Bifurcation and stability of a two d.o.f. system under simultaneous parametric, external and self-excitation

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SUMMARY. In this paper the analysis of self-excited structures under turbulent wind, taking into account the possible occurrence of multiple Hopf bifurcations, is carried out. In particular, a two d.o.f. nonlinear system, describing the dynamics of two poles, exposed to turbulent wind flow and linked by a strongly nonlinear viscous device, is considered. The stationary wind is responsible of self-excitation, while the turbulent part provides both parametric and external excitations. Thus the simultaneous presence of those excitations is taken into account, in a specific resonance condition. The periodic and quasi-periodic solutions are studied after the application of a perturbation scheme and the possible interaction between the two different d.o.f. is highlighted.

1 INTRODUCTION

Nonlinear dynamical systems can experience a variety of bifurcation and instability phenomena, which can be related to different kind of excitations. Multi-parameter families of self-excited autonomous systems, under a suitable combination of parameters, undergo, for instance, multiple Hopf bifurcations, in which several degrees of freedom are involved in the post-critical dynamics. Multi-modal responses can also been triggered by internal resonances occurring between unstable and stable modes. This is the case, for example, of internally-resonant cables subjected to steady wind [1] where, in addition to uni-modal steady solutions, multi-modal stable oscillations appear. On the other hand, when dealing with non-autonomous systems, the parametric and external excitations also lead to interesting behaviors, such as, for example, Neimark bifurcations, fold bifurcations and jump phenomena [2].

Depending on the nature of the loads, the three kinds of excitation can interact, as in the case of structures exposed to turbulent wind. In particular, the steady part of the flow is responsible for the self-excitation, while the unsteady flow brings on parametric and external forces.

In the framework of the cable dynamics, a first attempt of analysis of the interaction between self and external excitations was done by Paolone [3], where the oscillations of a cable in a turbulent flow were investigated through a one d.o.f. system. It was shown the existence of Neimark bifurcations from the stationary mono-periodic solution, conveying to stable quasi-periodic oscillations.

In this framework, Abdel-Rohman [4] considered a one d.o.f. self-excited system, descending from a Galerkin projection of a continuous model, to study the galloping phenomena of tall cantilever structures. There, the Multiple Scales Method (MSM) was used to analyze the response of the system to a mono-frequent unsteady wind flow, bearing the simultaneous presence of the self, parametric and external excitations, in case of primary and secondary resonances. It was shown that the unsteady component can cause a significant decrease in the critical wind speed at which galloping occurs; moreover, the contribution of the unsteady component is less prominent at high wind velocities, where the amplitude of galloping oscillations is very similar to the case of steady wind flow.

This paper aims to extend the analysis of self-excited structures under turbulent wind, taking into account the possible occurrence of multiple Hopf bifurcations and, more generally, the interaction of the different degrees of freedom. To this end, a two d.o.f. nonlinear system, under the simultaneous presence of the self, parametric and external excitations, is considered. That is figured to describe the dynamical behavior of a structure constituted by two poles, linked by a strongly nonlinear viscous device, subjected to wind flow. The combined effect of the fluctuating component of wind and the (nonlinear) motion of the structure leads to the appearance of time-varying coefficients (parametric excitation) and known terms (external excitation).

The Multiple Scale Method [2] is applied, considering a specific resonance condition. A set of coupled Amplitude Modulation Equations (AME) is obtained, describing the slow dynamics of the system in terms of amplitudes and phase-differences. The equilibrium solutions, representing steady oscillations, are analyzed, although not exhaustively, in the space of the bifurcation parameters. Their stability is discussed, highlighting the influence on the response of the three components of the excitations in different regions of the parameter space.

2 THE MODEL

Two slender clamped-clamped poles of non-circular cross-sections, linked on the mid-span by a strongly nonlinear viscous device, are opened to unsteady wind. The two sub-structures lie in a plane, the wind blows orthogonally to (see Fig. 1). The out-of-plane stiffness of the poles is assumed much larger than the one in the in-plane direction.

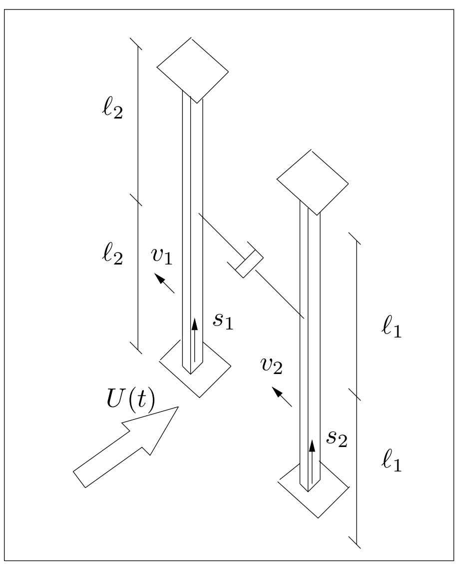

Figure 1: The two poles opened to the wind

The structure is symmetric with respect to the axis passing through the mid-span of the poles, therefore half structure is considered (by accounting for half of the mechanical characteristics of the device). If $\{v_1(s_1,t), v_2(s_1,t)\}$ are the in-plane transverse displacements of the two poles, respectively, where s_1, s_2 are the local abscissas and t the time, the approximate continuous nonlinear model of the system is the following (see [5]):

$$
\begin{cases}
\rho_1 \ddot{v}_1 + EI_1 v_1'''' + \dfrac{EA_1}{2\ell_1} v_1'' \displaystyle\int_0^{\ell_1} v_1'^2(\zeta) d\zeta = b_1 \\
\rho_2 \ddot{v}_2 + EI_1 v_2'''' + \dfrac{EA_2}{2\ell_2} v_2'' \displaystyle\int_0^{\ell_2} v_2'^2(\zeta) d\zeta = b_2
\end{cases}
\tag{1}
$$

where ρ_i the mass density, EI_i the bending stiffness, EA_i the axial stiffness, ℓ_i the length and b_i the aerodynamic forces, the prime denoting differentiation with respect to s_1 or s_2 and the dot with respect to the time. The boundary conditions are

$$
\begin{cases}
v_1(0) = v_2(0) = 0 \\
v_1'(0) = v_2'(0) = 0 \\
v_1''(\ell_1) = v_2''(\ell_2) = 0 \\
EI_1 v_1'''(\ell_1) = -EI_2 v_2'''(\ell_2) = f(\dot{v}_1(\ell_1) - \dot{v}_2(\ell_2))
\end{cases}
\tag{2}
$$

where f represents the force exerted by the viscous device. The viscous device is figured to get a constitutive law of type $f(\Delta) = \kappa_1 \dot{\Delta} + \kappa_3 \dot{\Delta}^3$, where $\Delta := \dot{v}_1(\ell_1) - \dot{v}_2(\ell_2)$, with $\kappa_3 \gg \kappa_1$. It provides coupling terms between the two poles in the boundary conditions.

The wind applies on the i-th pole ($i = 1, 2$) a lift force, lying on the plane of the structure, of type $b_i = \bar{b}_i(U(t)) + c_{i1}(U(t))\dot{v}_i + c_{i2}(U(t))\dot{v}_i^2 + c_{i3}(U(t))\dot{v}_i^3$, where c_{ij} are the linear ($j = 1$), quadratic ($j = 2$) and cubic ($i = 3$) aerodynamic coefficients, respectively, which depend on the relative wind velocity $U(t)$ (and on its powers) and on the shape of the cross-section of the pole. The wind velocity can be decomposed as $U(t) = \bar{U} + u(t)$, where $\bar{U}$ is a constant (average) part, representing the steady component, and $u(t)$ is a periodically time-dependent part, representing the turbulence. Therefore, the aerodynamic forces provide, by means of its steady part, constant-coefficient velocity-dependent terms, which can be responsible for galloping; moreover they furnish periodic time-dependent terms, responsible for both external and parametric excitations (see [4]). In this work, the turbulent part is just considered in its fundamental frequency, as $u(t) = \sin(\Omega t)$.

The system (1)-(2) is discretized via the Galerkin method, by assuming one in-plane mode for each poles, evaluated in absence of the wind and viscous device. The two resulting second-order, non-homogeneous, time-periodic, ordinary differential equations are coupled in the linear and nonlinear velocities, containing quadratic (in the velocity) and cubic (in the velocity and displacement) nonlinearity:

$$
\begin{cases}
\ddot{x} - (\mu + b_0 u(t))\dot{x} + \omega_1^2 x + (b_{11} + b_{12} u(t))\dot{x}^2 + (b_{21} + b_{22} u(t))\dot{x}^3 + \\
\qquad + b_3(\dot{x} - \dot{y}) + b_4(\dot{x} - \dot{y})^3 + c_1 x^3 = \eta_1 u(t) \\
\ddot{y} - (\nu + b_5 u(t))\dot{y} + \omega_2^2 y + (b_{61} + b_{62} u(t))\dot{y}^2 + (b_{71} + b_{72} u(t))\dot{y}^3 + \\
\qquad - b_3(\dot{x} - \dot{y}) - b_4(\dot{x} - \dot{y})^3 + c_2 y^3 = \eta_2 u(t)
\end{cases}
\tag{3}
$$

where $x(t), y(t)$, are the unknown amplitudes of the tips of the two poles, respectively. The coefficients μ and ν are the aerodynamic plus structural modal damping of the poles, respectively: $\mu = 2\xi_{s1}\omega_1 + \xi_{a1}\bar{U}$, $\nu = 2\xi_{s2}\omega_2 + \xi_{a2}\bar{U}$, where ξ_{si} are the structural modal damping ratios and ξ_{ai} the aerodynamic damping ratios.

The two sub-structures independently undergo (at $\mu = \mu_c$ and $\nu = \nu_c$ respectively) a Hopf bifurcation when the turbolent wind is absent and the small linear coupling due to the viscous device is neglected. The quantities μ and ν are taken as bifurcation parameters (a further bifurcation parameter σ, accounting for resonance detuning, will be introduced later). They are linear combinations of two physical parameters, e.g. the wind velocity $\bar{U}$ and one of the two structural damping coefficients ξ_{si}, the remaing parameters being kept fixed. Thus, by varying μ and ν, a family of physical systems is spanned, in which each member posseses different mechanical properties and undergoes different wind loads. The terms b_i, c_i, η_i are auxiliary (fixed) parameters. The natural frequencies ω_1, ω_2 are assumed to be incommensurable, representing a case of different stiffness of the two poles.

3 THE MULTIPLE SCALES

To apply the Multiple Scales Method, a dimensionless small parameter ϵ is introduced and the dependent variables $\{x, y\}$ are expanded as

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \epsilon^{\frac{1}{2}} \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} + \epsilon^{\frac{3}{2}} \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} \tag{4}$$

The linear damping terms are assumed small; the coefficients of the external and parametric excitation, as well as the nonlinearity, are ordered so that they appear at the last order perturbation equations. Therefore the coefficients $\{\mu, \nu, b_0, b_3, b_5\}$ are of order ϵ, the coefficients of $\{\eta_1, \eta_2\}$ of order $\epsilon^{\frac{3}{2}}$ and the coefficients $\{b_{11}, b_{12}, b_{61}, b_{62}\}$ of order $\epsilon^{\frac{1}{2}}$. The other coefficients are of order 1. After introducing two independent time scales $t_0 := t$ and $t_1 := \epsilon t$, the derivative with respect to the time assumes the expression $d/dt = d_0 + \epsilon d_1$, where $d_i := \partial/\partial t_i$. As a consequence, the perturbation equations, multiplied by $\epsilon^{\frac{1}{2}}$, read:

$$\begin{aligned}
\mathcal{O}(\epsilon) &: \begin{cases} d_0^2 x_1 + \omega_1^2 x_1 = 0 \\ d_0^2 y_1 + \omega_2^2 y_1 = 0 \end{cases} \\
\mathcal{O}(\epsilon^3) &: \begin{cases} d_0^2 x_2 + \omega_1^2 x_2 = & - b_{12}\sin(\Omega t_0) d_0 x_1^2 + b_0 \sin(\Omega t_0) d_0 x_1 - b_{22} \sin(\Omega t_0) d_0 x_1^3 + \\ & + 3b_4 d_0 x_1^2 d_0 y_1 - 3 b_4 d_0 x_1 d_0 y_1^2 - b_4 d_0 x_1^3 - b_{21} d_0 x_1^3 - b_{11} d_0 x_1^2 + \\ & - b_3 d_0 x_1 + b_3 d_0 y_1 + b_4 d_0 y_1^3 + \mu d_0 x_1 - 2 d_0 d_1 x_1 - c_1 x_1^3 + \eta_1 \sin(\Omega t_0) \\ d_0^2 y_2 + \omega_2^2 y_2 = & - b_{62} \sin(\Omega t_0) d_0 y_1^2 - b_{72} \sin(\Omega t_0) d_0 y_1^3 - 3 b_4 d_0 x_1^2 d_0 y_1 + \\ & + 3 b_4 d_0 x_1 d_0 y_1^2 + b_4 d_0 x_1^3 + b_3 d_0 x_1 - b_3 d_0 y_1 - b_4 d_0 y_1^3 + \\ & - b_{61} d_0 y_1^2 - b_{71} d_0 y_1^3 + \nu d_0 y_1 - 2 d_0 d_1 y_1 - c_2 y_1^3 + \eta_2 \sin(\Omega t_0) \end{cases}
\end{aligned} \tag{5}$$

Equation $(5)_1$ admits the following solution:

$$\begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} = \begin{Bmatrix} A_1(t_1)\exp(i\omega_1 t_0) \\ A_2(t_1)\exp(i\omega_2 t_0) \end{Bmatrix} + cc \tag{6}$$

where cc denotes the complex conjugate, i is the imaginary unit and A_1, A_2 are unknown complex amplitudes. By substituting Eq. (6) in Eq. $(5)_2$ and by zeroing the secular terms which arise in the right hand side, a set of differential equations in A_1, A_2 is obtained:

$$\begin{aligned} d_1 A_1 &= f_1(A_1, A_2) \\ d_1 A_2 &= f_2(A_1, A_2) \end{aligned} \tag{7}$$

Hence, by coming back to the true time t, Eq. (7) provides the Amplitude Modulation Equations (AME). In case of external resonance of type 1:1 with the first frequency, namely $\Omega = \omega_1 + \epsilon\sigma$ (σ is the detuning parameter), the AME read:

$$\begin{aligned} \dot{A}_1 =& \frac{\mu - b_3}{2} A_1 - \frac{\eta_1}{4\omega_1} e^{i\sigma t} + \frac{b_{12}\omega_1}{2} A_1 \bar{A}_1 e^{i\sigma t} + \frac{b_{12}\omega_1}{2} A_1^2 e^{-i\sigma t} + \beta_1 A_1^2 \bar{A}_1 - 3b_4\omega_2^2 A_1 A_2 \bar{A}_2 \\ \dot{A}_2 =& \frac{\nu - b_3}{2} A_2 - 3b_4\omega_1^2 A_1 A_2 \bar{A}_1 + \beta_2 A_2^2 \bar{A}_2 \end{aligned} \tag{8}$$

where $\beta_1 = -\frac{3}{2}\omega_1^2(b_{21} + b_4) + i\frac{3c_1}{2\omega_1}$ and $\beta_2 = -\frac{3}{2}\omega_2^2(b_4 + b_{71}) + i\frac{3c_2}{2\omega_2}$.

It is worth noticing how, among the parametric terms, just the quadratic ones are resonant when $\Omega \simeq \omega_1$; moreover the coupling between the two d.o.f. is due to the cubic nonlinearity of the damping device, while the linear part is responsible for a shift of the critical condition for galloping.

The polar form of the (8), obtained posing $A_1 := \dfrac{1}{2} a_1 e^{i\vartheta_1}$, $A_2 := \dfrac{1}{2} a_2 e^{i\vartheta_2}$ and $\varphi_1 := \sigma t - \vartheta_1$, and referred as Reduced Amplitude Modulation Equations (RAME), is:

$$\begin{aligned} \dot{a}_1 =& \frac{1}{2}(\mu - b_3) a_1 - \frac{3}{4} b_4 \omega_2^2 a_1 a_2^2 - \frac{3}{8}\omega_1^2 a_1^3 (b_4 + b_{21}) + \frac{3}{8} b_{12}\omega_1 a_1^2 \cos\varphi_1 - \frac{\eta_1 \cos\varphi_1}{2\omega_1} \\ \dot{a}_2 =& \frac{1}{2}(\nu - b_3) a_2 - \frac{3}{4} b_4 \omega_1^2 a_1^2 a_2 - \frac{3}{8}\omega_2^2 a_2^3 (b_4 + b_{71}) \\ a_1 \dot{\varphi}_1 =& \sigma a_1 - \frac{3c_1}{8\omega_1} a_1^3 - \frac{1}{8} b_{12}\omega_1 a_1^2 \sin\varphi_1 + \frac{\eta_1 \sin\varphi_1}{2\omega_1} \end{aligned} \tag{9}$$

3.1 *Fixed points*

Fixed points of Eq. (9), obtained posing $\dot{a}_1 = \dot{a}_2 = \dot{\varphi}_1 = 0$, represent stationary oscillations of the poles. Is this Subsection, analytical expressions of them are sought.

In case of absence of turbulence ($b_{12} = \eta_1 = 0$), the case of non-resonant double Hopf bifurcation is obtained [6]. Just Eq. $(9)_{1,2}$ are interesting, being the phase φ_1, as well as ϑ_2, a slave variable. In this case, besides the trivial solution $a_1 = a_2 = 0$, indicated as O, the self-excitation is responsible for galloping, and the classical mono-modal solutions occur. One of them, indicated as I, is the following: $\{a_{1e} = 2\sqrt{\frac{\mu - b_3}{\omega_1^2(b_{21}+b_4)}},\ a_{2e} = 0\}$ which occurs when $\mu \geq b_3$, $\forall\nu$. Its stability is ruled by the sign of the real part of the eigenvalues of the Jacobian matrix, that read:

$$\lambda_{1,2} = \frac{1}{16}\left(-3\left(5b_4 + 3b_{21}\right)\omega_1^2 a_{1e}^2 - 8b_3 + 4(\mu + \nu) \pm \sqrt{\left(3\left(b_4 + 3b_{21}\right)\omega_1^2 a_{1e}^2 - 4(\mu - \nu)\right)^2}\right) \tag{10}$$

A second mono-modal solution, indicated as II, is $\{a_{1e} = 0, a_{2e} = 2\sqrt{\frac{\nu - b_3}{\omega_1^2(b_{71}+b_4)}}\}$ and occurs when $\nu \geq b_3$, $\forall\mu$. Its stability is governed by the following eigenvalues of the Jacobian matrix:

$$\lambda_{1,2} = \frac{1}{16}\left(-3\left(5b_4 + 3b_{71}\right)\omega_2^2 a_{2e}^2 - 8b_3 + 4(\mu + \nu) \pm \sqrt{\left(3\left(b_4 + 3b_{71}\right)\omega_2^2 a_{2e}^2 + 4(\mu - \nu)\right)^2}\right) \tag{11}$$

A bi-modal solution, indicated as III, is

$$
\begin{aligned}
a_{1e} &= \left[- \frac{\left(\frac{\mu}{2} - \frac{b_3}{2}\right)\left(-\frac{3}{8} b_4 \omega_2^2 - \frac{3}{8} b_{71} \omega_2^2\right) + \frac{3}{4} b_4 \omega_2^2 \left(\frac{\nu}{2} - \frac{b_3}{2}\right)}{\left(-\frac{3}{8} b_4 \omega_1^2 - \frac{3}{8} b_{21} \omega_1^2\right)\left(-\frac{3}{8} b_4 \omega_2^2 - \frac{3}{8} b_{71} \omega_2^2\right) - \frac{9}{16} b_4^2 \omega_1^2 \omega_2^2} \right]^{\frac{1}{2}} \\
a_{2e} &= \left[- \frac{\frac{3}{4} b_4 \omega_1^2 \left(\frac{\mu}{2} - \frac{b_3}{2}\right) + \left(\frac{\nu}{2} - \frac{b_3}{2}\right)\left(-\frac{3}{8} b_4 \omega_1^2 - \frac{3}{8} b_{21} \omega_1^2\right)}{\left(-\frac{3}{8} b_4 \omega_1^2 - \frac{3}{8} b_{21} \omega_1^2\right)\left(-\frac{3}{8} b_4 \omega_2^2 - \frac{3}{8} b_{71} \omega_2^2\right) - \frac{9}{16} b_4^2 \omega_1^2 \omega_2^2} \right]^{\frac{1}{2}}
\end{aligned}
\tag{12}
$$

Both solutions I and II indicate periodic oscillations of one poles, while the other one is at rest, at this order. The solution III represents quasi-periodic oscillations of the two poles.

In case of presence of turbulence, two nonlinear algebraic equations can be drawn in the following way: $\cos\varphi_1$ is obtained by zeroing the right-hand side of Eq. $(9)_1$, $\sin\varphi_1$ is obtained by zeroing the right-hand side of Eq. $(9)_3$, and then the variable φ_1 is condensed by the relation $\cos^2\varphi_1 + \sin^2\varphi_1 = 1$. The resulting equation, where only a_1 and a_2 appear and valid when η_1 or b_{12} is different fro zero, is the following:

$$
\frac{\omega_1^2 \left(6 b_4 \omega_2^2 a_1 a_2^2 + 3 b_4 \omega_1^2 a_1^3 + 3 b_{21} \omega_1^2 a_1^3 + 4 b_3 a_1 - 4 \mu a_1\right)^2}{(4\eta_1 - 3 b_{12} \omega_1^2 a_1^2)^{\,2}} + \frac{\left(8 \sigma \omega_1 a_1 - 3 c_1 a_1^3\right)^2}{\left(b_{12} \omega_1^2 a_1^2 - 4\eta_1\right)^2} = 1 \tag{13}
$$

It is sided by the equation obtained taking the right-hand-side of Eq. $(9)_2$ equal to zero. Solutions of that system are sought by numerical procedures and are discussed in the following Section.

4 NUMERICAL RESULTS

4.1 *Absence of turbulent wind*

In case of absence of turbulence, the behavior chart of the system is shown in Fig. 2, when $\omega_1 = 1, \omega_2 = 1.7, b_4 = -\frac{1}{2}, b_{21} = 1, b_{71} = 1, b_3 = 0.1$. The solution O exists in the whole plane $\{\mu, \nu\}$. The lines indicated as I and II are the boundary limits of the mono-modal solutions I and II, respectively. The line indicated as III represents the boundary limit of the solution III. The line 1 is the path of the section reported in Fig. 3.

Figure 3_1 shows the variable a_1, while Fig. 3_2 shows the amplitude a_2 varying μ, along the section 1. In particular, the trivial solution O loses stability at the bifurcation point B_1, where the stable galloping solution I appears. When μ is further increased, a secondary bifurcation occurs (B_2), the solution I loses stability and the bi-modal, unstable, branch III appears.

4.2 *Presence of turbulent wind*

The turbulent wind produces, in case of resonance $\Omega \simeq \omega_1$, two kind of excitations: external, of amplitude η_1, and parametric, of amplitude b_{12}. For the same values of the auxiliary parameters assumed in the case of absence of turbulent wind and for $\nu = 0$, $c_1 = 1$, the effects on the response are evaluated here.

The case where the external excitation prevails against the parametric one is shown in Fig. 4. There, a mono-modal branch of stable equilibria in a_1 (referred as I) is present; it corresponds to a branch of periodic oscillations in the variable $x(t)$, of amplitude a_1. When the parameter μ is increased, a divergence occurs at B_1, corresponding to a Hopf bifurcation for $x(t)$. At B_1, a bi-modal branch of equilibria (referred as III) starts and the branch I becomes unstable. The bi-modal solution is initially stable, but it loses stability on a secondary bifurcation referred as B_3. Then,

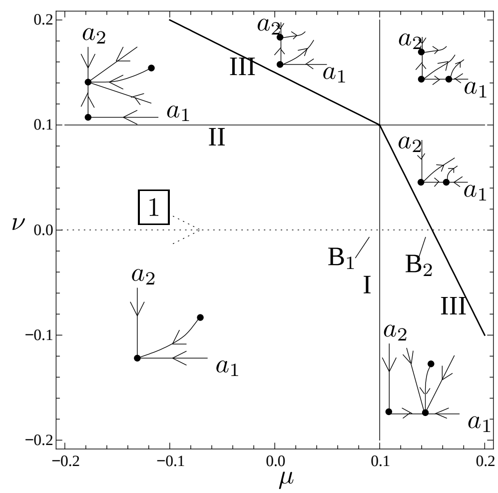

Figure 2: Stability diagram in absence of turbulent wind

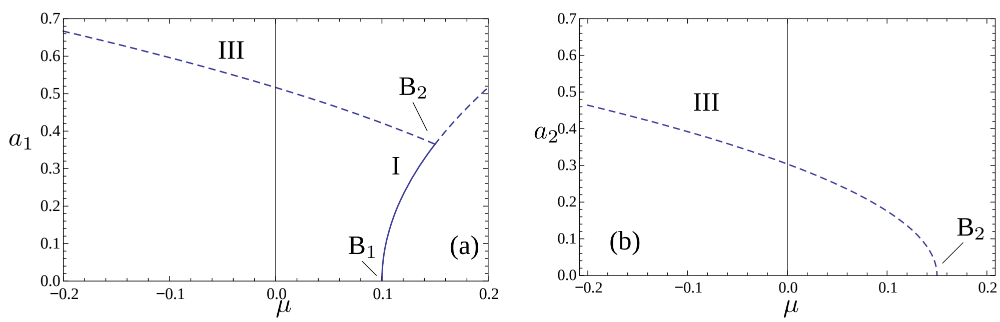

Figure 3: Equilibrium branches in absence of turbulent wind for $\nu = 0$ (section 1). (a) amplitude a_1; (b) amplitude a_2.

increasing μ, the branch III dies on the branch I when the divergence point B_2 is reached. A Hopf bifurcation occurs at B_4 on the branch I, but any periodic solutions branching off this point are not sought in this paper. It is worth noticing how the backbone of these two branches (I and III) are the corresponding branches obtained in absence of turbulence (dotted lines in Fig. 4). For the same values of Fig. 4, a section in the planes $\{\sigma, a_1\}$ and $\{\sigma, a_2\}$ is shown in Fig. 5 when $\mu = 0.06$.

When the parametric excitation is prevailing against the external one, the sections are shown in Figs. 6 and 7, for different values of σ. When $\sigma = 0$ (see Fig. 6), the equilibrium branch I exist for all the values of μ, and near the value $\mu = 0.1$ a loop and change in stability occur. The backbone curve is again the corresponding branch I obtained in absence of turbulence (dotted line). When $\sigma = 0.06$ (see Fig. 7), the behavior is more complicated, since the branch I is divided into two parts, one of them is an island. On the lower part, instability occurs when the Hopf bifurcation point B_4 is reached (any periodic soultions branching off this point are not sought in this paper). On the island, the two divergence points B_1 and B_2 represent the limits of existence on the bi-modal branch III.

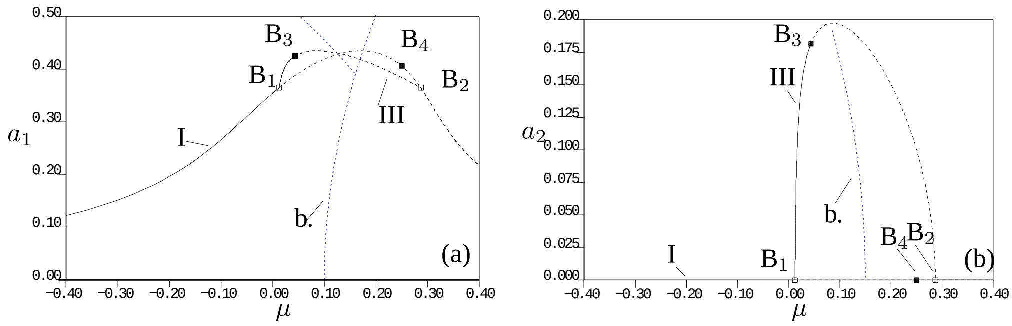

Figure 4: Equilibrium branches for $\eta_1 = 0.06$, $b_{12} = 0.005$, $\sigma = 0$. Continuous line: stable; dashed line: unstable; dotted line: backbone (absence of turbulent wind). (a) amplitude a_1; (b) amplitude a_2.

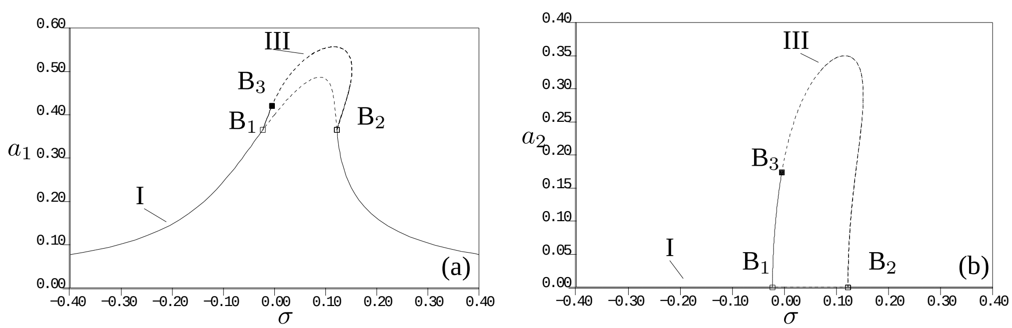

Figure 5: Equilibrium branches for $\eta_1 = 0.06$, $b_{12} = 0.005$, $\mu = 0.06$. (a) amplitude a_1; (b) amplitude a_2.

The backbone is again highlighted. In case of $\mu = 0.099$, a section in the planes $\{\sigma, a_1\}$ and $\{\sigma, a_2\}$ is shown in Fig. 8, where the two parts of branch I and the bi-modal branch III are evident.

5 CONCLUSIONS

In this paper a 2 d.o.f. nonlinear dynamical system, drawn by a Galerkin projection of a continuous structure, constituted by two poles and a viscous nonlinear device, opened to turbulent wind, is considered. The system is subjected to simultaneous self-excitation, external and parametric excitations, the first due to the steady part of the aerodynamic force, the last two due to the turbulent part of the wind. The Multiple Scales are used to obtain Amplitude Modulation Equations, under the 1:1 resonance condition between the fundamental component of the turbulent wind and the first d.o.f., and in absence of internal resonance. For fixed values of the auxiliary parameters, the dynamical behavior of the system is studied and comparison among cases of preponderance of different kind of excitations are analyzed, in terms of equilibrium branches of the amplitudes. In particular, when the turbulence is present, both mono-modal and bi-modal branches occur, and they are drawn on the backbones constituted by the corresponding branches obtained without turbulence. The parametric

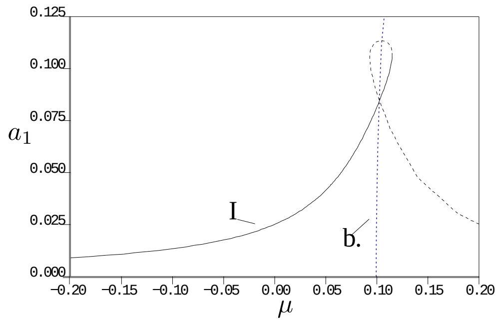

Figure 6: Equilibrium branches for $\eta_1 = 0.003$, $b_{12} = 0.51$, $\sigma = 0$. Continuous line: stable; dashed line: unstable; dotted line: backbone (absence of turbulent wind). (a) amplitude a_1; (b) amplitude a_2.

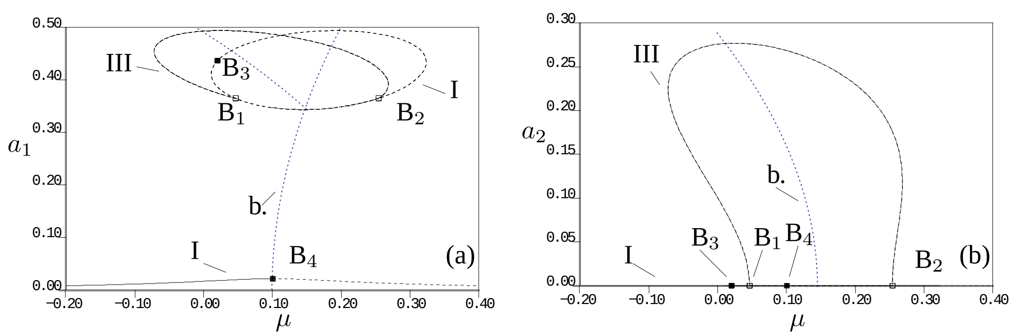

Figure 7: Equilibrium branches for $\eta_1 = 0.003$, $b_{12} = 0.51$, $\sigma = 0.06$. Continuous line: stable; dashed line: unstable; dotted line: backbone (absence of turbulent wind). (a) amplitude a_1; (b) amplitude a_2.

excitation is also responsible for the separation of the mono-modal branch in two parts, one of them is an island.

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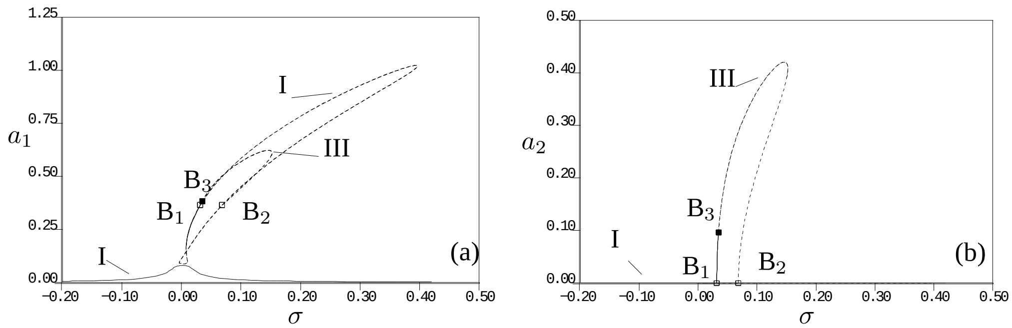

Figure 8: Equilibrium branches for $\eta_1 = 0.003$, $b_{12} = 0.51$, $\mu = 0.099$. Continuous line: stable; dashed line: unstable. (a) amplitude a_1; (b) amplitude a_2.

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