

Spectral Optimization Problems

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Abstract

In the lecture I shall present some classes of shape optimization problems of the form

$$\min \{F(\Omega) : \Omega \in \mathcal{A}\}$$

where the unknown Ω varies in a suitable family \mathcal{A} of domains and the cost function $F(\Omega)$ depends on Ω through the solution u_Ω of an elliptic PDE in Ω . Two main cases fall in this scheme: the case of integral functionals

$$F(\Omega) = \int_{\mathbb{R}^d} j(x, u_\Omega(x)) \, dx$$

and the case of functions of the spectrum of an elliptic operator

$$F(\Omega) = \Phi(\lambda(\Omega)).$$

One of the main issues is concerned with the existence of optimal domains; while in general this cannot be expected, as some simple counterexamples show, in several interesting cases optimal domains actually exist.

In the lecture several open problems and new direction of research will be addressed,

References

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