Handling state variable constraints and actuator saturation in structural control strategies

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SUMMARY. Structural control is a very promising technique for the seismic protection of strategic structures and infrastructures against severe earthquakes. However, the conditions for making structural control a key point of modern seismic engineering are still far from being satisfied. In past years, the demand for a continuous power supply limited the number of active control applications in the field of civil engineering. Today, having overcome this drawback by introducing the concept of semi-active control, engineers involved in this field are increasingly called to satisfy reliability and robustness requirements. In light of these needs, the inherent limits of classic control devices, such as actuator saturations and stroke stops, cannot be neglected, especially when dealing with strong seismic demands. On this respect, the paper discusses some control strategies that allow to handle a variety of constraints directly in the control law. Applications to a case study are also presented via numeric simulations.

1 INTRODUCTION

Structural control is an active research field in the technical literature [1, 2, 3, 4] with considerable potentialities within the seismic engineering field. Particularly, active or semi-active control strategies can be designed with the purpose of keeping fully operational those structures that are important for civil protection, even after very strong earthquakes, as required by technical standards (e.g. [5]). Yet, the long return period of severe earthquakes poses the problem of the system's reliability and generally limits the applicability of non-passive control strategies to the seismic protection of structures and infrastructures. Indeed, the control system is required to perfectly operate in the (unpredictable) moment when the seismic event occurs, after remaining in "stand-by" for a long time. Moreover, strong seismic events entail severe control demands that the system is not always able to satisfy. This is, for instance, the case in which state variable constraints (such as the physical limits to the stroke of movable masses in inertial actuators) and actuator saturations limit the operation of the control devices.

The problem of actuator saturation is well-known in system engineering as it can lead to the so-called wind-up instability when integrators are adopted in the controller (e.g. [6]). However, integral terms are usually not necessary to stabilize the motion of elastic structures thus making wind-up instability a secondary task in structural control. Nonetheless, actuator saturation may significantly reduce the control effectiveness and even cause damage in the system. On the other hand, the problem of state variable constraints is less explored in the literature and it is usually neglected in structural control applications.

A possible way of handling actuator saturation consists of coupling a "low-gain" controller with a "bang-bang" one (e.g. [7]). However, the most general tool for handling control limitations is probably represented by the so called "State Dependent Riccati Equation" (SDRE) (e.g. [8]) method, in the general framework of nonlinear regulation. An application of such a method to the case of state variable constraints can for instance be found in [9]. Here, the SDRE method is briefly reviewed, at first, in view of its application in active control strategies. Then, the method is applied in a typical structural context, with the aim of handling state variable constraints and actuator saturations. A numerical case study is finally considered to validate the proposed approaches.

2 GOVERNING RELATIONS: THE STATE DEPENDENT RICCATI EQUATION

A structural system equipped with some active or semi-active controller is considered. The system is inherently linear but, after introducing control limitations, the dynamics of the controlled system becomes globally nonlinear. The equations of motion of the controlled system can thus be written as $\dot{x} = f(x) + g(x)u$, where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector and f and g are suitable vector fields. The nonlinear regulator problem for the given system can be written in standard form as the minimization of the following performance index J:

$$J = \frac{1}{2}\int_{t_0}^{\infty}(x^T Q(x)x + u^T R(x)u)dt \tag{1}$$

subjected to the constraint given by the equations of motion $\dot{x} = f(x) + g(x)u$. In equation (1) the weight matrices $Q(x)$ and $R(x)$ contain the penalties on state variables and control forces and must be chosen such that $Q(x) = D^T(x)D(x) \geq 0$ and $R(x) > 0, \forall x$.

The SDRE method is a very convenient tool for obtaining suboptimal solutions for the above stated problem. This approach simply requires the solution of a state dependent form of the Riccati equation that can be derived by expressing the equations of motion $\dot{x} = f(x) + g(x)u$ in state-dependent coefficient form $\dot{x} = A(x)x + B(x)u$ through direct parametrization. Since this last system posses a linear structure, albeit being nonlinear, it is possible to construct a feedback as $u = -R^{-1}(x)B^T(x)P(x)x$ where matrix $P(x)$ solves the SDRE given by:

$$A^T(x)P + PA(x) - PB(x)R^{-1}(x)B^T(x)P + D^T(x)D(x) \tag{2}$$

As it can be recognized looking at equation (2), the SDRE method is formally similar to the classic linear quadratic regulator, with the exception that all coefficient matrices are state dependent. This means that the Riccati equation (2) must be solved online to calculate the feedback.

Some interesting features of the SDRE method were demonstrated in [10]. These properties include local asymptotic stability and asymptotic satisfaction of the necessary conditions for optimality of the nonlinear regulator problem. It is also worth noting that the parametrization $\dot{x} = A(x)x + B(x)u$ is not unique in the multi-variable case. The performance of the system strongly depends on such parametrization and on the peculiar choice of the control weights $Q(x)$ and $R(x)$.

3 ACTIVE CONTROL STRATEGIES IN PRESENCE OF CONTROL LIMITATIONS

Without loss of generality, the use of active mass dampers (AMDs) as control actuators is under specific investigation in this work. Indeed, AMDs are widely applied in structural control strategies due to their relative simplicity and effectiveness. Two are the main constraints that usually affect AMDs: the stroke stops that limit the motion of the movable mass and the control force bound u_{max}. An example of AMD subjected to the considered kind of constraints, experimentally applied to the protection of a mast for mobile phone networks against wind action in a previous work [1], is shown in the picture of Figure 1.

In order to apply the considered device in a typical structural context, let us specialize the problem reported in Section 2 to the one of a single degree of freedom elastic structure equipped with the AMD and subjected to base acceleration $\ddot{x}_0$. The case study is represented in Figure 2, with obvious meaning of the structural parameters. Due to the stroke stops, the relative displacement

Figure 1: Bidirectional active mass damper with stroke stops and actuator saturation.

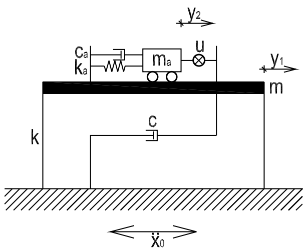

Figure 2: Structure equipped with AMD subjected to base acceleration.

$y_r = y_2 - y_1$ between the mass and the structure is bounded by a physical constraint placed at y_{max}. The equations of motion of the considered system simply read as:

$$\begin{array}{c} m\ddot{y_1} + c\dot{y_1} + ky_1 = -m\ddot{x_0} + c_a\dot{y_r} + k_a y_r + \phi(y_r) + u \\ m_a\ddot{y_r} + c_a\dot{y_r} + k_a y_r = -m_a\ddot{x_0} - m_a\ddot{y_1} - \phi(y_r) - u \\ \phi(y_r) = (\frac{y_r}{y_{max}})^{(2N+1)} \\ |u| \leq u_{max} \end{array} \tag{3}$$

where the nonlinear force $\phi(y_r)$ has been introduced to simulate the presence of the hard physical constraint [9], N being an integer number. In the case of an unbounded control signal ($u_{max} = \infty$) a possible parametrization of equation (3) is the following:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 \quad 0 & 1 \quad 0 \\ 0 \quad 0 & 0 \quad 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \\ -\frac{1}{m_a} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix} \ddot{x_0} \tag{4}$$

where the state vector has been defined as $x = [y_1, y_r, \dot{y_1}, \dot{y_r}]^T$ and the following matrices have been introduced:

$$M = \begin{bmatrix} m & 0 \\ m_a & m_a \end{bmatrix} \quad C = \begin{bmatrix} c & -c_a \\ 0 & c_a \end{bmatrix} \quad K(x) = \begin{bmatrix} k & -k_a - (\frac{x_2^{2N}}{{y_{max}}^{2N+1}}) \\ 0 & k_a + (\frac{x_2^{2N}}{{y_{max}}^{2N+1}}) \end{bmatrix} \tag{5}$$

It is noteworthy that M, C and K in equation (5) do not represent the mass, damping and stiffness matrices of the structural system because, for convenience, equations (3) are written in terms of transformed coordinates $(y_1, y_r = y_2 - y_1)$ instead of absolute displacements (y_1, y_2). This transformation also explains the asymmetry of such matrices in equation (5).

According to equation (3), when the movable mass reaches the physical limits, a collision occurs (reproduced by the force $\phi(y_r)$) which causes a sudden loss of control effectiveness and may even produce damages in the system. In order to design the nonlinear controller in such a way to avoid penetration of the movable mass into the forbidden region ($|x_2| > y_{max}$), a nonlinear state dependent penalization on $x_2 = y_r$ or on its first time derivative $\dot{x_2} = \dot{y_r}$ can be introduced in the performance index J. A possible choice of the weight matrix $Q(x)$ appearing in equation (1) can thus be written as suggested in reference [9]:

$$Q(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (x_2/y_{max})^{2N} \end{bmatrix} \tag{6}$$

where, without loss of generality, the structural velocity $\dot{y_1}$ is the controlled variable.

In the case of a bounded control signal ($u_{max} < \infty$) it is necessary to substitute the inequality constraint $|u| \leq u_{max}$, in equation (3), by a smooth function. This can be achieved by introducing the *saturation sine function* which is defined as [10]:

$$u = satsin(u_{max}, x_5) = \begin{cases} u_{max}, & for \;\; x_5 > \frac{\pi}{2} \\ u_{max} \cdot sin(x_5), & for \;\; -\frac{\pi}{2} \leq x_5 \leq \frac{\pi}{2} \\ -u_{max}, & for \;\; x_5 < -\frac{\pi}{2} \end{cases} \tag{7}$$

where x_5 is an additional state variable. Now, by introducing the pseudo-control u_1, the equations of motion can be easily rewritten in terms of augmented state by adding the equation $\dot{x}_5 = u_1$. A convenient parametrization of this system can be written as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \end{bmatrix} = \begin{bmatrix} 0 \quad 0 & 1 \quad 0 & 0 \\ 0 \quad 0 & 0 \quad 1 & 0 \\ -M^{-1}K & -M^{-1}C & \begin{matrix} \frac{satsin(u_{max}, x_5)}{m x_3} \\ -\frac{satsin(u_{max}, x_5)}{m_a x_4} \end{matrix} \\ 0 \quad 0 & 0 \quad 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \\ 0 \end{bmatrix} \ddot{x_0} \tag{8}$$

in which the controller regulates the pseudo-control u_1. This allows to obtain suboptimal control without violating the saturation constraint. Indeed, according to equation (7), the force $u = satsin(u_{max}, x_5)$ effectively exerted by the control actuator does never exceed the saturation limit u_{max} whatever is the value of u_1. In case the physical constraint on y_r can be neglected a possible choice of $Q(x)$ is as follows:

$$Q(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_1 \end{bmatrix} \tag{9}$$

where r_1 is a small penalty on u_1 that is introduced in order to avoid a singular problem. The SDRE also allows to handle actuator saturations and state variable constraints all at once. This can simply be achieved through the parametrization reported in equation (8) and by defining the weight matrix $Q(x)$ as:

$$Q(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & (x_2/y_{max})^{2N} & 0 \\ 0 & 0 & 0 & 0 & r_1 \end{bmatrix} \tag{10}$$

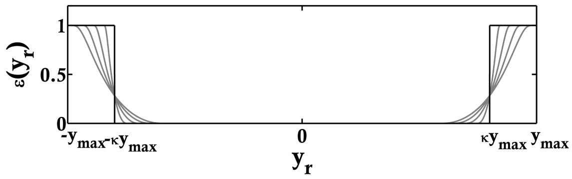

Figure 3: Regularization function $\varepsilon(y_r)$ (the black line represents the Heaviside function while the grey lines represent $\varepsilon(y_r)$ for different values of κ_0, κ_1 approaching κ).

4 ENHANCED NONLINEAR DAMPING APPROACH

The problem of state variable constraints can be treated as described in Section 3 by assigning a state dependent nonlinear weight to the first time derivative of the constrained variable y_r in equation (6). Nonetheless, it must be mentioned that assuming very large exponents N in equation (6) might reflect on numerical difficulties in solving the Riccati equation. On the other hand, if N is small, the effectiveness of the said strategy in preventing the movable mass from attaining the limit is low. A possible alternative way to prevent the mass from approaching the physical limits is to slow down its motion in the vicinity of the stroke stops. This can be easily achieved by introducing in the system an additional nonlinear derivative term in $\dot{y_r}$ with state-dependent gain $g_d \cdot \varepsilon(y_r)$, g_d being a specified constant gain and $\varepsilon(y_r)$ being a suitable nonlinear function. Clearly, $g_d \cdot \varepsilon(y_r) \cdot \dot{y_r}$ is a force that must be provided, when necessary, by the actuator. Thus, this approach is obviously applicable only when actuator saturation does not occur.

The most trivial function $\varepsilon(y_r)$ to be adopted in the controller is the Heaviside unit step function which is nil when $|y_r|$ is less than κ times (with $0 < \kappa < 1$) the stroke stop y_{max} and it is equal to unity elsewhere. However, this function is discontinuous at $|y_r| = \kappa \cdot y_{max}$ which requires an abrupt application of the breaking force which, theoretically, would need an infinite power. Practically, the unit step function does not solve the problem of the abrupt stop of the actuator. In order to overcome this drawback, it is necessary to define a function $\varepsilon(y_r)$ that varies smoothly between 0 and 1 within the interval $[\kappa_0 y_{max}, \kappa_1 y_{max}]$ with $0 < \kappa_0 < \kappa_1 < 1$. A function of this type can be defined as follows:

$$\varepsilon(y_r) = \begin{cases} 1, & |y_r| \geq \kappa_1 y_{max} \\ 1 - exp(\frac{-1}{1-(|y_r|-\kappa_0 y_{max})^2/(\kappa_1 y_{max}-\kappa_0 y_{max})^2} + 1), & \kappa_0 y_{max} < |y_r| < \kappa_1 y_{max} \\ 0, & |y_r| \leq \kappa_0 y_{max} \end{cases} \tag{11}$$

A convenient parametrization of the equations of motion of the system with the enhanced nonlinear damping term $g_d \cdot \varepsilon(y_r) \cdot \dot{y_r}$ is formally identical to equation (3) in which matrix C is also state dependent and it is expressed as:

$$C(x) = \begin{bmatrix} c & -c_a - g_d \varepsilon(x_2) \\ 0 & c_a + g_d \varepsilon(x_2) \end{bmatrix} \tag{12}$$

It is noteworthy that the function $\varepsilon(y_r)$ defined in equation (11) is a C^∞ function (it can be also shown that it tends to the Heaviside function in the functional space L^2 as κ_0, $\kappa_1 \rightarrow k$). This ensures a gradual application of the breaking force and the reduction of the required power (the slope of $\varepsilon(y_r)$ is always finite). A plot of this function is shown in Figure 3.

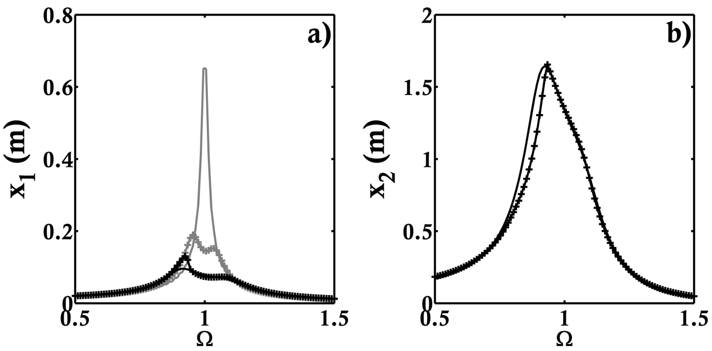

Figure 4: $frcs$ of the system (uncontrolled: grey solid; ideal actively controlled: black solid; passively controlled: grey crossed; actively controlled with force saturation: black crossed).

5 NUMERICAL EXAMPLE

The reduced scale one-storey building with AMD considered by Mongkol *et al.* in reference [7] is here adopted as the case study to test the effectiveness of the proposed approaches. The following mechanical parameters characterize the considered structure-AMD system: $m = 750\ kg$, $k = 1.76{\times}10^5\ N/m$, $\xi = 0.01$, $m_a = 7.45\ kg$, $k_a = 1.72{\times}10^2\ N/m$ and $\xi_a = 0.06$, while $u_{max} = 250\ N$ and $y_{max} = 1.2\ m$ are assumed as the control limitations. The frequency response curves ($frcs$) of the system subjected to a sinusoidal base excitation with a peak ground acceleration (PGA) of $0.35g$, g being the gravity acceleration, are here numerically calculated. The $frcs$ represent the variations of the steady amplitudes of state variables versus the excitation frequency ratio $\Omega = \omega/\omega_1$, ω being the circular frequency of the base acceleration signal and $\omega_1 = \sqrt{k/m}$ being the natural circular frequency of the structure without the AMD.

5.1 System with actuator saturation

The system subjected to the force saturation limit $u_{max} = 250\ N$ is under attention at first. This constraint is handled following the approach described in Section 3, which relies on the regulation of the pseudo-control u_1 by rewriting the equations of motion in terms of augmented state (see equation (8)). The weight matrix adopted in the SDRE is given by equation (10). After performing several attempts, the values $R(x) = 0.0001$ and $r_1 = 0.07$ have been chosen in the simulations. For comparative purposes, the uncontrolled, the passively controlled and the ideally controlled cases have also been considered. The ideally controlled case (also called unconstrained solution) is here chosen as a reference and corresponds to the unconstrained active control solution (linear system) assuming $R(x) = R = 2 \cdot 10^{-6}$. This last value, representing the penalty on the control force in equation (1), has been chosen in order to have a maximum required control force of about twice the limit $u_{max} = 250\ N$ imposed in the constrained solution. Clearly, assuming $R < 2 \cdot 10^{-6}$ would allow to obtain as large control performances as desired at the expense of larger control forces.

Figure 4 shows the $frcs$ of the considered systems, while Figure 5 (a) shows the $frcs$ of the control forces. The presented results clearly outline that, when the actuator saturates, the saturation control solution allows to achieve control performances that are only slightly worse than the unconstrained case. In any case, the constrained solution is always at least as effective as the passively controlled case. The results presented in Figure 5, besides showing that the saturation limit $u_{max} = 250\ N$ is never exceeded in the constrained solution, also emphasize that this last

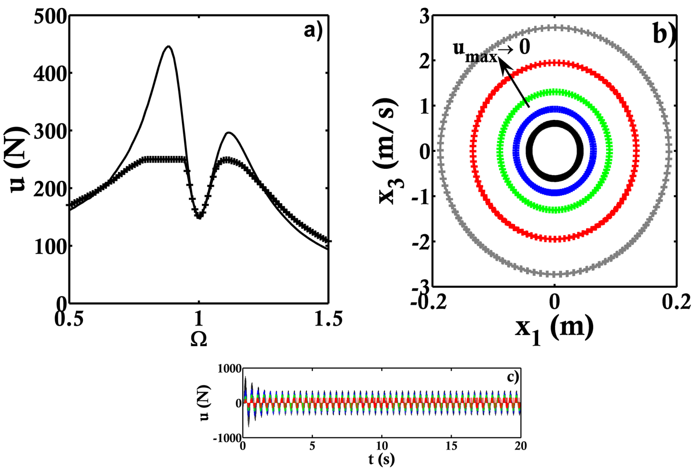

Figure 5: $frcs$ of control force u (ideal actively controlled: black solid; actively controlled with force saturation: black crossed): (a). Periodic orbits for $u_{max} \rightarrow 0$ (passively controlled case in grey): (b). Time histories of control forces for different values of u_{max} ($\Omega = 0.95$): (c).

solution requires much lower control forces than the unconstrained one in a wide range of the frequency spectrum. It is also worth mentioning that, as the control limitation becomes more penalizing ($u_{max} \rightarrow 0$), the system is always able to calculate the feedback and the control performance tends to the one of the passively controlled case. This obviously suggests to tune the passive parameters of the AMD to those of the optimal passive case (this choice is sometimes called "Active Tuned Mass Damper"). The limit of the controlled solution for $u_{max} \rightarrow 0$ is emphasized in Figure 5 (b) which shows the periodic attractors of the system near resonance ($\Omega = 0.95$) for different values of u_{max} approaching 0. On the contrary, likewise in the unconstrained problem, no theoretical limit to the control performance exists for $u_{max} \rightarrow \infty$. Indeed, when $u_{max} \rightarrow \infty$, the saturation control solution behaves as an unconstrained active solution with a very small penalty on the control force, i.e. the vibration of the structure is highly reduced at the expense of very large control forces. Finally, Figure 5 (c) shows the time history plots of the control forces for different values of u_{max} near resonance ($\Omega = 0.95$).

5.2 System with stroke stops

The system subjected to stroke stops is now worth investigating. The control limit is handled following the approach described in Section 3, which is based on a state depending penalty on the relative velocity $\dot{x_2}$ between the mass and the structure. The weight matrix adopted in the SDRE is given by equation (6) with $N = 8$ and $R(x) = 2 \cdot 10^{-6}$. The value $N = 8$ has been chosen after performing several attempts. Generally speaking, a greater N entails an improved control of the relative displacement between the mass and the structure. However, as N is increased, the SDRE tends to become ill conditioned and the controller might be unable to calculate the feedback. Clearly, an optimum value of N must be sought.

Figure 6 shows the $frcs$ of the uncontrolled, the passively controlled, the ideally controlled and the constrained systems. The results emphasize that, similarly to the saturation case, the performance of the system is intermediate between the ideal and the passive cases, at least for the considered constraint severity. It is also worth noting that, as clearly shown in Figure 6 (b), the constrained solution effectively allows to prevent x_2 from exceeding the limit y_{max} although, in the most demanding cases, small penetrations in the forbidden region might arise. This circumstance clearly entails that the value y_{max} must be chosen in such a way to be safely below the physical limit.

The effectiveness of the enhanced nonlinear damping approach, described in Section 4, is also worth investigating. On this respect, the results presented in Figure 7 show that such an approach (assuming $\kappa_0 = 0.8$, $\kappa_1 = 0.9$ and $g_{d1} = 20 \cdot c_a$) allows to slightly improve the control performance of the system at the expense of a lower effectiveness in preventing x_2 from approaching the limit. To better understand this mechanism, the resonant solution ($\Omega = 1.0$) is investigated in Figure 8. Particularly, Figure 8 (a) shows the periodic attractors of the ideal case and the constrained cases, while Figure 8 (b) shows the time histories of the control forces. These results clearly show that the reduction of x_2 with respect to the ideal active control case is obtained at the expense of a significant increment of control force. On this respect, it is also worth noting that the enhanced damping requires much larger control forces than the approach based on equation (6). Indeed, the enhanced damping produces large peaks of control force which, for the sake of clarity, are even out of the scale of Figure 8 (b). This, in fact, makes the approach based on equation (6) preferable for technical applications.

5.3 System with both actuator saturation and stroke stops

The results presented so far essentially show that the SDRE method allows to effectively handle force saturations and stroke stops, separately. As discussed in Section 3, it is theoretically possible to include physical constraints and actuator saturation in the SDRE, at the same time. However, controlling the relative displacement between the mass and the structure may require large control forces, with the obvious consequence that the two classes of control limitations are in competition with each other. In the presented case, assuming a force limit $u_{max} = 250\ N$ required to choose $y_{max} = 1.8\ m$ as the minimum possible constraint that did not cause numerical problems. The results of this case are presented in Figure 9 and show that the system is capable of reducing x_2, with respect to the unconstrained case, when force saturation does not occur. Indeed, in the region of the primary resonance, x_2 is reduced at the expense of an increment of the control force with respect to the unconstrained case. It is also worth mentioning that, assuming $y_{max} = 1.8\ m$ in the fully constrained solution, reflects on maximum values of x_2 appreciably below y_{max} (approximately $1.5\ m$). A similar profitable behavior is quite the opposite than the one observed in the case without force saturation.

6 CONCLUSIONS

The results presented in this paper demonstrate that control limits, which are inherent of real systems, can be directly considered in the nonlinear regulator design through the standard tool of the state dependent Riccati equation. This permits to avoid, for instance, an emergency shut down of the system which might be necessary in order to save the control devices when limits are exceeded. The feasibility of the proposed approach is here demonstrated by application to the problem of a single degree of freedom structure equipped with an AMD and subjected to a sinusoidal base acceleration. However, the generalization of the proposed approach to the case of a multi-degrees-of-freedom system with limited access to the state is straightforward.

Numerical simulations results demonstrate that state variable constraints and actuator saturations can be effectively handled when not contemporary present. Generally speaking, as the severity of the force limit is increased, the performance of the system reduces up to that of the passive case.

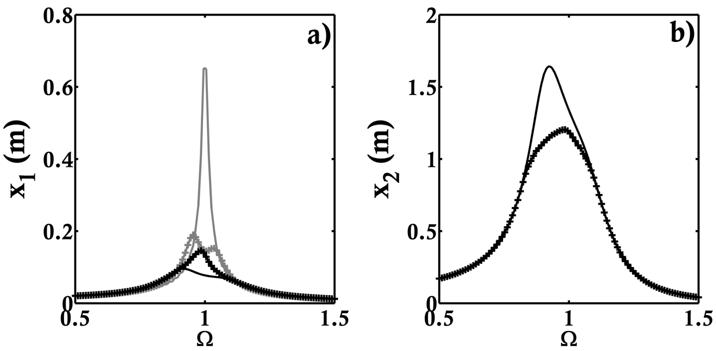

Figure 6: $frcs$ of the system (uncontrolled: grey solid; ideal actively controlled: black solid; passively controlled: grey crossed; actively controlled with limit stops: black crossed).

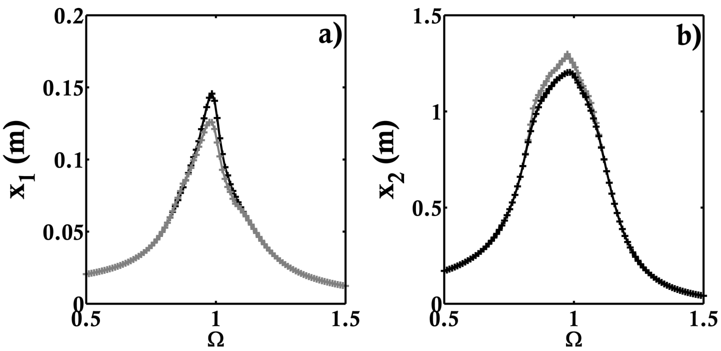

Figure 7: $frcs$ of the system (actively controlled with limit stops: black crossed; actively controlled with limit stops and enhanced damping: grey crossed).

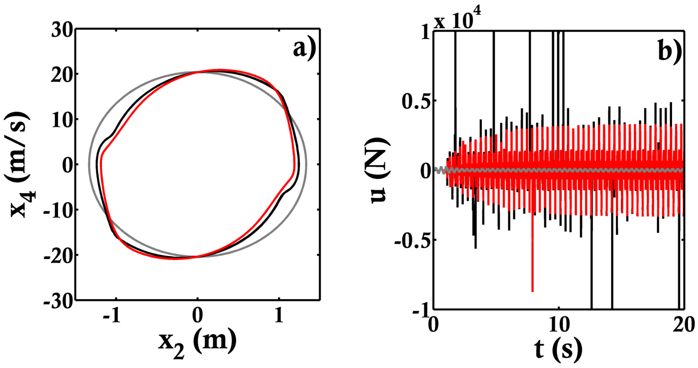

Figure 8: Periodic attractors of resonant solutions ($\Omega = 1$) (ideal actively controlled: grey solid, actively controlled with limit stops: red solid, actively controlled with limit stops and enhanced damping: black solid): (a). Time histories of control forces u ($\Omega = 1$) (ideal actively controlled: grey solid; actively controlled with limit stops: red solid; actively controlled with limit stops and enhanced damping: black solid): (b).

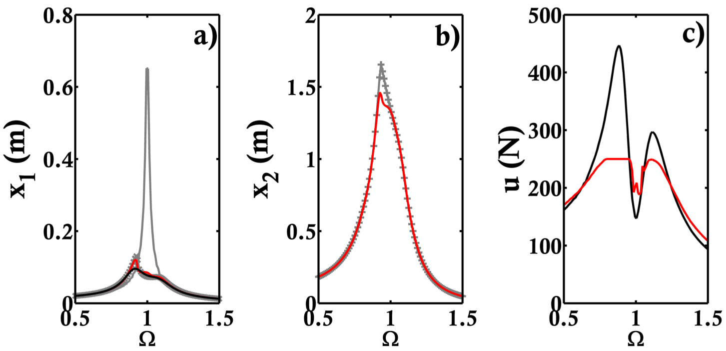

Figure 9: $frcs$ of the system (uncontrolled: grey solid; ideal actively controlled: black solid; actively control with actuator saturation: grey crossed; actively controlled with both limit stops and force saturation: red solid).

Similarly, the effectiveness of the system is deteriorated as the state variable constraint becomes more penalizing. The numerical results also confirm, to some extent, the possibility of handling the contemporary presence of force saturation and state variable constraints. In this case, however, force saturation is seen to strongly limit the state variable control.

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