Multi-Objective Parameter Identification via ACOR algorithm

Giuseppe Fileccia Scimemi[1], Santi Rizzo[1]
[1]*Department of Structural, Aerospatial and Geotechnical Engineering, University of Palermo, Italy*
E-mail: giuseppe.filecciascimemi@unipa.it, santi.rizzo@unipa.it

Keywords: parameter identification, ACOR, multi-objective optimization.

SUMMARY The spreading of advanced constituive models, needed to model complex phenomena, makes necessary to solve difficult parameter identification problems. The need of multiple tests to fully characterize the experimental behaviour makes the parameter identification problem a multi objective one. Unlike conventional techniques, based on the formulation of an aggregate scalar objective function, in the present work the problem is addressed using a new multi objective algorithm obtained extending the continuous Ant Colony Optimization algorithm. Mathematical tests and application to a real world problem are performed and different performance measures are used to asses the performance of the approach.

1 INTRODUCTION

In many fields of engineering the use of advanced materials and the necessity to better understand and model complex phenomena makes necessary the adoption of advanced constitutive models. These models are generally characterized by complex analytical formulations and many control parameters that not always had a clear counterpart on the observed experimental behavior. For this reason, control parameters could be directly evaluated from simple experimental results only for few cases. To overcome this difficulties ad hoc procedures are required for the correct parameters identification.

In general the parameter identification problem can be treated as an inverse problem [1] and formulated as an optimization problem. For an assigned system the experimental response is given, and making use of the assigned model, the control parameters are searched minimizing an assigned measure of the distance between the numerical response and the experimental one.

For simple constituive models it is possible to set up experimental tests where some parameters has null effect, considering a sequence of uncorrelated single-objective problems, $\min D_i(\mathbf{k})$. When this is not possible the identification problems is a true multi objective problem but usually is solved by combining the multiple objectives into one scalar objective. A common approach is the construction of an aggregate function that combines all of the objective functions, usually a weighted linear sum of the objectives [2], $min(F(D_1(\mathbf{k}), D_2(\mathbf{k}), ...D_{nexp}(\mathbf{k})))$, other approach consider the minimization of only one objective while constraining the remaining objectives to be less than given target values [3]. These simplified approaches have many drawbacks:

a) the choice of the weights or the choice of the principal objective strongly influences the solution found

b) only a solution is found and no information is provided on the multi-modality and epistasis

The parameter identification problem could be stated as a multi-objective problem where it is unlikely that the different objectives, different experimental and numerical response, would be optimized by the same control parameter choices. For this reason some trade-off between different objectives is needed.

In this work the identification problem is approached by a Pareto-compliant ranking method, where no a priori information on the problem is needed and the concept of non-dominated solutions is used. Non dominated solutions are those for which improvement in one objective can only occur with the worsening of at least one other objective when compared with other solutions. Thus, the multi-objective problem has not a unique solution, but a set of solutions.

In the field of bio-inspired computational optimization, different classes of methods have been used to deal with this kind of problems like MOEAs NSGA-II and SPEA2. Ant Colony Optimization, ACO, was initially developed for combinatorial optimization [4], and has been recently adapted to continuous optimization [5]. Ant Colony Optimization is inspired by the ants foraging behavior and requires that the problem is partitioned into a finite set of components, these being intermediate targets before reaching the ultimate goal. In Ant Colony Optimization for continuous optimization, ACOR, the partition of the problem into finite set is given by the intrinsic search space decomposition into different dimensions.

In the present work an ACOR extension [6] proposed by the authors to treat multi-objective problems is applied to a real parameter identification problem. The paper is organized as follows: after presenting the basic of the ACOR, the proposed multi-objective extension to ACOR algorithm is illustrated and the identification problem is stated. Then, the results for standard benchmark problems and for real case of experimental data available in literature [7] are discussed.

2 PARAMETER IDENTIFICATION PROBLEM DEFINITION

Let us consider *nexp* experiments and let $\bar{\mathbf{s}}_\mathbf{i}$ the experimental measurements obtained

$$\bar{\mathbf{s}}_\mathbf{i} \in \mathbb{R}^{ndat}; \quad i = 1, 2, ..., nexp; \tag{1}$$

Let $\mathbf{x}$ the *npar*-dimensional vector of the unknown control parameters and $\mathbf{s}_i(\mathbf{x})$ the numerical results computed for the experiment *i* using the assigned model:

$$\mathbf{s}_\mathbf{i}(\mathbf{x}) \in \mathbb{R}^{ndat}; \quad i = 1, 2, ..., nexp; \tag{2}$$

Let us define a measure of the distance between the experimental and numerical response for the *i*-experiment:

$$D_i(\mathbf{x}) = D_i(\bar{\mathbf{s}}_\mathbf{i}, \mathbf{s}_\mathbf{i}(\mathbf{x})) \tag{3}$$

The inverse problem can be assumed coincident with the following multi-objective optimization problem:

$$\min_{k \in \mathbb{R}^{npar}} (D_1(\mathbf{k}), D_2(\mathbf{x}), ..., D_{nexp}(\mathbf{x})) \tag{4}$$

As a matter of fact the parameter identification inverse problem, in terms of experimental measurement, lacks, in general, of two of the three criteria for being well- posed: there is not a unique solution, and furthermore the solution does not depend continuously on the data. Hence is usually an ill-posed problems in Hadamard sense [8] and exhibits the usual difficulties like nonlinearity, multimodality, epistasis, high sensitivity of the solution to noise in the measured data and, moreover, the modelization error has to be taken into account.

3 MULTI OBJECTIVE ANT COLONY OPTIMIZATION FOR CONTINOUS DOMAINS

The Ant Colony Optimization has been proposed by Dorigo M. and Di Caro G. [4]. It was first proposed for combinatorial optimization problems. Since its emergence many attempts have been made to use it for tackling continuous problems. More recently, Socha K. and Dorigo M. [5], have proposed the natural extension of the ACO algorithm to continuous domains, ACOR. The idea that is central to the way ACOR works is the incremental construction of solutions based on the biased (by pheromone) probabilistic choice of solution components. At each construction step, the ant chooses a Probability Density Function. In what follows a brief description of the ACOR algorithm is reported. Further details could be found in [5].

Let us defines a general optimization problem:

$$\min_{x \in \mathbb{R}^{npar}} (f(\mathbf{x})); f : R^{npar} -> R \tag{5}$$

where $\mathbf{x}$ is the vector of the assumed design variables. ACOR uses an archive $\mathbf{T}$, of assigned size k, in which the solutions, vectors $\mathbf{x}$, are stored. These solutions are ordered according to the fitness value, i.e. best solutions first. The search space is explored using a repeating cycle of actions that is summarised in Box 1 below. For each cycle an ant is stocastically selected from the archive and a new ant is constructed by performing npar construction steps. At construction step i, the ant chooses a value for the variable xi using only the information related to the i-th dimension. The probability p_l of choosing the l^{th} ant is given by:

$$p_l = \frac{\omega_l}{\sum_{r=1}^{k} \omega_r}; \qquad \omega_l = \frac{1}{qk\sqrt{2\pi}} e^{\frac{(l-1)^2}{2q^2k^2}} \tag{6}$$

where ω_l represents the weight associated to the solution l.

Box 1 - ACOR pseudo-code

```
while termination not met
   for i = 1 to m
      choose l^th ant
      generate new ant through g_i
      insert ant in archive
   end i
   choose best k ants
end
```

After the choice of the l^{th} solution a new ant $\mathbf{x}^{new}$ is generated, modifyng every parameter $\mathbf{x_i}^{new}$ by a gaussian function g_i characterized by standard deviation σ_i and mean μ_i for i-parameter:

$$\sigma_i = \xi \sum_{e=1}^{k} \frac{|\mathbf{x}_i^e - \mathbf{x}_i^l|}{k-1}; \qquad \mu_i = \mathbf{x}_i^l \tag{7}$$

This process is repeated m-times and the new solutions are inserted into the archive, at the end of the cycle only the better k solutions are kept and the worst m are discarded. The cycle is repeated until the termination solution is met. ξ, q and k are parameters of the algorithm, further details on the effect of this parameters are reported in [5].

To extend ACOR algorithm to multi-objective problems [6] the scalar concept of optimality is replaced by the notion of Pareto optimality. A solution is said to be Pareto optimal for a multi objective problem if all other solutions have a higher value for at least one of the objective functions, or else have the same value for all objectives. If we consider two solutions $\mathbf{x_1}$ and $\mathbf{x_2}$ the solution $\mathbf{x_1}$ is said to dominate the other solution $\mathbf{x_2}$, if both the following conditions are true:

a)the solution $\mathbf{x_1}$ is no worse than $\mathbf{x_2}$ in all objectives

$$f_j(\mathbf{x_1}) \leq f_j(\mathbf{x_2}) \quad for \quad all \quad j = 1, m \tag{8}$$

b)the solution $\mathbf{x_1}$ is strictly better than $\mathbf{x_2}$ in at least one objective

$$f_j(\mathbf{x_1}) < f_j(\mathbf{x_2}) \quad for \quad at \quad least \quad one \quad j \in 1, m \tag{9}$$

If any of the above conditions is violated, the solution $\mathbf{x_1}$ does not dominate solution $\mathbf{x_2}$. Typically, there is an entire curve or surface of Pareto points or non-dominated points and the shape of this curve indicates the nature of the tradeoff between different objectives.

Hence for multiobjective problems the archive of solutions is ranked based on the non domination level. In the same non domination level a further ranking is introduced usign the concept of crowding of solutions, [9]. In order to preserve diversity of solutions, and differently from standard ACOR where the ants (solutions) are created and are refused immediately or end up in the archive, the ants have the chance, if certain conditions are met, to keep living for more than one iteration till when they are added into the archive or definetely refused. The decision about saving a solution in the archive (SAVE) or rejecting it (REPLACE) or even letting it run for other iterations (CONTINUE) is given to a special acceptance function A. The archive has an adjustable size that varies between k and k+nants, where nants is the number of ants constructed per iteration. In Box 2 the algorithm pseudo-code is reported.

Box 2 - MultiObjective ACOR pseudo-code

```
Non dominance/crowding ordering (archive);
Calculation of weights
while termination not met
  Set current archive size to k
  Repeat
    If not(continue) then
      a) choose new solution from archive using (6)
    Else
      b) set current solution to new solution
    Perturb the components of current solution
    Evaluate new solution
    Accept new solution with probability A(new solution)
    for one of the following operations:
      1)Save
      2)Replace
      3)Continue
  Until archive size is ≥ k+nants
  Non dominance/crowding ordering (archive of k+nants);
  Take the first k solutions
  Increase number of iterations
end
```

As pointed out after the generation of a new solution a function A is used to decide the operation to be performed. The function A takes an exponential form similar to the one appearing in the Simulated Annealing algorithm [10]. If the new solution dominates the current solution then A takes the following form:

$$A = e^{-sign \frac{\Delta dom}{nA}} \tag{10}$$

Where sign is a variable that can take two values: -1 and 1. It takes value -1 if the objective functions calculated in the new solution take the extreme values. In this way extremal values, that gives more diversity to solutions set, are preserved. Δdom is the average amount of domination defined as:

$$\Delta dom = \sum_{i=1}^{nA} \prod_{j=1,nobj} \frac{|f_j(newsolution) - f_j(\mathbf{T}_i)|}{R_j}; \quad f_j(newsolution) \neq f_j(\mathbf{T}_i) \tag{11}$$

where R_j is the maximum range of objective function j, nA is the number of archive solutions that dominate newsolution.

When new solution is dominated by current solution or are non dominated the acceptance funtion assumes the following form:

$$A = e^{\Delta dom_{min}} \tag{12}$$

where Δdom_{min} is the minimum amount of domination between new solution and the archive solutions dominating new solution.

4 NUMERICAL RESULTS

The first numerical results reported regards well known mathematical benchmark functions for multi objective problems, both convex and non convex, in table 1 are reported the functions chosen ant the limit imposed on the variables.

In order to assess the performance of the algorithm the tests were performed also using NSGAII [11]. For the two algorithms the following paramters were adopted: NSGAII $p_{mut} = 0.11$; $p_{cross} = 0.95$; MO_ACOR $q = 0.2$, $\xi = 0.6$;

The algorithms have been compared on the basis of four performance measures [12]:

algorithm effort (AE), can be defined as the ratio of the total number of function evaluations N_{eval} over a fixed period of simulation time T_{run}:

$$AE = \frac{N_{eval}}{T_{run}} \tag{13}$$

ratio of non-dominated individuals (RNI), the ratio of non-dominated individuals found in the set of solutions:

$$RNI = \frac{nondominated}{k} \tag{14}$$

size of Space Covered (SSC), a quantitative measure of the dominated volume in the objective domain. It can be calculated first ordering the nc non dominated solutions solutions based on the values of one of the two objectives (say f1) and then summing up the contributions:

$$SSC = \sum_{i=1}^{nc} f_i(\mathbf{x}_i) - f_1(\mathbf{x}_{i-1}) * f_2(\mathbf{x}_i) \tag{15}$$

measure of uniform distribution (UD), measures the distribution of nc non-dominated solutions. The index UD could be defined as:

$$UD = \frac{1}{1 + S_{nc}} \tag{16}$$

Table 1: Mathematical test functions

function		min	max
SCH	$f_1 = x^2$ $f_2 = x - 2^2$	-10^3	10^3
FON	$f_j = 1 - exp\left(-\sum_{i=j}^{3}\left(x_j - \frac{1}{\sqrt{3}}\right)^2\right)$ $j = 1, 3$	-4	4
KUR	$f_1 = \sum_{i=1}^{n-1} -10exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)$ $f_2 = \sum_{i=1}^{n} \lvert x_i^{0.8}\rvert + 5sinx_i^3$	-5	5
ZDT1	$f_1 = x_1$ $f_2 = g\left[1 - \sqrt{\frac{x_1}{g}}\right]$ $g = 1 + 9\frac{\sum_{i=2}^{n} x_i}{n-1}$ $n = 30, i = 1, n$	0	1
ZDT6	$f_1 = 1 - exp(-4x_1)sin^6(6\pi x_1)$ $f_2 = g\left[1 - \left(\frac{f_1}{g}\right)^2\right]$ $g = 1 + \left(9\frac{\sum_{i=2}^{n} x_i}{n-1}\right)^0 .25$ $n = 10, i = 1, n$	0	1

where S_{nc} is the standard deviation of crowding of the overall set of non-dominated individuals measured by:

$$S_{nc} = \sqrt{\frac{\frac{\sum_{i=1}^{nc} crowd(i)}{nc} - crowd(i)}{nc - 1}} \tag{17}$$

where crowd(i) is the crowding measure defined in (NSGAII).

In table 2 are reported the results obtained.

The performance in terms of RNI and SSC indicator are comparable, but for UD indicator MO_ACOR gives better performances assuring a more uniform cover of solutions space. The values of AE gives contrastant values and more investigation is needed to correctly intepretate these results.

The same approach was used to solve the parameter identification problem of displacement-controlled tests conducted on double cantilever beam (DCB) specimens, aiming to investigate the time-dependent response of the adhesive joint [7]. Details about constituive model used and about the numerical set-up could be found in [13]. To fully characterize the model behaviour 4 different experimental tests were considered with different velocities of imposed displacement. In this way the parameter identification coincides with the following multiobjective optimization problem:

$$\min_{x \in \mathbb{R}^{npar}} (f_i(\mathbf{x})); f : R^{npar} -> R \quad i = 1, 4 \tag{18}$$

Table 2: Mathematical test functions: simulation results

	SCH		FON		KUR	
	NSGAII	MO_ACOR	NSGAII	MO_ACOR	NSGAII	MO_ACOR
AE	3.9*10-4	5.7*10-5	3.5*10-4	1.14*10-4	2.8*10-4	1*10-4
RNI	1	1	1	1	1	1
UD	0.76	0.86	0.76	0.86	0.76	0.86
SSC	2.6	2.6	0.65	0.65	26.5	26.1

	ZDT1		ZDT6	
	NSGAII	MO_ACOR	NSGAII	MO_ACOR
AE	4.2*10-4	4.8*10-4	5.6*10-4	1*10-4
RNI	1	1	1	1
UD	0.73	0.90	0.67	0.84
SSC	0.32	0.33	0.91	0.90

the parameters adopted for multi objective algorithms are the same of precedent tests. In table 3 are reported the results of the optimization runs.

Table 3: Interface Parameter Identification: simulation results

	NSGAII	MO_ACOR
AE	2.4	3.7
RNI	1	1
UD	0.65	0.82
SSC	0.89	0.87

Also in this case MO_ACOR shows a better performance on the UD indicator while mantaining comparable results on the other indicators.

5 CONCLUSIONS

The present work addressed the parameter identification problem by a new multiobjective evolutionary algorithm obtained extending the ACOR algorithm. The algorithm was compared with the classical NSGAII over some well known mathematical test functions. The indicators used to asses the performance of the algorithm shows an increase in distribution of solutions found and comparable performance on other indicators. The same algorithm was successfully applied to a real parameter identification problem showing the applicability of the proposed approach in real world problems. Further studies are needed to investigate the effect of case-study properties on the overall performances. Furthermore more experimentation is needed to explore the effect of multi-modality, isolation, and deception on the algorithm results.

References

[1] Beck J, Woodbury K., "Inverse problems and parameter estimation: integration of measurements and analysis." *Measure Science Technology*, **9**, 839847 (1998).

[2] Milania A.S., Dabboussib W., Nemesb J.A., R.C. Abeyaratnec, “An improved multi-objective identification of JohnsonCook material parameters.” *International Journal of Impact Engineering*, **36**, 294-302 (2009).

[3] Ignizio J. P., “A Review of Goal Programming: A Tool for Multiobjective Analysis” *The Journal of the Operational Research Society*, **29**, 1109-1119 (1978).

[4] Dorigo M., Gambardella L. M., “Ant colony System: A cooperative learning approach to the traveling salesman problem.” *IEEE Trans. on Evol. Comp.*, **1**, 5366 (1997).

[5] Socha, Krzysztof, Dorigo, “Ant colony optimization for continuous domains.” *Eur. Journal of Operational Research*, **185**, 1155-1173 (2008).

[6] Riva Sanseverino E., Zizzo G., Fileccia Scimemi G., “Optimal dispatch of dispersed and renewable generating units in power distribution systems by a new meta-heuristic multiobjective approach” submitted for possible pubblication in *Knowledge-Based Intelligent System Advancements:The Journal of the Operational Research Society*.

[7] Xu C, Siegmund T, Ramani K., “Rate-dependent crack growth in adhesives II. Experiments and analysis.” *Int. Journal of Adhesion and Adhesives*, **23**, 152 (2003).

[8] Mahnken R, Stein E., “Parameter identification for viscoplastic models based on analytical derivatives of a least-squares functional and stability investigations.” *International Journal of Plasticity*, **12**, 451479 (1996).

[9] Deb K., Agrawal S., Pratap A., Meyarivan T. A., “Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II” *Parallel Problem Solving from Nature VI*, (2000).

[10] Smith K., Everson R., Fieldsend J., “Dominance measures for multi-objective simulated annealing.” *2004 IEEE Congr. Evol. Comput.*,23-30 (2004).

[11] Deb K., “Multiobjective Optimization using Evolutionary Algorithms” John Wiley and Sons Ltd. (2001).

[12] Tan K.C., Lee T.H., Khor E.F., “Evolutionary algorithms for multi-objective optimization: performance assessments and comparisons” *Intl. Conf. on Evolutionary Computation*, (2001).

[13] Giambanco G., Fileccia Scimemi G., “Mixed mode failure analysis of bonded joints with rate-dependent interface models” *International Journal for Numerical Methods in Engineering*, **67**, 1160-1192 (2006).