Structural damage detection using dynamic response: state-of-the-art and prospects

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SUMMARY. The performance of the structural assessment methods based on the observation of the dynamic response is discussed. After a brief literature survey, the directions which seem more promising from the point of view of future developments are outlined.

1 INTRODUCTION

During their service life, civil and mechanical engineering structures are unavoidably subject to be damaged. Damage may be due to changes in the material properties and integrity (as a consequence of aging and chemical-physical interaction with the environment) and to the time-varying applied loads. Often these phenomena can take place simultaneously, giving rise to particularly unfavourable conditions.

The corresponding damage can prejudice the structural serviceability or can even lead to the decrease of the structural safety and can reduce the duration of the service life.

Pointing the attention on civil engineering structures, the economical consequences of these phenomena are especially relevant in the case of “strategic” structures, like infrastructure systems and school buildings, but can be significant, although not always easily quantifiable, also in the architectural heritage area. The related economic problem is a topical issue, as many infrastructures urgently will require a reliable evaluation of their integrity and safety [1] since many of them are approaching the end of their expected service life, usually set between 50 and 100 years.

Hence a significant increment of interest has been observed, during the last decade, for the structural health assessment through the continuous or discrete monitoring of the dynamic response. Such interest was encouraged by the methodological and operational evolution of the ambient vibration testing and operational modal analysis techniques.

In the present study a brief literature background on the topic is outlined, then the performance of the different classes of assessment methods is discussed, giving an outlook to those technique which seem more promising from the point of view of future developments.

2 BACKGROUND ON STRUCTURAL DAMAGE DETECTION

2.1 *Introduction*

The dramatic improvements that took place in the last few decades in the fields of low-cost displacement and acceleration transducers, signal conditioning and sampling hardware, electronic data acquisition systems, have driven the interest of the scientific community toward the use of the dynamic response of structural systems as a tool to assess the damage and the safety.

So the current tendency of the scientific research is towards using the dynamic response data to identify possible changes in the structural properties, with the aim to deduce information on the structural integrity of the relevant cross sections.

Anyway an effective structural damage detection system must fulfill some basic and self-evident requirements: first of all it must be able to find those kinds of damage which are relevant to serviceability or safety, then it must reach this result before than the damage itself becomes so severe that it can be observed using more simplified methods, like, for example, visual inspection.

2.2 *Literature background*

A wide literature survey on the topic of the structural damage detection through vibration-based methods can be found in Doebling et al. (1996) [2], in Farrar et al. (2003) [3], in Hsieh et al. [4], in Morassi and Vestroni [5], and in the monographic issue of Structural Control and Health Monitoring journal, that collects the main findings of the Italian Research Project (PRIN) VinCES - Vibration in Civil Engineering Structures [6] financed by the Italian Ministry of the University and Research (MIUR) in 2004.

The structural identification, fundamental diagnosis technique to assess the integrity of structures, has been lately developed with particular reference to the issue of the “output-only” identification.

The main difference with the classical approach is that the input excitation is not known or hardly available. Currently, two are the most commonly used techniques for the output-only identification. The first one, called Enhanced Frequency Domain Decomposition (EFDD) (Brincker et al. [7]), can be considered the evolution of the Peak–Picking technique (Douglas and Reid [8], Bendat and Piersol [9]) and consists in the singular values decomposition of the Spectral matrix for each frequency of interest. Another technique, that requires more significant computational efforts, is the Stochastic Subspace Identification (Van Overschee and De Moor [10], Peeters and De Roeck [11], Katayama [12]).

More recently the development of transducers systems having good performances and relatively low cost (displacement Bragg grating transducers, MEMS accelerometers, wireless sensors, radar sensors etc.), together with the spread of acquisition systems with high velocity and elevated number of channels allowed the installation of permanent monitoring systems in many of the new long span bridges in Japan, North America and Europe (Brownjohn et al. (2005) [13]). Among them some are worth recalling such as: the suspension bridge over the Humber in the United Kingdom, that from 1984 to 1998 has been the longest bridge of the world (Brownjohn et al. (1987) [14]), the suspension bridge over the Great Belt in Denmark (Jensen, (2006) [15]) and the Akashi-Kaikyo in Japan (Fujino and Kashima (2006) [16]), that is currently the longest existing bridge, with a maximum span length of 1991 m.

Damage identification in bridges and, more generally, in civil engineering structures through dynamic tests is however a complex task, as the parameters involved in the investigation process (natural frequencies of vibration, modal shapes, damping ratios) depend upon the overall structural stiffness and are therefore slightly influenced by the damage effects, that are usually localized. Often the variation of the dynamic behavior is so small that it can be confused with the effect due to changes of the ambient temperature, the humidity content and the masses added during the service life.

Moreover, the correlation between the stiffness of the structural elements and the mechanical strength of the cross sections, in most of the real cases, is weak and therefore the safety estimate is difficult. It must be pointed out that some investigation techniques require the knowledge of the dynamic properties of the undamaged structures, information that is rarely available.

In 2001 Brincker et al. [17] reported their experience of 15 dynamic tests carried out on a bridge in Switzerland, before it was razed and replaced by a new bridge, realizing an artificial state of progressive damage. The Authors applied the EFDD technique, based on the determination of eigenfrequencies, eigenvectors and damping changes and, neglecting the influence of environmental factors, believed that they could determine the presence of damage even for small changes of the modal parameters.

The case of damage due to local cracks has been studied by Patil and Maiti [18], Lin et al. [19], Kim and Stubbs [20], who limited their attention to small size steel structures, adopting the hypothesis that cracks are always open.

On the contrary, just a few studies concern the behavior of damaged prestressed or reinforced concrete structures, which is more complex than the one of structures made of homogeneous materials. The main reason of this difficulty is that the cracks, crossed by the steel reinforcements, open and close alternatively during the motion, giving rise to changes in the dynamic response of the damaged structural element (Breccolotti and Materazzi [21], Breccolotti et al. [22]).

In 2000 Maeck et al.[23] described two methods for the determination of the stiffness decrease of structural concrete beams due to damage. The first method uses an updating algorithm based on the stiffness characteristics, which are damage-dependent, and on the comparison between the undamaged and the damaged structural response. It is based only on the eigenfrequencies and, therefore, it cannot detect asymmetric damage in a symmetric structure. The second one, called Direct Stiffness Calculation, is able to detect and localize damage through the modal shape determination.

The non-linearity of damaged reinforced concrete structural elements was recently studied in the joint time-frequency domain by Owen et al. [24] and by Neild et al. [25]. A finite element model for damaged reinforced concrete elements based on the theory of the Fracture Mechanics was proposed by Saavedra and Cuitiño [26].

3 REMARKS ON VIBRATION-BASED STRUCTURAL ASSESSMENT METHODS

The well-known classification of damage-identification methods proposed by Rytter [27] defines four levels of damage identification:

- Level 1: Damage detection;
- Level 2: Damage localization;
- Level 3: Damage quantification;
- Level 4: Prediction of the remaining service life of the structure.

The methods available to achieve this goals belong to few general families:

- Methods based on frequency changes;
- Method based of mode shape changes;
- Statistical and other methods.

The first two families of methods are discussed in the present section, while the other are addressed in the next section as they appear mode promising for future developments.

3.1 *Methods based on frequency changes*

The assessment methods based on the observation of the changes of the structural natural frequencies compare the frequencies evaluated trough experimental modal analysis with respect to a known reference condition (Cerri and Vestroni [28], Breccolotti et al. [29]).

The simple mathematical model, presented in [29], with reference to simple beams, can be useful to evaluate the sensitivity of these methods. The Authors consider a simple supported beam (see Figure 1). The structural damping and the temperature effects are neglected. The structural behavior is represented using a linear elastic model. The damage is supposed to be spread over a unique portion of the beam of length *b*, centered at a distance *a* from the left hand side support. It is modeled as a suitable reduction of the beam flexural stiffness.

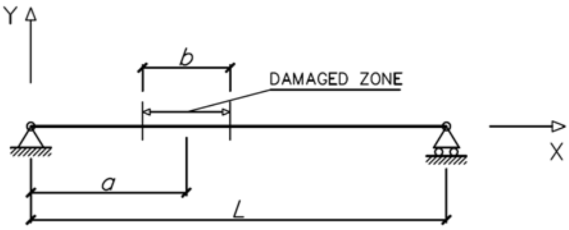

Figure 1: Sketch of the model of the simple supported beam.

The eigenvalue equation for the undamaged beam is:

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2Y}{dx^2}\right]=\lambda m(x)Y, \tag{1}$$

where $Y(x)$ is the modal shape, E is the modulus of elasticity, $J(x)$ the section moment of inertia, $m(x)$ is the mass per unit length and the purely imaginary eigenvalue λ is the square of the pulsation ω. Obviously the boundary conditions are:

$$Y(0)=Y(L)=0,\ \frac{d^2Y}{dx^2}(0)=\frac{d^2Y}{dx^2}(L)=0, \tag{2}$$

A numerable infinity of eigenvalues, along with the corresponding modal shapes, can be obtained.

In the case of a prismatic beam having constant cross section ($J(x) = J$, $m(x) = m$) the pulsations are:

$$\omega_r = r^2\pi^2\sqrt{EJ/mL^4}, \tag{3}$$

and the corresponding normalized modal shapes are:

$$Y_r(x)=\sqrt{2/mL}\,\sin(r\pi/L), \tag{4}$$

where r represents the number of the mode.

The damage is modeled as a variation of the structural characteristics, considering a decrement of the moment of inertia $\Delta J(x)$. As a consequence the eigenvalues and the mode shapes undergo the variation $\Delta\lambda_r$ and $\Delta Y_r(x)$, respectively.

The eigenvalue equation for the damaged beam becomes:

$$\frac{d^2}{dx^2}\left[E\left[J(x)+\Delta J(x)\right]\frac{d^2\left[Y_r+\Delta Y_r\right]}{dx^2}\right]=m(x)\left[\lambda_r+\Delta\lambda_r\right]\left[Y_r+\Delta Y_r\right], \tag{5}$$

which can be developed as:

$$\frac{d^2}{dx^2}\left\{E\left[J(x)\frac{d^2Y_r}{dx^2}+\Delta J(x)\frac{d^2Y_r}{dx^2}+J(x)\frac{d^2\Delta Y_r}{dx^2}+\Delta J(x)\frac{d^2\Delta Y_r}{dx^2}\right]\right\}= \\ = m(x)\left[\lambda_r Y_r + Y_r\Delta\lambda_r + \lambda_r\Delta Y_r + \Delta Y_r\Delta\lambda_r\right], \tag{6}$$

Taking into account the Eq. (1) and neglecting the higher order terms, the Eq. (6) becomes:

$$\frac{d^2}{dx^2}\left\{E\left[\Delta J(x)\frac{d^2Y_r}{dx^2}+J(x)\frac{d^2\Delta Y_r}{dx^2}\right]\right\}= \\ = m(x)\left[Y_r\Delta\lambda_r + \lambda_r\Delta Y_r\right], \tag{7}$$

Multiplying by Y_r, integrating over the whole length of the beam and considering the boundary conditions, the Eq. (7) becomes:

$$\int_0^L E\Delta J(x)\left(\frac{d^2Y_r}{dx^2}\right)^2 dx + \int_0^L EJ(x)\frac{d^2Y_r}{dx^2}\frac{d^2\Delta Y_r}{dx^2}dx = \\ = \lambda_r\int_0^L m(x)Y_r(x)\Delta Y_r(x)dx + \Delta\lambda_r\int_0^L m(x)Y_r(x)^2 dx, \tag{8}$$

Using the normalization conditions the following expression can be obtained:

$$\Delta\lambda_r = \int_0^L E\Delta J(x)\left(\frac{d^2Y_r}{dx^2}\right)^2 dx, \tag{9}$$

Under the assumed hypotheses, the Eq. (9) determines the increment of the *r*-th eigenvalue with the variation of the structural characteristics. The same result stands still for fixed-fixed beams and for fixed-pinned beams.

Assuming finally that the variation of the structural characteristics, ΔJ (x), may be modeled as a constant decrement of the moment of inertia ΔJ in a portion of length b, centered at $x = a$, the Eq. (9) simplifies as:

$$\Delta\lambda_r = \frac{2E\Delta J}{mL}\left(\frac{r\pi}{L}\right)^4\int_{a-b/2}^{a+b/2}\left(\sin\frac{r\pi x}{L}\right)^2 dx, \tag{10}$$

Taking into account that $\Delta\lambda_r = 2\omega_r\Delta\omega_r$, the relative variation of the pulsation is:

$$\frac{\Delta\omega_r}{\omega_r} = \frac{1}{2\pi r}\frac{\Delta J}{J}\left(\beta - \sin\beta\cos\alpha\right), \tag{11}$$

where the dimensionless parameters $\alpha = 2r\pi a/L$ and $\beta = r\pi b/L$ have been introduced.

Since the term

$$F(\alpha,\beta) = \frac{\beta - \sin\beta\cos\alpha}{2\pi r} \tag{12}$$

is always positive for $\beta > 0$, the decrement of the structural characteristics ΔJ leads to a relative decrement of the frequency $\Delta\omega_r$ of the r-th mode .

In order to quantify the sensitivity and the accuracy of the frequency-based methods, the variation of the function $F(\alpha,\beta)$ (Eq. 12) with the dimensionless parameters a/L and b/L is investigated.

The analysis is carried out with reference to the simple structural system shown in Figure 1 and is limited to the first three modes. The ratio a/L is varied between 0 and 1 and the ratio b/L ranges between 0 and the maximum value compatible with its location. For b/L , the values 0.01, 0.05, 0.10, 0.15 and 0.20 were considered, as representative of possible damage situations.

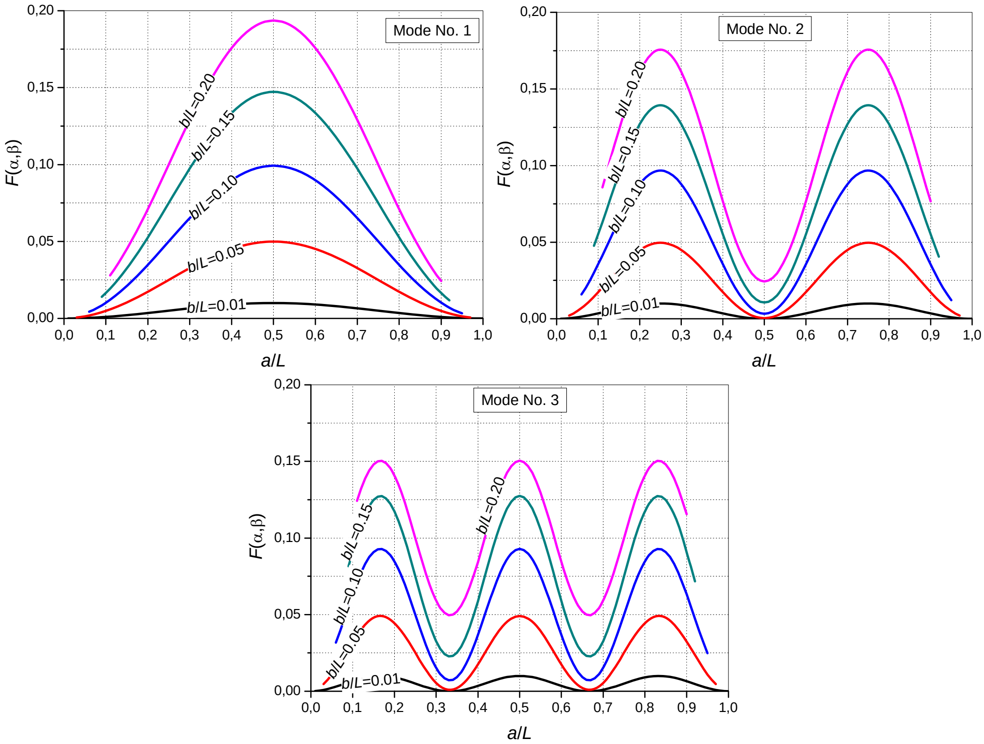

Figure 2: Function $F(\alpha,\beta)$.

The graphs (Figure 2 for the modes no. 1, 2 and 3, respectively) show the sensitivity of the method in finding the location of damage (a/L). After having observed that the shapes of $F(\alpha,\beta)$ roughly resemble those of the absolute values of the first three modal shapes, it must be pointed out that the maximum reached by the function in the first mode $F(\alpha,\beta) = 0.194$ slightly decreases to 0.176 and 0.150 while the mode number increases. Moreover the sensitivity in finding the location (Level 2 in the list at the beginning of this Section 3) varies with the location itself and with the mode number.

Pointing the attention on the reasonable case where $b/L = 10\%$, the sensitivity is at the best (mode 1 with damage at midspan) equal to 0.10. If the relative variation of $\Delta J/J$ is 0.30, which may be considered as a maximum in most practical cases, the corresponding relative variation of $\Delta\omega/\omega$ is 0.10 x 0.30 = 0.030, which can be very difficult to identify and to distinguish from the thermal effects.

3.2 *Methods based on mode shape changes*

The most widely used methods belonging to this family are the Modal Assurance Criterion (MAC) and the Coordinate Modal Assurance Criterion (COMAC) methods. The MAC is defined as follows:

$$MAC\left(\phi_i,\phi_j\right)=\frac{\left|\phi_i^T\phi_j\right|^2}{\phi_i^T\phi_i\phi_j^T\phi_j}, \qquad (13)$$

where ϕ_i and ϕ_j are two mode shapes. The COMAC is defined as:

$$COMAC\left(i\right)=\frac{\sum_{j=1}^{N}\left[\left|\phi_{i,j}^A\phi_{i,j}^B\right|\right]^2}{\sum_{j=1}^{N}\left(\phi_{i,j}^A\right)^2\sum_{j=1}^{N}\left(\phi_{i,j}^B\right)^2}, \qquad (14)$$

where N is the number of mode shapes and $\phi_{i,j}^A$ and $\phi_{i,j}^B$ are the values of the *j*-th mode shape at the point *i* for the measurement sets *A* and *B*.

The MAC and the COMAC determine the level of correlation between the measured mode shapes obtained from two sets of tests. If the two sets of eigenvectors are identical, the corresponding MAC value will be close to unity, thus indicating the full correlation between the two modes. The deviation of these factors from 1 could be interpreted as a damage indicator.

The COMAC factors are generally used to identify where the mode shapes coming from two sets of measurements are not correlated and so they are more suitable in order to localize the position of damage.

The basic limitation of these methods is that they need the knowledge of a reference set of modal shapes. As a consequence they must be used in a framework of periodical controls, like in a monitoring system.

Moreover the COMAC method gives its better results when a high number of modes are considered, with the obvious difficulties connected with the excitation of the upper modes, in terms of energy requirements and accuracy of measurement.

Anyway from the literature [30] it comes that they are not sensitive enough to find the damage at an early stage, before it can be detected by visual inspections, even if this conclusion is only demonstrated for specific structures and not proven in a more formal and general way.

4 PERSPECTIVES AND DEVELOPMENTS

The considerations developed in Section 3 lead to the quantitative conclusion that the frequency-based methods of assessment display poor sensitivity to the changes of the structure characteristics that may lead to failure.

Even the methods based on the analysis of the eigenvector shape (MAC, COMAC, and so on) do not seem completely adequate to find the effects of those kinds of damage that can lead to failure.

Both frequency based method and mode shape based methods can reach the level 1 and 2 of the classification proposed by Rytter (Section 2) only when the damage is so severe that it can be found by simple visual inspections and so they cannot fulfill completely the requirements outlined in section 2.1.

More promising seems the search for those types of damage that are widely spread over the structures and, consequently, are not directly related to failure, but prominently involve serviceability and durability. That is the case that may occur in structures made up using composite materials, like fiber-reinforced composites damage and reinforced or prestressed concrete, that may be subjected to delamination and cracking, respectively. Anyway major improvements are needed on the accuracy of the measurement protocol, in order to consider the environmental effects, like the temperature variation, the variation of the characteristics of the supports and so on.

4.1 *Damage detection through the observation of mode-shape curvature*

Some advanced damage detection methods are based on the local changes of the curvature in correspondence to the location of damage. Among these methods may be included the *Modal shape curvature method*, the *Damage index method* and the *Frequency-response-function curvature method*.

The Modal shape curvature method [31] is based on the numerical estimate of the curvature by a central difference approximation:

$$\phi_i^{"} = \left(\phi_{i+1} - 2\phi_i + \phi_{i-1}\right) / h^2, \tag{16}$$

where *h* is the distance between to measuring points *i* and *i +1*.
The location of damage is identified as the place where the absolute difference between curvatures of the damaged and the undamaged structure reaches its maximum.

The damage index method [32] uses a suitable damage index based on the differences of the modal strain energy of the damaged and the undamaged structure. The methods can use the numerical derivatives of the displacement measurements or the direct measurements of the local strain. The second possibility makes the method interesting as this kind of measurement, even if most complicate than the standard acceleration measurements, is comparatively more sensitive.

The frequency-response-function curvature method [33] is an extension of the modal shape curvature method to all frequencies in the measurement range. In fact it modifies the Eq. 16 as:

$$\alpha^{"}(\omega)_{i,j} = \left(\alpha(\omega)_{i+1,j} - 2\alpha(\omega)_{i,j} + \alpha(\omega)_{i-1,j}\right) / h^2, \tag{17}$$

where $\alpha_{i,j}$ is the ordinate of the frequency response function measured at the point *i*, as the force is applied at point *j*.

4.2 *Damage detection through the probabilistic analysis of the response*

This family of methods is based on the probabilistic analysis of the dynamic response of structures subjected to Gaussian white noise. The method relies on the principle that the response of a non-linear system to a Gaussian excitation is no longer Gaussian. Higher order statistical moments can be used as a measure of the non-Gaussian response and thus as a damage indicator.

The main advantage of the method is that the knowledge of the structural response before the onset of damage is no longer needed, as the undamaged system is characterized by a linear (and thus Gaussian) response. The procedure is accounted for in several works [34], [35], [36].

In Breccolotti and Materazzi [36] are presented the results of a set of experimental tests on artificially damaged RC beams 4.30 m long and having cross-section 20 x 30 cm. A control loop electrodynamic shaker was used to apply a white noise Gaussian load in a three point bending configuration (Figure 3).

The schedule of the laboratory tests is reported in Table 1. It included 9 increasing damage levels realized by means of concrete cover removal, rebars partial cuts and application of an increasing quasi-static external load to induce cracking in the beams.

In correspondence of the damage steps no. 4, 7 and 8 several dynamic tests were repeated progressively increasing the value of the RMS Gaussian excitation from 10,81 N until 65.97 N.

The asymmetry of the distribution of the measured response acceleration was synthetically described by means of the third standardized statistical moment (the skewness, γ_1) defined as:

$$\gamma_1 = \frac{1}{\sigma^3} \frac{\sum_i (x_i - \mu)^3}{N} = \frac{\mu_3}{\sigma^3} \tag{15}$$

where μ is the mean value of the distribution, σ is its standard deviation and μ_3 is the third statistical moment. The results are reported in Figure 4.

For every damage step it was observed that the structure behaves linearly under low intensity external forces, even if it is damaged and cracked. In fact the skewness values are very close to zero (first point of each line in Figure 4). An increase of the structural non-linearity under increasing external load can be observed without any change in the actual damage condition. For the damage steps no. 4 and 7 the values of the skewness were very similar, even if the beam underwent severe localized damages (step no. 4: partial cut of the rebars; step no. 7: upper concrete cover removal). On the contrary the results from step no. 8 displayed lower values of the skewness coefficient.

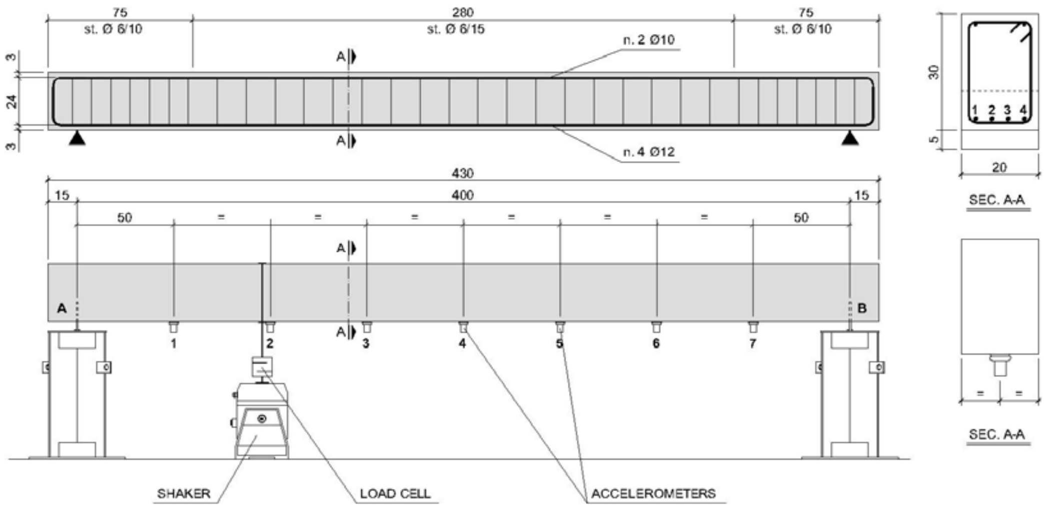

Figure 3: Setup of the RC beams test.

Table 1: Definition of the damage steps.

Step no.	*Damage step*
0	Undamaged beam
1	Applied load of 23.55 kN
2	Lower concrete cover removal at midspan
3	Partial cut (50%) of rebar no. 1
4	Partial cut (50%) of rebar no. 4
5	Partial cut (50%) of rebar no. 2
6	Partial cut (50%) of rebar no. 3
7	Upper concrete cover removal at midspan
8	Applied load of 39.25 kN

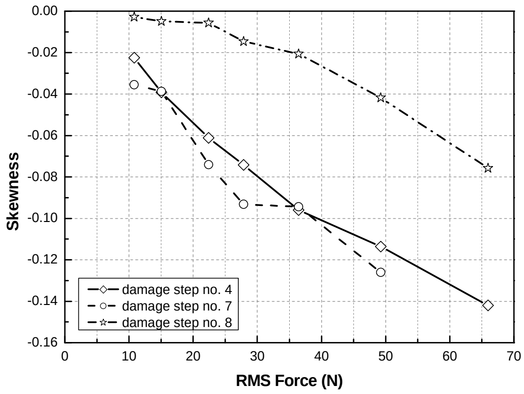

Figure 4: Skewness values for damage steps 4, 7 and 8.

These results encourage the expectation that the skewness may be correlated to the state of damage in a testing procedure based on the application of band-limited white noise loads.

5 CONCLUSIONS

The performance of the structural assessment methods based on the measurement of the dynamic response has been investigated.

The classical methods based on the observation of the frequency changes and on the study of the modal shapes have been first considered. Both techniques display some drawbacks. First of all they need the knowledge of the dynamic structural properties before the onset of damage, data that are hardly available if the measurements are not carried out in the frame of a permanent monitoring system. Moreover, pointing the attention on their practical application, their sensitivity does not seem adequate to find damage in an early stage, before it can become dangerous from the point of view of safety. The most accurate methods based on the observation of changes in the modal shapes can give anyway interesting results, but require the excitation of a great number of upper modes, task that is very difficult to accomplish in most practical cases.

More promising seem the perspective of finding special forms of diffuse damage, like the state of cracking in reinforced concrete and prestressed concrete structures.

For this purpose some advanced techniques based on the analysis of the modal curvature and of the stochastic responses, that the structures display when excited by random noise loads, seem to give promising results.

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