Continuity of solutions for a problem in the Calculus of Variations

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We consider the following problem in the Calculus of Variations :

$$\text{To minimize } u \mapsto \int_\Omega F(\nabla u(x)) + G(x, u(x))\, dx \quad , \ u \in W^{1,1}(\Omega),$$

under a Dirichlet boundary condition : $u_{|\partial\Omega} = \phi$. Here, Ω is a bounded open set in $\mathbb{R}^n$, $F : \mathbb{R}^n \to \mathbb{R}$ is convex, $G : \Omega \times \mathbb{R} \to \mathbb{R}$ is smooth and $\phi : \partial\Omega \to \mathbb{R}$ is continuous. We do not assume any growth assumption from above on F.

We address the question of the continuity of a solution u when Ω satisfies further geometric assumptions.