

# Title

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**SUMMARY.** With the development of nanoscale semiconductor devices (nanostructures), quantum transport has become a major tool of solid state physics and a very active area of research among physicists and mathematicians.

## 1 INTRODUCTION

Among the possible approaches that can be used for the kinetic description of quantum statistical systems [1], the Wigner-function approach is the one which adopts ideas and concepts that are most similar to the ones of classical transport theory [2]. In particular, a quantum phase space can be introduced, which possesses many of the properties of the classical phase space. Wigner functions can be considered the quantum analogues of classical distribution functions and enjoy many of their properties. The moments of the Wigner function give the macroscopic quantities (observables), such as densities, currents, fluxes and energies; the Wigner function, however, is not positive definite and thus cannot be interpreted as a probability distribution in phase space. Another important aspect of this analogy is that the transport equations that govern the behaviour of the Wigner function tend to the equations of classical kinetic theory in the semiclassical limit. In the absence of collisions, for example, the behaviour of the Wigner function is governed by the Wigner equation (or the Liouville-von Neumann equation) which, in the classical limit, becomes the Vlasov equation of plasma physics. Also, in those cases in which self-consistent fields are important, the Wigner equation is coupled with the Poisson equation, yielding the Wigner-Poisson [3] system which, in the classical limit, becomes the Vlasov-Poisson system.

## 2 THE MODEL

From the arguments above, it appears evident that an investigation of the solutions of the quantum transport equations that govern the behaviour of the Wigner function in the semiclassical limit is important and very desirable for the understanding of the structure of the full quantum solutions. The analysis can be carried out by perturbation techniques, once the proper asymptotic parameter is chosen. Perturbation schemes certainly carry their own limitations, but they also have the advantage of making the analysis simpler, so that it can be carried out farther by analytical means, before the numerical solution is needed. In particular, a perturbation method allows to study, in a relatively simple manner, the structure of the quantum phase space and to compare it with the structure of the corresponding classical phase space.

### 2.1 *The Vlasov-Poisson system*

In this work, we apply the concepts described above to the study of the quantum corrections to the Bernstein-Greene-Kruskal equilibria [4] (BGK modes) that arise in the nonlinear theory of

collisionless plasma waves. These phenomena are usually described by the Vlasov-Poisson system

$$\frac{\partial f}{\partial t} + p \frac{\partial f}{\partial x} - E \frac{\partial f}{\partial p} = 0, \quad (1)$$

$$\frac{\partial E}{\partial x} = 1 - \int f dp, \quad (2)$$

which governs the time evolution of the electron distribution function  $f(x, p, t)$  and of the self-consistent electric field  $E(x, t)$  generated by the space-charge separation due to the motion of the electrons with respect to the fixed background of the ions.

## 2.2 Boundary conditions

When the Vlasov-Poisson system (1)-(2) is used to describe plasma waves, periodic conditions in space are imposed,  $f(x, p, t) = f(x + L, p, t)$  and  $E(x, t) = E(x + L, t)$ , where  $L$  is the periodicity length.

## 3 NUMERICAL RESULTS

The steady-state solutions of the Vlasov-Poisson system (1)-(2) are called BGK equilibria (see Figure 1). They can be lifted to time-dependent solutions of the Vlasov-Poisson system by Galilean transformations with arbitrary phase speed.

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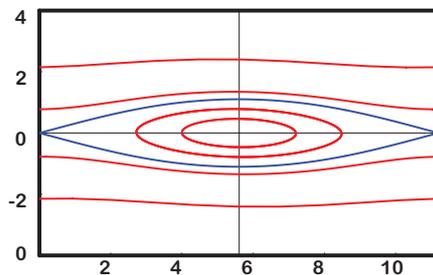


Figure 1: Phase-space portrait of a typical BGK equilibrium.

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## 4 CONCLUSIONS

We have investigated the steady-state solutions of the Wigner-Poisson system. We have computed the lowest order quantum corrections to the classical BGK modes.

**Remark 1.** References should be listed in order of citation. The examples below show the format for [1] Conference Proc., [2] books and [3, 4] journal papers.

**Remark 2.** Use Times New Roman, 10 pt (apart from the title, where 14 pt should be used).

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*References*

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