Partial Continuity for Vectorial Problems

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Abstract

In this talk, I will describe some recent work on the development of a low-order regularity theory for elliptic and parabolic problems in the vectorial setting. To be more precise, consider the elliptic system

$$\operatorname{div}[a(x, u, Du)] = 0 \quad \text{in } \Omega,$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain. The vector field $a : \Omega \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to \mathbb{R}^N$ satisfies natural p-growth and ellipticity assumptions with $p \geq 2$. I will present a result establishing that there is an open set of full measure in Ω on which the solution is Hölder continuous. The key assumption for the problem being considered is that the vector field a is only assumed to be continuous with respect to the arguments x and u. This distinguishes the result from others which provide regularity for the gradient of the solution while requiring at least Hölder continuity of the field a with respect to x and u. The analogous results for variational problems and parabolic systems will also be discussed. The work to be presented was done in collaboration with V. Bögelein (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany)and G. Mingione (University of Parma, Italy).