energy prescribed solutions for some class of semilinear elliptic equations.*

  We use an Energy constrained variational method, based on Hamiltonian type identities, to find prescribed Energy solutions of semilinear elliptic equations (or systems) of the form $\Delta u + W_u(u) = 0$ on R^n. We describe in particular applications to Allen Cahn and Non Linear Schroedinger equations.

- **Giampiero Palatucci** (Universitè de Nimes): *Sobolev embeddings and Concentration-compactness alternative for fractional Sobolev spaces*

  We extend the usual Concentration-compactness alternative to the fractional Sobolev spaces H^s for any $0 < s < N/2$. We study optimizing sequences for corresponding Sobolev embedding, showing that they are compact up to translations and dilations. In case of bounded domains, we also investigate a natural approximation by subcritical Sobolev quotients, using variational techniques. We show that for such approximations, optimal functions always exist and exhibit a concentration effect of the H^s energy at one point with an explicit concentration profile. (Coauthor: Adriano Pisante)

- **Vincenzo Vespri** (Università di Firenze): *Long term behaviour for solutions of quasilinear parabolic equations*

  In collaboration with S. Piro Vernier and F. Ragnedda, we use DiBenedetto's techniques to study the long term behaviour of solution to degenerate and singular parabolic equations. Thanks to this powerful tool, we avoid to use comparison functions or to minimize suitable functionals. These techniques can be applied also to generalize an estimate due to Bonforte-Vazquez (joint work with S. Fornaro).