Toward a novel approach for damage identification and health monitoring of bridge structures

Paolo Martino Calvi[1], Paolo Venini[1]
[1]*Department of Structural Mechanics, University of Pavia, Italy*
E-mail: paolomartino.calvi@unipv.it, paolo.venini@unipv.it

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SUMMARY. Moving from a couple–stress beam model coupled to a three-parameter damage estimation scheme, a frequency-domain damage identification approach is proposed that is tested on a numerical basis. Early stages of application to a real cable-stayed footbridge are eventually outlined in view of a more extensive investigation.

1 INTRODUCTION

Aim of this paper is the development of a damage identification scheme with sound mechanical basis but applicable to real bridge structures. To this goal three tools are exploited and linked together:

- a continuous weak formulation for the microstructure-dependent beam model presented in [3] along with a suitable finite element discretization;
- a damage model proposed in [1] along with a damage identification approach operating in the frequency domain. Such an approach is herein extended to handle the microstructured beam model developed in [3],
- an experimental evaluation of a cable-stayed footbridge in Bormio (Italy), [2].

The paper is therefore organized as follows. In section 2 the mechanical features of the beam model used in this paper are introduced after [3]. A few considerations on the discretization approach are then outlined focussing attention on the thin Euler-Bernoulli case. Afterwards, the three-parameters damage model introduced in [1] is briefly recalled and extended to handle the microstructured beam due to [3]. A system identification approach of least-square type is eventually developed. Not only the actual state of damage is assessed, if any, but also the internal length–scale constitutive parameter is computed. Finally, a real footbridge is considered [2] and relevant experimental data presented along with preliminary numerical results.

2 THE PHYSICAL MODEL AND ITS FEM APPROXIMATION

2.1 *Model mechanics and Timoshenko beam*

Referring to [3] for an exhaustive treatment, it suffices here to say that the couple–stress beam model is characterized by a generalized strain energy that reads:

$$U = \frac{1}{2}\int_{\Omega}(\underline{\underline{\sigma}} : \underline{\underline{\varepsilon}} + \underline{\underline{m}} : \underline{\underline{\chi}})dV \tag{1}$$

where $\underline{\underline{\varepsilon}}$ and $\underline{\underline{\sigma}}$ are the classical Green and Cauchy strain and stress tensors, respectively, $\underline{\underline{\chi}}$ is the symmetric curvature tensor and $\underline{\underline{m}}$ its work–conjugate tensor, i.e. the deviatoric part of the couple–

stress. In the framework of isotropic elasticity, the relevant constitutive law may be written as

$$\left\{ \begin{array}{lcl} \underline{\underline{\sigma}} & = & \lambda \mathrm{tr}(\underline{\underline{\varepsilon}}) + 2\mu\underline{\underline{\varepsilon}} \\ \underline{\underline{m}} & = & 2\ell^2\mu\underline{\underline{\chi}} \end{array} \right. \tag{2}$$

where λ and μ are the Lamé constants and ℓ is a material length scale parameter that measures the effect of couple stress. By averaging over the cross section A, stress resultants may be defined as

$$N = \int_A \sigma_{xx} dA, \;\; M_x = \int_A \sigma_{xx} z dA, \;\; Y_{xy} = \int_A m_{xy} dA, \;\; Q = \int_A \sigma_{xz} dA, \tag{3}$$

so as to arrive at the following Timoshenko–beam constitutive relationships:

$$\left\{ \begin{array}{lcll} N & = & \frac{E(1-\nu)A}{(1+\nu)(1-2\nu)}\frac{\partial u}{\partial x} & \qquad M_x = \frac{E(1-\nu)I}{(1+\nu)(1-2\nu)}\frac{\partial \phi}{\partial x} \\ Y_{xy} & = & \frac{1}{2}\ell^2\mu A\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) & \qquad Q = K_s\mu A\left(-\phi + \frac{\partial w}{\partial x}\right) \end{array} \right. \tag{4}$$

where u and w are the axial and transverse displacements, respectively, and ϕ is the rotation. Noticeably, the material lengthscale ℓ appearing in (4) may not be measured experimentally and an identification procedure for its evaluation shall be presented in the sequel of the paper. After some algebraic manipulations the equations of motion of the beam may be written as

$$\left\{ \begin{array}{l} \dfrac{E(1-\nu)A}{(1+\nu)(1-2\nu)}\dfrac{\partial^2 u}{\partial x^2} + f = m_0\dfrac{\partial^2 u}{\partial t^2} \\[3ex] K_s\mu A\left(-\dfrac{\partial \phi}{\partial x} + \dfrac{\partial^2 w}{\partial x^2}\right) - \dfrac{1}{4}\ell^2\mu A\left(\dfrac{\partial^3 \phi}{\partial w^3} + \dfrac{\partial^4 w}{\partial x^4}\right) + \dfrac{1}{2}\dfrac{\partial c}{\partial x} + q = m_0\dfrac{\partial^2 w}{\partial t^2} \\[3ex] \dfrac{E(1-\nu)I}{(1+\nu)(1-2\nu)}\dfrac{\partial^2 \phi}{\partial x^2} + K_s\mu A\left(-\phi + \dfrac{\partial w}{\partial x}\right) + \dfrac{1}{4}\ell^2\mu A\left(\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^3 w}{\partial x^3}\right) - \dfrac{1}{2}c = m_2\dfrac{\partial^2 \phi}{\partial t^2} \end{array} \right. \tag{5}$$

where m_0 and m_2 are the mass and moment of inertia, respectively, f, q are the axial and transverse body loads per unit length, and c is the body couple per unit length.

2.2 *The Euler-Bernoulli approximation*

For simplicity sake, a simpler Euler–Bernoulli idealization shall be adopted in this paper that may written as

$$\left\{ \begin{array}{l} \dfrac{E(1-\nu)A}{(1+\nu)(1-2\nu)}\dfrac{\partial^2 u}{\partial x^2} + f = m_0\dfrac{\partial^2 u}{\partial t^2} \\[3ex] \left[\dfrac{E(1-\nu)I}{(1+\nu)(1-2\nu)} + \ell^2\mu A\right]\dfrac{\partial^4 w}{\partial x^4} - \dfrac{\partial c}{\partial x} - q = m_2\dfrac{\partial^4 w}{\partial x^2 \partial t^2} - m_0\dfrac{\partial^2 w}{\partial t^2} \end{array} \right. \tag{6}$$

Focussing on transverse vibrations, one may notice that the main novel features of the adopted model are the presence of the internal lengthscale ℓ as well as a Poisson effect that affects the bending rigidity that is no longer EI but

$$\left[\frac{E(1-\nu)I}{(1+\nu)(1-2\nu)} + \ell^2\mu A\right].$$

3 CERRI AND VESTRONI'S DAMAGE MODEL

3.1 *The three–parameters damage model*

Cerri and Vestroni [1] developed a diffused cracking model for thin beams that establishes an analytical relationship between the the first three eigenfrequencies of the system and the three parameters of the model that may be listed as:

1. x = normalized location of the damaged area centroid;
2. b = normalized width of the damaged area;
3. $\beta = \frac{EI^U - EI^D}{EI^U}$ = non dimensional bending stiffness drop.

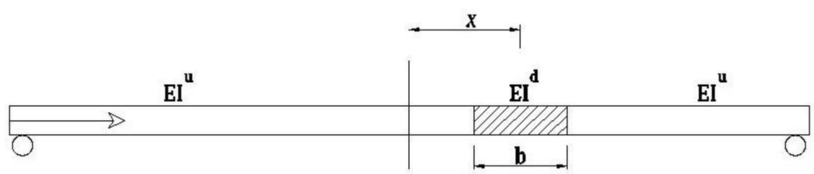

Figure 1: Cerri and Vestroni's model

With reference to Figure 1, the elastica for the damaged beam is obtained by first treating separately the three beam segments that are governed by the Euler-Bernoulli equation

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = 0, \tag{7}$$

that are subsequently linked together via classical compatibility and static continuity requirements. The result is an analytical position of the characteristic equation of the structure that allows for a fast solution of the nonlinear (eventually over–determined) system of equations of type

$$g_i(\lambda_i, x, b, \beta) = 0, \quad i = 1, \ldots, N. \tag{8}$$

3.2 *The identification approach*

The main idea of this paper is to apply the same damage model proposed in [1] to the microstructured beam proposed in [3]. To do so, analytical computations are dropped and the nonlinear system (8) is set up and solved numerically. As to space discretization, a standard finite element approach is used whereas the nonlinear system (8) is solved via a least-square optimization strategy. By denoting with ω_i^e and $\omega_i(x, b, \beta, \ell)$ the actual i–th eigenfrequency for a given damage state and the relevant estimation corresponding to the current guess (x, b, β, ℓ), respectively, the following min-square optimization problem is written:

$$\begin{cases} \min\limits_{x,b,\beta,\ell} F(x,b,\beta,\ell) & = & \dfrac{1}{2}\displaystyle\sum_{i=1}^{N\geq 4}\left(\dfrac{\omega_i(x,b,\beta,\ell)-\omega_i^e}{\omega_i^e}\right)^2 \\ \\ s.t. & & [\underline{\underline{K}}(x,b,\beta,\ell)-\omega_i^2(x,b,\beta,\ell)\underline{\underline{M}}]\underline{\Phi}_i = 0,\ \ i=1,\ldots,N \end{cases} \tag{9}$$

where one may write

$$\begin{aligned} K_{ij} &= \int_0^L \left(\frac{E(1-\nu)I}{(1+\nu)(1-2\nu)} + \ell^2\mu A\right)\frac{\partial^2\psi_i}{\partial x^2}\frac{\partial^2\psi_j}{\partial x^2}dx \\ M_{ij} &= \int_0^L \left(m_0\psi_i\psi_j + m_2\frac{\partial\psi_i}{\partial x}\frac{\partial\psi_j}{\partial x}\right)dx \end{aligned} \tag{10}$$

in which ψ_i's are the classical Hermite polynomial basis functions. For this paper's sake, the numerical solution of the optimization problem has been pursued by using the Optimization Toolbox available under the software Matlab, [4].

3.3 *Numerical example*

Before moving to a real-life footbridge, to get the flavor of the proposed approach results from a numerical simulation are presented. Geometrical and physical parameters of the beam under investigation read as follows: Young modulus $E = 2.1 \times 10^5$, Poisson ratio $\nu = 0.3$, beam span $L = 1000$, mass density $\rho = 0.78 \times 10^{-8}$, rectangular cross section dimensions $b = 3$, $h = 5$. Cerri and Vestroni's damage model to be identified were set as $x = 150$, $b = 100$, $\beta = 0.5$. Furthermore, the characteristic length $\ell = h/50 = 0.1$ is a parameter to be identified as well. The capability of the proposed approach to solve the problem is checked with respect to the distance between the initial guess and the actual solution, within a procedure that aims to determine the convergence region in the design parameter space. Another quantity of interest is the number $N \geq 4$ of eigenmodes to be used in order to ensure convergence of the approach. It is found that $N = 6$ represents a good compromise between performance and numerical cost. Each column of the following matrix $\underline{\underline{X}}_0$ represents an initial guess vector that leads to the correct solution of the problem:

$$\underline{\underline{X}}_0 = \begin{bmatrix} x_0 \\ b_0 \\ \beta_0 \\ \ell_0 \end{bmatrix} = \begin{bmatrix} 0 & 75 & 75 & 225 & 225 \\ 0 & 200 & 50 & 50 & 50 \\ 0.025 & 0.25 & 0.1250 & 0.1250 & 0.005 \\ 0.30 & 0.015 & 0.075 & 0.075 & 0.075 \end{bmatrix} \rightarrow \underline{x} = \begin{Bmatrix} 150 \\ 100 \\ 0.5 \\ 0.1 \end{Bmatrix} \tag{11}$$

Generally speaking, the method appears to be robust in that convergence is guaranteed even when the initial guess is far from the solution. This is then true when the initial estimates of the parameters approximate the solution from below as well as form above. However one should notice that the robustness of the approach depends on the number of available eigenmodes, and $N = 6$ may be hard to be evaluated experimentally in a reliable fashion. Therefore, the very same simulations were re-run by using the minimal number $N = 4$ of eigenfrequencies. By so doing, only the following initial vectors allows to compute the correct solution vector, i.e:

$$\underline{\underline{X}}_0 = \begin{bmatrix} x_0 \\ b_0 \\ \beta_0 \\ \ell_0 \end{bmatrix} = \begin{bmatrix} 0 & 75 & 75 \\ 0 & 200 & 50 \\ 0.025 & 0.25 & 0.1250 \\ 0.30 & 0.015 & 0.075 \end{bmatrix} \rightarrow \underline{x} = \begin{Bmatrix} 150 \\ 100 \\ 0.5 \\ 0.1 \end{Bmatrix} \tag{12}$$

It is therefore clear that more extensive simulations are needed to assess the robustness of the proposed approach and the relevant cost-effectiveness ratio to make available a few guidelines in view of the application to real-life structures as the one investigated next.

4 THE BORMIO FOOTBRIDGE

4.1 *The footbridge structure*

A single-span footbridge of about 66 m is investigated, Figure 2. The reinforced concrete deck is curved in plan and elevation, and has a thickness varying from 280 mm to 450 mm and is sustained by a single pylon, hinged at the base. Dynamic tests were performed in situ during the various construction phases that are herein confronted with the numerical simulations. Preliminary results on hypothetical damage states that may affect the bridge performance are also presented. The variations of the first eigenfrequencies of the system with respect to the damage parameters introduced in [1] are sketched as a first stage of an health monitoring methodology currently under development.

Figure 2: A view of the Bormio footbridge

4.2 *Frequency domain experimental data*

Reference is made to [2] for a more complete description of the experimental campaign and relevant measurements and data. For the main outcomes, one may look at Figure 3 that shows the locations of the accelerometers as well as the first modal shapes detected experimentally. As to the relevant eigenfrequencies, they were found to be:

$$\omega_1 = 1\,hz,\;\; \omega_2 = 1.75\,hz,\;\; \omega_3 = 2.75\,hz,\;\; \omega_4 = 3.75\,hz,\;\; \omega_5 = 4.5\,hz. \tag{13}$$

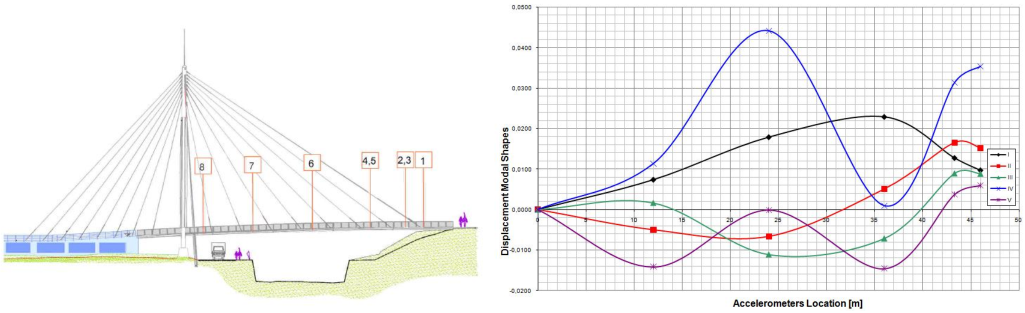

Figure 3: Accelerometers location and modal shapes

4.3 *Simulating numerical damage and relevant identification*

The numerical analysis of the bridge structure was performed in two subsequent stages. Firstly, an accurate model of the as–built was realized with the goal of having at disposal a numerical replica of the real undamaged structure. Figures 4, 5 and 6 show the first, second and fourth modal shapes computed by the numerical model. As to the numerical eigenfrequencies they were found to be

$$\begin{array}{ccccccc} \omega_1\ [hz] & \omega_2\ [hz] & \omega_3\ [hz] & \omega_4\ [hz] & \omega_5\ [hz] & \omega_6\ [hz] & \omega_7\ [hz] \\ 1.062 & 2.006 & 2.085 & 2.698 & 3.204 & 3.485 & 4.398 \end{array} \tag{14}$$

and an excellent agreement with the experimental data may be observed. In the second stage of the

Figure 4: First displacement eigenmode

investigation, a sequence of damage scenarios modeled after [1] were introduced along the bridge span to assess the sensitivity of the eigenfrequencies of the bridge. This is the first step of an inverse health monitoring procedure that shall move from new in situ measurements of the frequencies of the system to be used within the least square optimization scheme proposed in Equation (9). The

Figure 5: Second displacement eigenmode

Figure 6: Fourth displacement eigenmode

goal is tough in that we no longer deal with a simply supported straight beam. Conversely, the pylon, the cables and the presence of a curved deck in both the horizontal and vertical planes bring into the analysis quite reacher damage scenarios than those one may encounter by studying the ideal case of a simply supported beam. Therefore, the numerical model should be accurate enough to capture all these peculiarities. The numerical model presented herein appears to be promising in this respect but a more refined one seems to be mandatory for a successful application to the evaluation of the health of the footbridge.

5 CONCLUSIONS

An approach for damage estimation and parameter identification operating in the frequency domain has been presented. Numerical investigations have been presented concerning a micro–structured Euler-Bernoulli damaged beam whereas experimental issues on the cable-stayed footbridge of Bormio (Italy) were discussed. Ongoing research includes the adoption of linked-interpolated discretization schemes in a Timoshenko-beam framework as well more extensive damage investigations on the cable-stayed footbridge that should include more complex damage models.

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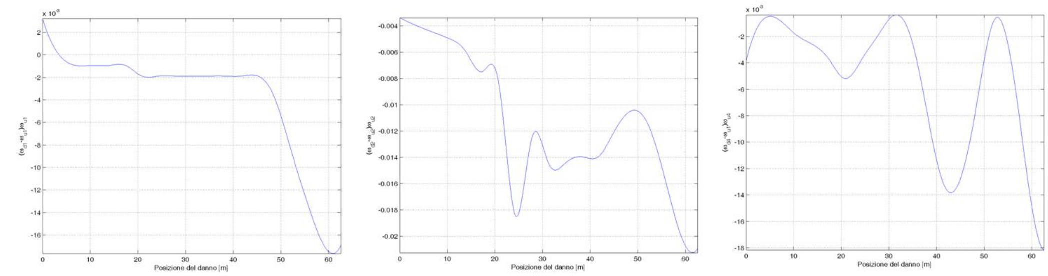

Figure 7: Normalized eigenfrequencies variation w.r.t. damage position on the bridge span

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