Wave Propagation In Axisymmetric Structures From Finite Element Analysis

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SUMMARY. This paper describes a wave and finite element (WFE) method for the numerical prediction of wave propagation in axisymmetric structures. A small segment of the structure is modelled using conventional finite element methods, commonly using a commercial package, and the mass and stiffness matrices found. This typically involves a single shell element or, especially for laminate structures, a stack of solid elements meshed through the thickness. Internal fluid can be included straightforwardly. Periodicity conditions are then applied. An eigenvalue problem results, the solutions of which yield the dispersion relations and the wave modes. The circumferential order of the wave can be specified in order to define the phase change a wave experiences as it propagates across the element in the circumferential direction. The resulting eigenproblem then relates the wavenumber and frequency. The WFE method is described and illustrated by application to cylinders in vacuo and filled with fluid and curved panels. These include various isotropic and laminated constructions. Complex dispersion curves and wave modes are presented and discussed. The method is seen to be simple in application and provides accurate results with very little computational cost.

1 INTRODUCTION

Axisymmetric structures are present as a component in a number of systems, both engineering and bio–mechanical. Examples include pipes, human circulation, acoustic ducts, aircraft fuselages and aerospace structures to name a few. Knowledge of the wave propagation characteristics in these kind of structures is of importance in many applications in science and industry. Typical applications include, amongst others, the transmission of structure–borne sound, statistical energy analysis, shock response and non–destructive testing. Theoretical understanding of wave propagation also provides the background necessary for the utilisation and a better implementation of many techniques. In particular knowledge of high frequency wave propagation is fundamental in ultrasonic and acoustic emission techniques. More generally the wave approach is valuable in any case for which mid–high frequencies are involved, that is when the size of the structure is large compared to the wavelength and the computational cost of Finite Element (FE) analysis of the structure as a whole becomes impractically large. The primary characteristics of these waves are the dispersion relationship, that is the relationship between frequency and wave heading to the wavenumber, and wave modes, which are related to the cross–sectional displacements and internal forces.

Analysis of wave motion typically involves assumptions and approximations concerning stress, strain and displacement states of the structure [1]. In simple cases, such as isotropic thin cylinders, analytical expressions for the dispersion curves can be found [2] but analysis of wave motion generally requires the development of a mathematical model that is difficult at best, especially for complex constructions. Moreover, the accuracy of the solutions depends upon whether the assumptions made are valid. In particular at high frequency the underlying assumptions and approximations concerning the stress–strain distribution in the solid generally break down. As the frequency increases and the wavelength starts to be comparable to the cross-section dimensions more refined models are required, e.g. [3, 4, 5]. Hence the analysis becomes increasingly complicated, while the computational cost of standard numerical approaches, such as the Finite Element Method, becomes impractically large. Difficulties in analytical approaches and limitations of standard FE formulations have motivated the development of numerical approaches to predict the dispersion properties, wave modes and group and phase velocities in both structure and acoustic fluid, e.g. [6, 8, 7, 9].

The main aim of this work is to describe a wave finite element (WFE) method for axisymmetric structures *in vacuo* and filled with fluid. Part of this work was already presented by the same authors in [10]. The method is an extension to axisymmetric structures of the WFE approach to 2–dimensional structures of Mace and Manconi [11]. Compared to similar methods, the WFE method proposes a systematic and straightforward approach which combines the theory of wave propagation in periodic structures with conventional FE analysis. As such, it is an application of FE analysis to periodic structures, although in the cases considered the structures are homogeneous and hence the periodicity of arbitrary length. The structures of interest in the present work are homogeneous in the axial and circumferential directions, but their properties can vary in an arbitrary manner in the radial direction. Examples include isotropic, laminated and sandwich cylinders, either *in vacuo* or filled with fluid, and also the general case of curved panels. The method requires the analysis of just a small segment of the structure. The segment is discretised using conventional FEs: just a single rectangular shell finite element or a stack of solid elements meshed through the cross–section. This allows a very substantial reduction in the number of DOFs involved in the computation, particulary when not only the structure but also the fluid is considered. The mass and stiffness matrices of the FE model are then typically obtained using commercial FE packages. These matrices are subsequently post–processed using periodicity conditions to obtain an eigenproblem whose solutions provide the dispersion curves and the wave modes. Hence, making use of a wave description, the approach enables the evaluation of the wave characteristics up to high frequencies with great accuracy. The form of the eigenproblem depends on the nature of the problem at hand. In particular for wave propagation in a closed cylinder, where the wavenumber around the circumference can only take certain discrete values, the eigenproblem is a quadratic eigenproblem. The general approach proposed is in contrast to the spectral finite element (SFE) method (e.g. [8, 9]) in which new elements, with a space-harmonic displacement along the axis of the waveguide, must be derived on a case-by-case basis. The simple algebra involved in the method and the possibility to use commercial FE packages makes the WFE technique also suitable for industrial applications.

2 WFE FORMULATION FOR AXISYMMETRIC STRUCTURES

A schematic representation of axisymmetric structures is shown in figure 1, where y, r and α are the cylindrical coordinates, R is the mean radius and h is the thickness. The structures are homogenous in the y and α directions but their proprieties can vary in an arbitrary manner in the r direction. A time harmonic disturbance at a frequency ω is assumed to propagate through the structure with a helical pattern so that

$$w(r, \alpha, y, t) = W(r)e^{i(\omega t - k_\alpha \alpha - k_y y)}. \tag{1}$$

In equation (1), $W(r)$ is the complex wave amplitude while k_α and k_y are the components of the wavenumber k in the circumferential and axial directions. For real wavenumbers $k_\alpha = k\cos\theta$ and $k_y = k\sin\theta$ where θ is the direction in which the wave propagates. Exploiting the periodicity of

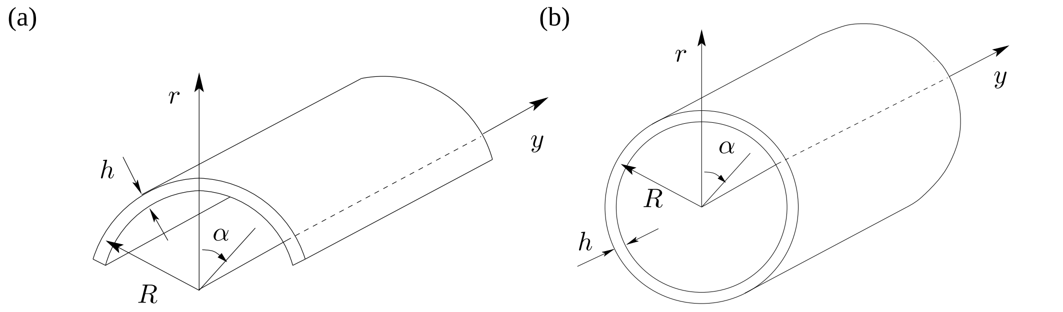

Figure 1: Examples of axisymmetric structures: (a) curved panel, (b) cylinder, *in vacuo* or filled with fluid.

the structure, a prismatic segment of length L_y subtending a small angle L_α is taken. Once the 'period' of the system is defined, it is meshed in such a way that it has an identical distributions of nodes on both the α and y faces. If the periodic lengths L_α and L_y are small enough, the simplest way to discretise the structural part of the segment is obtained using either just one shell element or generally using a stack of solid elements meshed through the cross section as shown in figure 2. If fluid is present 8–noded acoustic finite element are used to mesh the fluid as shown in figure 2(b).

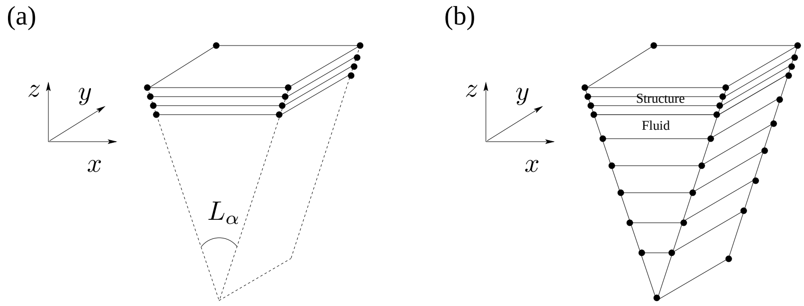

Figure 2: FE mesh of a small rectangular prismatic segment of the axisymmetric structure: (a) *in vacuo* and (b) with fluid.

The standard FE equations of motion for the segment, assuming time harmonic behaviour, are

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{q} = \mathbf{f}. \tag{2}$$

When the fluid is considered, the mass and stiffness matrix in equation (2) are those obtained from FE formulation for acoustic fluid–structure interaction. They can be found in a number of texts on the FE method, e.g. [12].

In the present analysis the curved segment in figure 2 has been modelled by FEA as being piece-wise flat. Hence, in order to model the curvature, a transformation matrix $\mathbf{R}$ should be defined. The mass and stiffness matrices in the global reference system for the curved segment then become

$$\begin{aligned} \mathbf{M} &= \overline{\mathbf{R}}^T \mathbf{M}_{LOC} \overline{\mathbf{R}}; \\ \mathbf{K} &= \overline{\mathbf{R}}^T \mathbf{K}_{LOC} \overline{\mathbf{R}}, \end{aligned} \tag{3}$$

where $\mathbf{M}_{LOC}$ and $\mathbf{K}_{LOC}$ are the mass and stiffness matrices in local coordinates, that is the mass and stiffness matrices of the flat FE model.

Following the WFE procedure for 2–dimensional structures [11], the degrees of freedom (DOFs) of the FE model are arranged in a column vector **q** as

$$\mathbf{q} = [\mathbf{q}_1^T \;\; \mathbf{q}_2^T \;\; \mathbf{q}_3^T \;\; \mathbf{q}_4^T]^T, \tag{4}$$

where the superscript T denotes the transpose and where $\mathbf{q}_j$ is the vector of the nodal DOFs of all the element nodes which lie on the jth corner of the segment as shown in figure 3. A similar expression is given for the nodal forces **f**

$$\mathbf{f} = [\mathbf{f}_1^T \;\; \mathbf{f}_2^T \;\; \mathbf{f}_3^T \;\; \mathbf{f}_4^T]^T. \tag{5}$$

The vectors **q** and **f** are then the concatenation of the nodal DOFs and forces. Internal and edge nodes can be included [11] but are not considered here. Since the structure is periodic in 2–dimensions of periodicity L_α and L_y, the free propagating wave in equation (1) can take the form of a Bloch wave [13]. The displacements **q** on each side of the periodic element are therefore related by

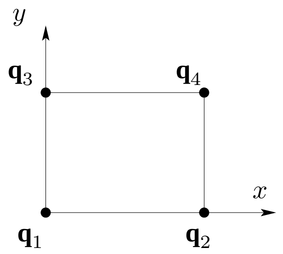

Figure 3: Node numbering.

$$\mathbf{q}_2 = \lambda_\alpha \mathbf{q}_1; \quad \mathbf{q}_3 = \lambda_y \mathbf{q}_1; \quad \mathbf{q}_4 = \lambda_\alpha \lambda_y \mathbf{q}_1, \tag{6}$$

where

$$\lambda_\alpha = e^{-i\mu_\alpha}; \;\; \lambda_y = e^{-i\mu_y} \tag{7}$$

and

$$\mu_\alpha = k_\alpha L_\alpha \text{ and } \mu_y = k_y L_y \tag{8}$$

are known as propagation constants. The nodal degrees of freedom are rearranged to give

$$\mathbf{q} = \Lambda_R \mathbf{q}_1, \tag{9}$$

where

$$\Lambda_R = [\mathbf{I} \quad \lambda_\alpha \mathbf{I} \quad \lambda_y \mathbf{I} \quad \lambda_\alpha \lambda_y \mathbf{I}]^T. \tag{10}$$

In the absence of external excitation, equilibrium at node 1 implies that the sum of the nodal forces of all the elements connected to node 1 is zero. Consequently

$$\Lambda_L \mathbf{f} = \mathbf{0}, \tag{11}$$

where

$$\Lambda_L = [\mathbf{I} \quad \lambda_\alpha^{-1} \mathbf{I} \quad \lambda_y^{-1} \mathbf{I} \quad (\lambda_\alpha \lambda_y)^{-1} \mathbf{I}]. \tag{12}$$

Substituting equation (9) in equation (2) and premultiplying both sides of equation (2) by Λ_L gives

$$\overline{\mathbf{D}}(\omega, \lambda_\alpha, \lambda_y)\mathbf{q}_1 = 0; \quad \overline{\mathbf{D}}(\omega, \lambda_\alpha, \lambda_y) = \overline{\mathbf{K}}(\mu_\alpha, \mu_y) - \omega^2 \overline{\mathbf{M}}(\mu_\alpha, \mu_y), \tag{13}$$

where

$$\overline{\mathbf{K}} = \Lambda_L \mathbf{K} \Lambda_R; \quad \overline{\mathbf{M}} = \Lambda_L \mathbf{M} \Lambda_R \tag{14}$$

are the reduced stiffness and mass matrices, i.e. the segment matrices projected onto the DOFs of node 1 under the assumption of disturbance propagation as in equation (1). If there are n DOFs per node, the nodal displacement and force vectors are $n \times 1$, the element mass and stiffness matrices are $4n \times 4n$ while the reduced matrices are $n \times n$.

Equations (13) gives different forms of the eigenproblem relating λ_α, λ_y and ω, whose solutions give FE estimates of the wave modes (eigenvectors) and dispersion relations for the continuous structure.

3 NUMERICAL EXAMPLES

In this section numerical examples are presented to illustrate the application of the WFE method. Damping is neglected. Results are obtained by post–processing FE models found using a commercial FE package.

3.1 *Laminated sandwich cylinder and laminated sandwich curved panel in vacuo*

Sandwich structures are often used for their low weight compared to their high mechanical performance. Modelling the dynamics of sandwich plates is very difficult and they have been studied much less than orthotropic or isotropic panels. The sandwich panel analysed in this section comprises two laminated skins sandwiching a foam core. The two skins each comprise 4 orthotropic sheets of graphite–epoxy with a lay–up of $[+45/-45/-45/+45]$ and a total thickness of 4mm. The material properties of the skins are $E_x = 144.48$GPa, $E_y = E_z = 9.63$GPa, $G_{xy} = G_{yz} = G_{xz} = 4.128$GPa, $\nu_{xy} = \nu_{xz} = 0.02$, $\nu_{yz} = 0.3$ and $\rho = 1389$kg/m^3. The core is a 10mm polymethacrylamide ROHACELL foam whose material properties are: Young modulus $E = 1.8 \cdot 10^8$Pa, Poisson's ratio $\nu = 0.286$, density $\rho = 110$Kg/m^3. The nondimensional thickness of the sandwich construction is $h/R = 0.018$ with mean radius $R = 1$m. The WFE model was realised using 18 SOLID45 elements in ANSYS, 4 for each skin and 10 for the core, resulting in 57 DOFs after the reduction.

Similar sandwich composite curved panels were studied by Heron [14] and later by Ghinet and Atalla [15] using a discrete layer theory. Heron assumed a classical theory for sandwich structures (a thick core that carries shear stress and thin skins that work in bending and extension) while Ghinet and Atalla used Flügge theory to describe the strain–displacement relations. The two models lead to 47^{th} and 42^{nd} order dispersion systems respectively, which were subsequently solved numerically to determine the dispersion relations. In general solutions to the dispersion equation are pure real wavenumbers, representing propagating waves, pure imaginary wavenumbers, representing evanescent waves, and complex wavenumbers which corresponds to attenuating oscillatory waves. Obtaining complex dispersion curves using the WFE method is not difficult. For cylindrical structures it implies just solving a standard linear eigenproblem considering a standard linear companion form of a quadratic polynomial eigenvalue.

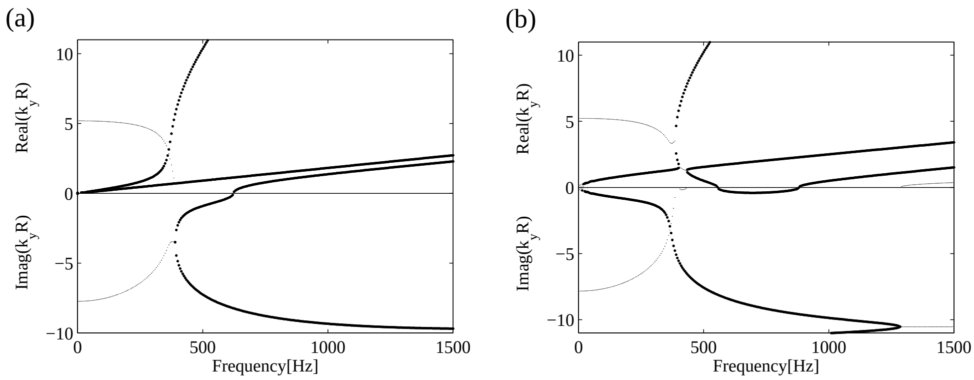

Figure 4: Dispersion curves for the laminated sandwich cylindrical shell. Circumferential modes (a) $n = 0$, (b) $n = 1$: ..... complex valued wavenumbers; **.....** pure real and pure imaginary wavenumbers.

Figure 4 shows the complex dispersion curves for waves propagating in the positive y direction for circumferential orders $n = 0$ and $n = 1$. The ring frequency for this sandwich cylinder is found at 622.7Hz. The dispersive behaviour is very complex and cannot be described simply in terms of torsional, extensional and flexural waves alone. However some features can be observed, particularly concerning cut–off and cut–on with non zero wavenumber and bifurcations between various wave modes. At higher frequencies, higher order waves start propagating which represent higher order wave modes across the thickness. Higher order branches are shown in figure 5 for $\theta = 90^o$, or equivalently for the breathing mode shape. The characteristics of the wave modes can

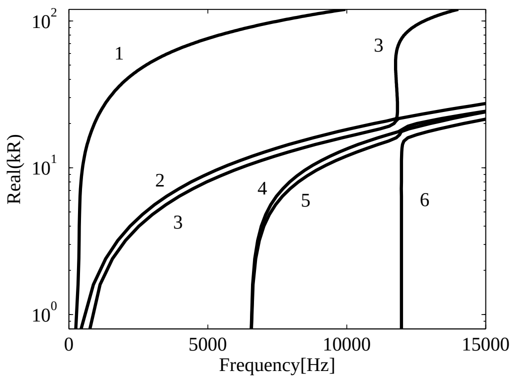

Figure 5: Dispersion curves for the laminated sandwich curved panel. Heading direction $\theta = 90^o$ corresponding to circumferential mode $n = 0$.

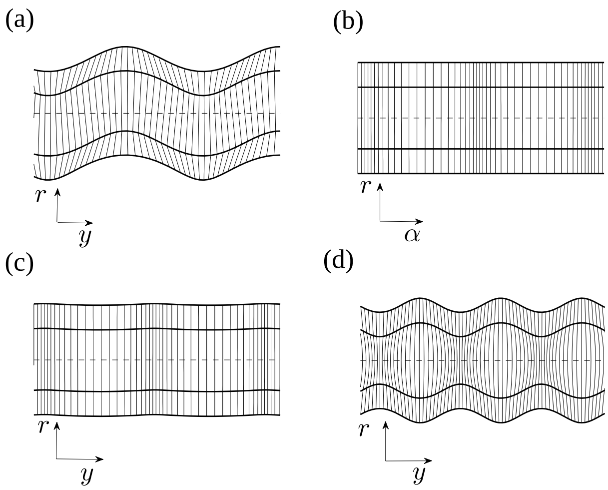

Figure 6: Wave modes for the laminated sandwich curved panel: (a) branch 1 at 2kHz, deformation in r and y directions; (b) branch 2 at 2kHz, deformation in r and α; (c) branch 3 at 2kHz, deformation in r and y directions; (d) branch 3 at 14kHz, deformation in r and y directions (not to scale).

be investigated by considering the eigenvectors ($\mathbf{q}_1$ in the WFE eigenproblems), which represent the deformations of the cross–section under the passage of the wave, and the kinetic (or strain) energy under the passage of the wave associated with the individual degrees of freedom. The kinetic (and strain) energy Γ follows from the mass (and stiffness) matrices in equation (3) and the eigenvectors $\mathbf{q}_1$ as

$$\Gamma = \frac{1}{2}[(i\omega\mathbf{V})^*\mathbf{M}(i\omega\mathbf{V})]; \;\; \mathbf{V} = [\mathbf{q}_1 \;\; \lambda_\alpha\mathbf{q}_1 \;\; \lambda_y\mathbf{q}_1 \;\; \lambda_\alpha\lambda_y\mathbf{q}_1]^T, \tag{15}$$

where the superscript * denotes the transpose conjugation. Figure 6 shows the deformations of the cross–section for the first three propagating wave branches in figure 5 at 2kHz. From figure 6(a) it can be noticed that the wave of branch 1 at 2kHz involves predominantly bending of the skins. Displacements in the radial direction are almost constant, in the circumferential direction are negligible while displacements in the axial direction arise from bending and rotation of the laminae. The skins and the core are moving in antiphase. For this wave the contributions in the α, y and r directions to the total kinetic energy are $\Gamma_\alpha \approx 0\%\Gamma$, $\Gamma_y = 0.25\%\Gamma$ and $\Gamma_r = 99.7\%\Gamma$ respectively. Fig 6(b) shows that wave of branch 2 is a quasi shear wave for which $\Gamma_\alpha = 100\%\Gamma$, $\Gamma_y \approx 0\%\Gamma$ and $\Gamma_r \approx 0\%\Gamma$. The wave of branch 3, figure 6(c), resembles a quasi–extensional wave mode involving primarily displacements in the y direction (some Poisson contraction can be noticed). The kinetic energies in the α, y and r directions are $\Gamma_\alpha \approx 0\%\Gamma$, $\Gamma_y = 93.4\%\Gamma$ and $\Gamma_r = 6.6\%\Gamma$. As the frequency increases, branch 3 crosses branch 2 and its behaviour changes: it approaches the first branch. The wave mode represented by branch 3 at high frequency is evaluated at 14kHz and it is shown in figure 6(d). This mode involves axial motion of the core and a significant out–of–plane displacement. Both axial and out–of–plane displacements are symmetric. The kinetic energy is predominantly in the r direction, $\Gamma_r = 97.3\%\Gamma$, while the contribution in the y direction is $\Gamma_y = 2.7\%\Gamma$. Deformations corresponding to higher order modes are shown in figure 7 for branches 4 and 5 at 10kHz and for branch 6 at 14kHz. Branch 4, figure 7(a), is the first antisymmetric quasi–

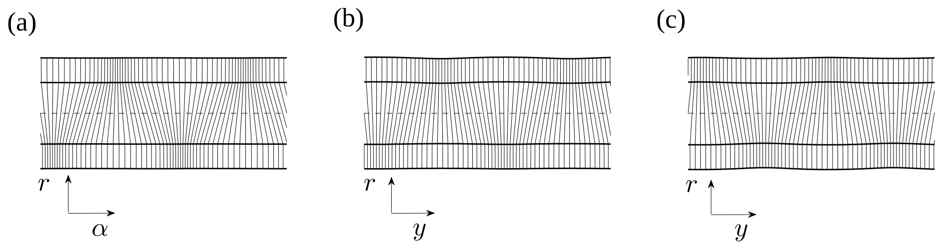

Figure 7: Wave modes for the laminated sandwich cylindrical shell: (a) branch 4 at 10kHz, deformation in α and r directions; (b) branch 5 at 10kHz, deformation in y and r directions; (c) branch 6 at 14kHz, deformation in y and r directions (not to scale).

shear wave mode. The kinetic energy for this wave is predominantly in the circumferential direction, while the contributions to the total kinetic energy in the y and r directions are negligible. Branch 5, figure 7(b), involves antisymmetric quasi–extensional motion in the skins with some shear in the core. The distribution of the kinetic energy is: $\Gamma_\alpha \approx 0\%\Gamma$, $\Gamma_y = 99.09\%\Gamma$ and $\Gamma_r = 0.91\%\Gamma$. Branch 6, figure 7(b), involves primarily axial motion of the core with Poisson contraction in the r direction. This wave resembles the wave of branch 5 at 10kHz as expected from figure 5.

The dispersion curves for the non–closed curved sandwich panel can be represented in the form $\omega = f(k_\alpha, k_y)$, that is the constant frequency contour in the wavenumber plane. As an example, figure 8 shows the dispersion contours at 200Hz and 500Hz. The curves with larger wavenumbers

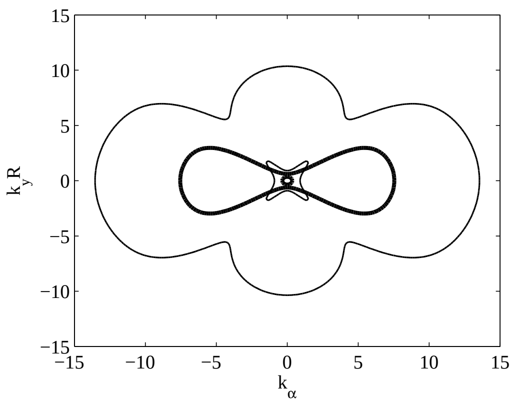

Figure 8: Dispersion contour curves for the laminated sandwich curved panel: bold line 200Hz; thin line 500Hz.

in figure 8 involves wave motion primarily in the r direction. The lower branches correspond to quasi–shear and quasi–extensional wave modes.

3.2 *Air–filled sandwich cylinder*

The sandwich panel analysed in this section is similar to those studied in section 3.2. The two skins each comprise 4 orthotropic sheets with a lay–up of $[+45/-45/-45/+45]$ and a total thickness of 2mm while the core is again a 10mm polymethacrylamide ROHACELL foam. The material properties of the skins are $E_x = E_y = 54$GPa, $E_z = 4.8$GPa, $G_{yz} = G_{xz} = 1.78$GPa, $G_{xy} = 3.16$GPa $\nu_{xy} = 0.06$, $\nu_{yz} = \nu_{xz} = 0.313$ and $\rho = 2000$kg/m^3. The nondimensional thickness of the sandwich construction is $h/R = 0.02$, $R = 0.7$m. The fluid considered is air at 20^oC. The WFE model is realised using 12 solid elements in ANSYS, 4 for each skin and 4 for the core, and 30 fluid elements in ANSYS, resulting in 70 DOFs after the WFE reduction. Figure 9 shows complex dispersion curves for the fluid–filled cylinder, acoustic fluid in rigid walled duct and soft walled duct. Circumferential mode orders $n = 0$ and $n = 1$ are shown. Figure 9 indicates how complicated wave propagation can be for such cases. The individual wave modes cannot be described simply in terms of shear, extensional, flexural and acoustic waves alone: all these motions become coupled. However, it can be noticed that the predominantly structure–borne flexural, torsional and axial waves are still clearly evident and the additional branches are due to the many predominantly fluid–borne waves that exist. For $n = 0$, figure 9(a), coupling occurs at low frequency between quasi–flexural branch and pressure release duct mode. As the frequency increases coupling then occurs between acoustic mode and quasi–extensional wave mode both for real and imaginary branches. When the pipe vibrates at $n = 1$, branches which are predominantly acoustic behave like acoustic modes in a rigid walled duct at low frequency. Structural branches which involve radial motion couple with acoustic modes, for example the first structural branch for $kR > 4$, which represents predominantly flexural wave modes, and the first branch in the imaginary sub–plane which represents predominately extensional motion (hence involves radial expansion and contraction).

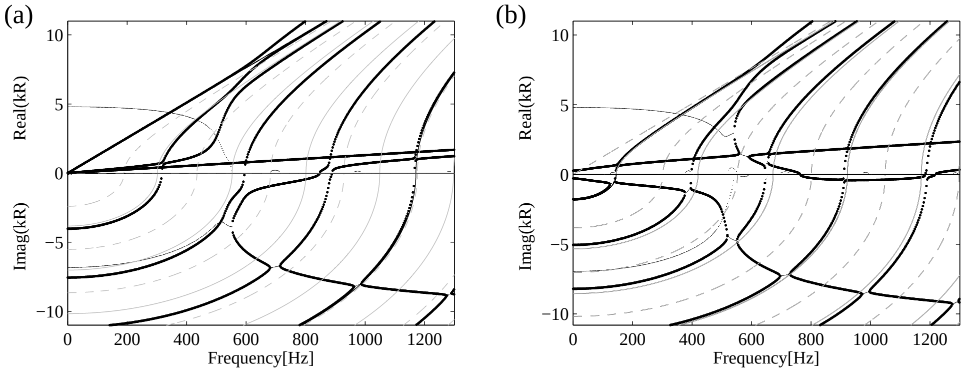

Figure 9: Air filled sandwich cylinder $h/R = 0.02$, (a) $n = 0$ and (b) $n = 1$: ..... complex valued wavenumbers, **.....** pure real and pure imaginary wavenumbers, ——— (grey thin) acoustic fluid in a rigid walled duct, - - - - - (grey thin) acoustic fluid in a soft walled duct.

4 CONCLUDING REMARKS

In this paper the free wave characteristics of axisymmetric structures were predicted using a Wave Finite Element (WFE) method. The WFE method is a systematic and straightforward approach which combines the theory of wave propagation in periodic structures with commercial finite element tools. The technique involves the post–processing of element matrices of a small segment of the structure, which is modelled using either a single shell element or a stack of solid elements meshed through the cross–section. Generally a 3–dimensional FE model of a small segment of the structure is realised and dispersion curves obtained at great accuracy and very small computation cost. The full power of existing FE packages and their extensive element libraries can be utilised. This makes the technique suitable for implementation in commercial finite element codes and suitable for industrial applications. In the first part of the paper the method was described. The method was then illustrated by application to laminated sandwich constructions *in vacuo* and filled with fluid. The application of the method was seen to be straightforward even in the complicated case of a laminated sandwich cylinder for which fluid–structure interaction was considered.

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