Time-depending behavior of PC beams externally plated with prestressed FRP laminates: a mechanical model

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SUMMARY. In this paper the authors present a mechanical model capable of predicting the long term behavior of PC beams externally plated with prestressed FRP laminates, starting from the rheological properties of composites. It is based on a mechanical approach that assumes there is a perfect adhesion between concrete core and external reinforcement.

1 INTRODUCTION

The strengthening of existing structures through the use of fibre-reinforced polymer (FRP) materials represents one of the most interesting and promising technique within the context of Civil Engineering.

More recently, prestressed FRP systems are being adopted in the rehabilitation of industrial structures or bridges, where PC beams need reinforcement interventions.

Within this context, a particularly important aspect is represented by the accurate modelling of the deferred behavior of reinforced elements [1-6].

In such cases, in fact, the different rheological properties of the components of the strengthened structure (e.g. concrete core and composite reinforcement) can lead to a migration of FRP stresses towards PC beam. The consequent increase of the stress state in beam can compromise the efficacy of the strengthening technique.

Theoretical and experimental studies, currently available in literature, refer to the failure behavior of these reinforced elements, while there are still few on their long term behavior. Several theoretical and experimental studies on the rheological behavior of FRPs have been performed by the authors, in order to characterize their viscous properties [7-13].

The guide-lines on the designing of FRP strengthening interventions drawn up within an international context, including the recently published Guidelines CNR-DT 200/2004, take into consideration the aforementioned rheological phenomena through the introduction of suitable reductive factors of the FRP design stresses.

In this paper the authors present a mechanical model capable of predicting the long term behavior of a PC beam strengthened with FRP prestressed reinforcement, once the viscous constitutive law of composite is known.

2 MECHANICAL MODEL

The mechanical model schematizes the viscous behavior of a pretensioned concrete beam, subject to a bending moment and strengthened with a FRP prestressed laminate (Fig. 1).

The basic hypotheses are:

- plane cross sections;
- perfect adhesion between FRP and prestressed concrete beam;
- external bending moment, M_{ex}, unvaried over time;
- linear viscoelastic behavior of the composite;

- perfect adhesion between concrete and steel tendons;
- elasticity moduli of the materials constituting the cross section, unvaried over time.
- negligible concrete creep behavior.

The last assumption is sustained by the fact that the strengthening intervention is generally realized on a beam that has been in service for several years.

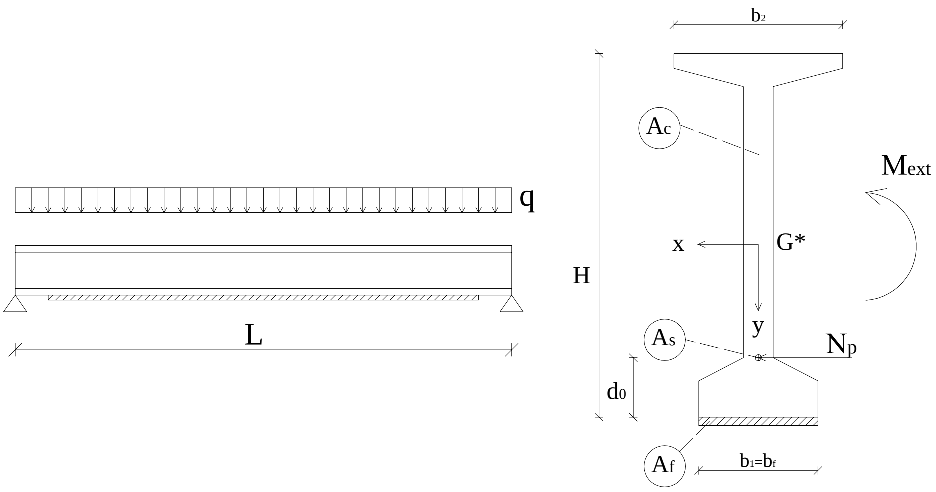

Figure 1: Prestressed concrete beam strengthened with a FRP laminate.

In reference to the cross section in Figure 2, defining the centroid of the transformed section, G*, as the origin of the axes, *x* and *y*, the axial strain over time, ε, in the cross section, is written:

$$\varepsilon(t,y) = \lambda(t) + \mu(t)\,y \tag{1}$$

where λ and μ represent the axial strain in G* and the curvature, respectively.

In addition, assuming that the concrete has an elastic behavior and that the strengthening is characterised by an initial Young modulus, E_f, equation (1) can be rewritten for the three materials constituting the cross section as follows:

$$\begin{cases} \varepsilon_f\left(t,y_f\right) = \lambda(t) + \mu(t)\,y_f & = \varepsilon_{fe}(t) + \varepsilon_{fv}(t) - \varepsilon_{f0} \\ \varepsilon_s(t,y) \quad = \lambda(t) + \mu(t)\,y & = \varepsilon_{se}(t) - \varepsilon_{s0} \\ \varepsilon_c(t,y) \quad = \lambda(t) + \mu(t)\,y & = \varepsilon_{ce}(t) \end{cases} \tag{2}$$

where:

- ε_{ce} and ε_{se} are the concrete and steel strain, respectively;
- ε_{fe} and ε_{fv} are the elastic and viscous contribution to FRP strain, respectively;
- ε_{s0} and ε_{f0} are the strain in tendons and in FRP corresponding to initial effective pretension, respectively.

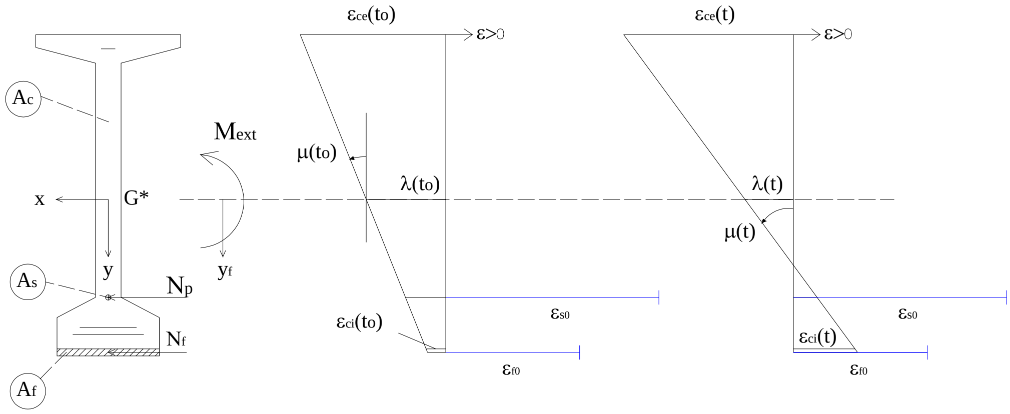

Figure 2: strain and curvature variation over time in the cross section.

From relations (2), the following expressions of the stresses in the materials are obtained:

$$\begin{cases} \sigma_f(t) = E_f \cdot \varepsilon_{fe}(t) = E_f\left[\left(\lambda(t)+\mu(t)y_f\right)-\varepsilon_{fv}(t)+\varepsilon_{f0}\right] \\ \sigma_s(t) = E_s \cdot \varepsilon_{se}(t) = E_s\left[\lambda(t)+\mu(t)y+\varepsilon_{s0}\right] \\ \sigma_c(t) = E_c \cdot \varepsilon_{ce}(t) = E_c\left[\lambda(t)+\mu(t)y\right] \end{cases} \tag{3}$$

Starting from the equilibrium equations of the cross section:

$$\begin{cases} \int_{A_f}\sigma_f(t)\,dA_f + \int_{A_c}\sigma_c(t)\,dA_c + \int_{A_s}\sigma_s(t)\,dA_s = 0 & \text{(translation)} \\ \int_{A_f}\sigma_f(t)\,y_f\,dA_f + \int_{A_c}\sigma_c(t)\,y\,dA_c + \int_{A_s}\sigma_s(t)\,y\,dA_s = M_{est} & \text{(rotation about the x axis)} \end{cases} \tag{4}$$

and using the expressions (3), the following is obtained:

$$\begin{cases} E_f\int_{A_f}\left[\left(\lambda(t)+\mu(t)y_f\right)-\varepsilon_{fv}(t)+\varepsilon_{f0}\right]dA_f + E_c\int_{A_c}\left[\lambda(t)+\mu(t)y\right]dA_c + \\ \qquad +E_s\int_{A_s}\left[\lambda(t)+\mu(t)y\right]dA_s = 0 \\ \\ E_f\int_{A_f}\left[\left(\lambda(t)+\mu(t)y_f\right)-\varepsilon_{fv}(t)+\varepsilon_{f0}\right]y_f\,dA_f + E_c\int_{A_c}\left[\lambda(t)+\mu(t)y+\varepsilon_{s0}\right]y\,dA_c + \\ \qquad +E_s\int_{A_s}\left[\lambda(t)+\mu(t)y\right]y\,dA_s = M_{ext} \end{cases}$$

Such equations can be written as follows:

$$
\begin{cases}
\left(E_f A_f + E_c A_c + E_s A_s\right)\lambda(t) + \left(E_f S_f + E_c S_c + E_s S_s\right)\mu(t) - E_f \int\limits_{A_f} \varepsilon_{fv}(t)\, dA_f + \\
\qquad + E_s \varepsilon_{s0} A_s + E_f \varepsilon_{f0} A_f = 0 \\
\left(E_f S_f + E_c S_c + E_s S_s\right)\lambda(t) + \left(E_f I_f + E_c I_c + E_s I_s\right)\mu(t) - E_f \int\limits_{A_f} \varepsilon_{fv}(t)\, y_f\, dA_f + \\
\qquad + E_s \varepsilon_{s0} S_s + E_f \varepsilon_{f0} S_f = M_{ext}
\end{cases}
\tag{5}
$$

where:

- A_f, A_c and A_s are the FRP, concrete and steel area, respectively;
- S_f, S_c and S_s are the FRP, concrete and steel first moment of area, respectively;
- I_f, I_c and I_s are the FRP, concrete and steel moment of inertia about the x axis, respectively.

Introducing the following symbols:

- $\overline{n}_c = \dfrac{E_c}{E_f}$, $\overline{n}_s = \dfrac{E_s}{E_f}$;
- $A^* = \left(A_f + \overline{n}_c A_c + \overline{n}_s A_s\right)$ (area of the transformed section);
- $S^* = \left(S_f + \overline{n}_c S_c + \overline{n}_s S_s\right) = 0$ (transformed section first moment of area about the x axis);
- $I^* = \left(I_f + \overline{n}_c I_c + \overline{n}_s I_s\right)$ (transformed section moment of inertia about the x axis);
- $N_p = E_s \varepsilon_{s0} A_s$ (prestressed tendons contribution to axial force);
- $N_f = E_f \varepsilon_{f0} A_f$ (prestressed FRP laminate contribution to axial force);
- $M_p = E_s \varepsilon_{s0} S_s$ (bending moment due to N_p the x axis);
- $M_f = E_f \varepsilon_{f0} S_f$ (bending moment due to N_f the x axis).

the equations (5) become:

$$
\begin{cases}
E_f A^* \lambda(t) + E_f S^* \mu(t) = E_f \int\limits_{A_f} \varepsilon_{fv}(t)\, dA_f - (N_p + N_f) \\
E_f S^* \lambda(t) + E_f I^* \mu(t) = M_{ext} + E_f \int\limits_{A_f} \varepsilon_{fv}(t)\, y_f\, dA_f - (M_p + M_f)
\end{cases}
\tag{6}
$$

The viscous deformation of the strengthening, $\varepsilon_{fv}(t)$, presents the following expression:

$$
\varepsilon_{fv}(t) = \int_{t_0}^{t} \frac{\sigma_f(\tau)}{E_f} f(\tau, t)\, d\tau, \tag{7}
$$

in which:

- $f(\tau,t) = -E_f \dfrac{\partial\Phi(\tau,t)}{\partial\tau}$;
- $\Phi(\tau,t) = \dfrac{1}{E_f}\left(1+\varphi(\tau,t)\right)$ (creep function);
- $\varphi(\tau,t)$ (creep coefficient).

Taking into account the equations (3), the relation (7) becomes:

$$\varepsilon_{fv}(t) = \int_{t_0}^{t}\left[\left(\lambda(\tau)+\mu(\tau)\,y_f\right)-\varepsilon_{fv}(\tau)+\varepsilon_{f0}\right] f(\tau,t)\,d\tau. \tag{8}$$

Substituting (8) in the equations (6), the following is obtained by simple algebra:

$$\begin{cases} \lambda(t)+\left(1-\dfrac{A_f}{A^*}\right)\displaystyle\int_{t_0}^{t}\lambda(\tau)\,f(\tau,t)\,d\tau-\dfrac{S_f}{A^*}\displaystyle\int_{t_0}^{t}\mu(\tau)\,f(\tau,t)\,d\tau=\lambda_e+\overline{\lambda}_e\displaystyle\int_{t}^{t}f(\tau,t)\,d\tau \\ \mu(t)-\dfrac{S_f}{I^*}\displaystyle\int_{t_0}^{t}\lambda(\tau)\,f(\tau,t)\,d\tau+\left(1-\dfrac{I_f}{I^*}\right)\displaystyle\int_{t_0}^{t}\mu(\tau)\,f(\tau,t)\,d\tau=\mu_e+\overline{\mu}_e\displaystyle\int_{t_0}^{t}f(\tau,t)\,d\tau \end{cases} \tag{9}$$

where:

- $\lambda_e = -\dfrac{(N_p+N_f)}{E_f A^*}, \quad \mu_e = \dfrac{M_{ext}-(M_p+M_f)}{E_f I^*}$;
- $\overline{\lambda}_e = -\dfrac{N_p}{E_f A^*}, \quad \overline{\mu}_e = \dfrac{M_{ext}-M_p}{E_f I^*}$.

Equations (9) represent a coupled system of two integral Volterra equations in the unknowns $\lambda(t)$ and $\mu(t)$.

Such a system can be solved using the Laplace transformation technique:

$$
\begin{cases}
L\left[\lambda(t)\right] = \dfrac{S_f}{A^*}\cdot L\left[\displaystyle\int_{t_0}^{t}\mu(\tau)\cdot f(\tau,t)\,d\tau\right] - \left(1-\dfrac{A_f}{A^*}\right)\cdot L\left[\displaystyle\int_{t_0}^{t}\lambda(\tau)\cdot f(\tau,t)\,d\tau\right] + \\
\qquad + \dfrac{\lambda_e}{s} + \overline{\lambda}_e \cdot L\left[\displaystyle\int_{t_0}^{t} f(\tau,t)\,d\tau\right] \\
L\left[\mu(t)\right] = \dfrac{S_f}{I^*}\cdot L\left[\displaystyle\int_{t_0}^{t}\lambda(\tau)\cdot f(\tau,t)\,d\tau\right] - \left(1-\dfrac{I_f}{I^*}\right)\cdot L\left[\displaystyle\int_{t_0}^{t}\mu(\tau)\cdot f(\tau,t)\,d\tau\right] + \\
\qquad + \dfrac{\mu_e}{s} + \overline{\mu}_e \cdot L\left[\displaystyle\int_{t_0}^{t} f(\tau,t)\,d\tau\right]
\end{cases}
\tag{10}
$$

Assuming t_0=0 and being evidently $\lambda(t_0)$=0, $\mu(t_0)$=0 for $t<t_0$, $f(t-\tau)$=0 for $t<\tau$, the convolution theorem allows:

$$
\begin{cases}
\dfrac{S_f}{A^*}\cdot L\left[\displaystyle\int_{t_0}^{t}\mu(\tau)\cdot f(t-\tau)\,d\tau\right] - \left(1-\dfrac{A_f}{A^*}\right)\cdot L\left[\displaystyle\int_{t_0}^{t}\lambda(\tau)\cdot f(t-\tau)\,d\tau\right] - \overline{\lambda}_e \cdot L\left[\displaystyle\int_{t_0}^{t} f(t-\tau)\,d\tau\right] = \\
\qquad = \dfrac{S_f}{A^*}\cdot\mu(s)\cdot F(s) - \left(1-\dfrac{A_f}{A^*}\right)\cdot\lambda(s)\cdot F(s) + \overline{\lambda}_e\cdot\dfrac{F(s)}{s} \\
\dfrac{S_f}{I^*}\cdot L\left[\displaystyle\int_{t_0}^{t}\lambda(\tau)\cdot f(t-\tau)\,d\tau\right] - \left(1-\dfrac{I_f}{I^*}\right)\cdot L\left[\displaystyle\int_{t_0}^{t}\mu(\tau)\cdot f(t-\tau)\,d\tau\right] + \overline{\mu}_e \cdot L\left[\displaystyle\int_{t_0}^{t} f(t-\tau)\,d\tau\right] = \\
\qquad = \dfrac{S_f}{I^*}\cdot\lambda(s)\cdot F(s) + \left(1-\dfrac{I_f}{I^*}\right)\cdot\mu(s)\cdot F(s) + \overline{\mu}_e\cdot\dfrac{F(s)}{s}
\end{cases}
\tag{11}
$$

where $F(s)$, $\lambda(s)$ and $\mu(s)$ represent the Laplace transforms of the function $f\,(t\text{-}\tau)$, $\lambda(t)$ and $\mu(t)$, respectively.

Substituting (11) in the equations (10), the unknown functions $\lambda(s)$ and $\mu(s)$ can be obtained by the following equation system:

$$
\begin{cases}
\lambda(s) = \dfrac{S_f}{A^*}\cdot\mu(s)\cdot F(s) - \left(1-\dfrac{A_f}{A^*}\right)\cdot\lambda(s)\cdot F(s) + \overline{\lambda}_e\cdot\dfrac{F(s)}{s} + \dfrac{\lambda_e}{s} \\
\mu(s) = \dfrac{S_f}{I^*}\cdot\lambda(s)\cdot F(s) - \left(1-\dfrac{I_f}{I^*}\right)\cdot\mu(s)\cdot F(s) + \overline{\mu}_e\cdot\dfrac{F(s)}{s} + \dfrac{\mu_e}{s}
\end{cases}
\tag{12}
$$

The inverse Laplace transforms of $\lambda(s)$ and $\mu(s)$ supply the solution of the viscous problem in the time domain.

3. NUMERICAL SIMULATIONS

A numerical analysis on the long term behavior of pretensioned concrete beams strengthened with prestressed FRP laminates has been developed.

In particular, it has been assumed that the reinforcement intervention is required to replace the first bottom row of tendons, due to their high corrosion.

Within this study, different plating types, characterized by the same initial prestressed axial force (N_f = 1.15 10^6 N), have been analyzed; the geometrical and mechanical properties, loading condition and strengthening geometry are shown in Figure 3.

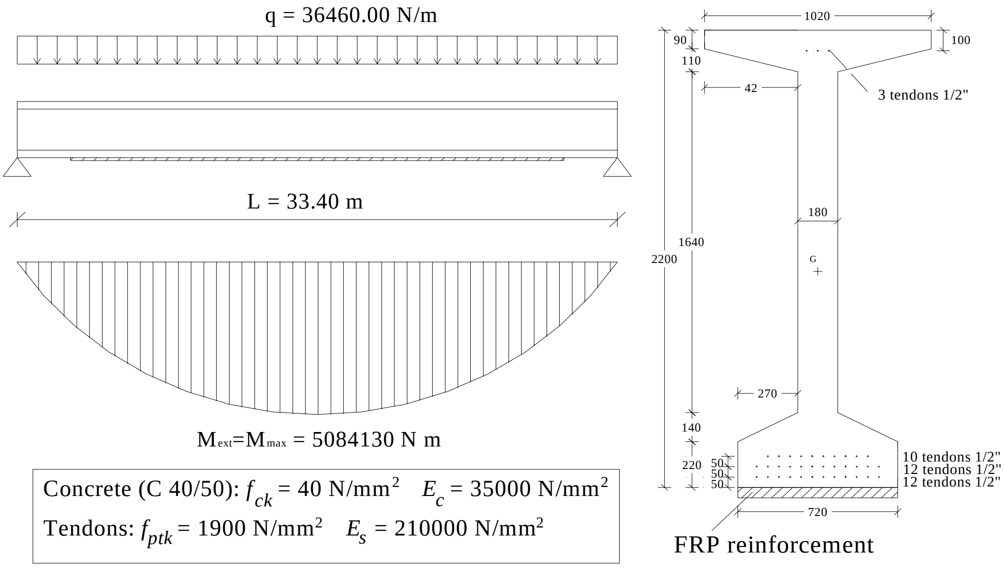

Figure 3: Details of the strengthened PC beam (dimensions in mm).

The long term behavior of adopted laminates have been characterized by using a micromechanical model recently proposed by the author [7-13], in the field of linear viscoelasticity, capable of predicting the viscous properties of a FRP laminate starting from those of the single phase (matrix and fibre).

The viscoelastic parameters of the matrices and aramid fibres, reported in Tables 1 and 2, have been evaluated by using data of Maksimov [14]. Instead, a linear elastic constitutive law has been assumed in the case of carbon fibres (Table 2).

Table 1: Rheological properties of the matrix.

$E_{1,m}$ [MPa]	$E_{2,m}$ [MPa]	$\eta_{1,m}$ [MPa h]	$\eta_{2,m}$ [MPa h]
2830	1610	1246	11300000

Table 2: Rheological properties of the fibres.

Type of Fibre	$E_{1,m}$ [MPa]	$E_{2,m}$ [MPa]	$\eta_{1,m}$ [MPa h]	$\eta_{2,m}$ [MPa h]
Aramid	122971	4.76E+10	150989	1.16E+08
Carbon	250000	1.00E+49	1.00E+50	1.00E+49

The numerical simulation, for a time period of 50 years, has been carried out on 4 types of laminates (Table 3), characterized by different volumetric fractions of fibres, made of same matrices (Table 1) and different fibres (Table 2). The initial stress value has been assumed variable between 10 and 40% of the failure strength of the fibres, assumed both for aramid and carbon fibres equal to: f_{fk} = 3500 MPa.

Table 3: Examined cases.

Type of laminate	Type of fibre	V_f [%]	V_m [%]	$\sigma_f(0)/f_{fk}$ [%]	$\sigma_f(0)$ [MPa]
I	Carbon	80	20	10	350
				20	700
				40	1400
II	Carbon	40	60	10	350
				20	700
				40	1400
III	Aramid	80	20	10	350
				20	700
				40	1400
IV	Aramid	40	60	10	350
				20	700
				40	1400

For each case, the initial and deferred stresses in the composite and in the bottom fibre of concrete, σ_{ci}, have been evaluated (Table 4).

Table 4: Instantaneous stresses in the concrete core and the FRP reinforcement.

Type of laminate	$\sigma_{ci}(0)$ [MPa]	σ_{ci} (50 years) [MPa]	$\sigma_f(0)$ [MPa]	σ_f(50 years) [MPa]	$\Delta\sigma_f/\sigma_f(0)$ [%]
I	-0.193541	-0.18163	350	349.083	-0.26
	-0.199630	-0.187275	700	698.098	-0.27
	-0.202842	-0.190252	1400	1396.12	-0.28
II	-0.199541	-0.126442	350	344.374	-1.61
	-0.202796	-0.128307	700	688.535	-1.64
	-0.204469	-0.129266	1400	1376.85	-1.65
III	-0.199714	2.94913	350	107.663	-69.24
	-0.202886	2.96746	700	212.016	-69.71
	-0.204515	2.9767	1400	420.686	-69.95
IV	-0.202793	3.00446	350	103.168	-70.52
	-0.204468	3.01378	700	204.643	-70.77
	-0.205317	3.01846	1400	407.585	-70.89

As an example, for type IV laminate, subject to an initial prestressed axial stress of $\sigma_f(0)$=1400.00 MPa, the diagrams of the instantaneous stresses in the composite (Figure 4) and in the bottom fibre of concrete (Figure 5) are reported.

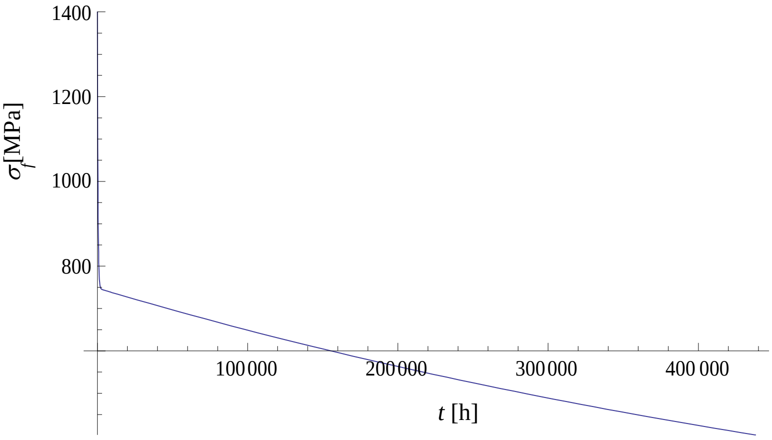

Figure 4: Instantaneous stresses in the AFRP composite.
(V_f = 0.4, V_m= 0.6)

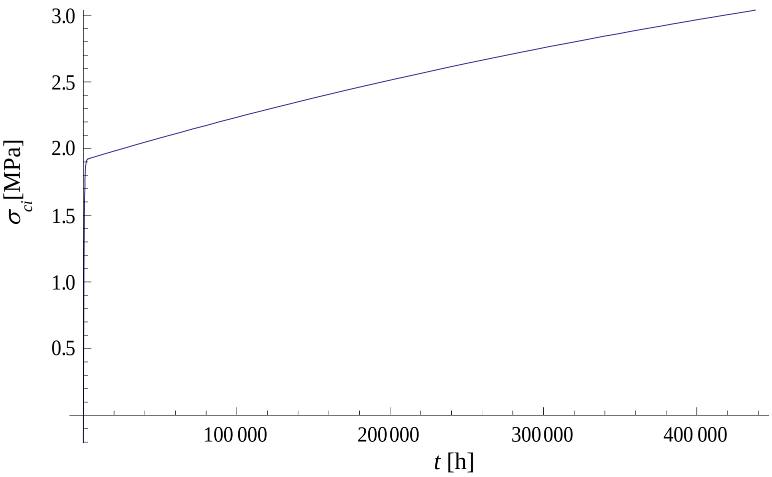

Figure 5: Instantaneous stresses in the bottom fibre of concrete.
(V_f = 0.4, V_m= 0.6)

5 CONCLUSION

This paper has presented a micromechanical model, formulated by the authors, that allows, within the context of linear viscoelasticity theory, the long term behavior of prestressed beams strengthened with FRP laminates to be analysed.

In particular, it is capable of obtaining the deferred global behavior of a strengthened element, starting from the rheological characterisation of the FRP laminate.

The results obtained within numerical simulations have highlighted a negligible stress migration in the case of CFRP laminates.

On the contrary, a marked stress variation has been observed with reference to the AFRP reinforcements, although the initial stress in the composites does not exceed the stress limit, suggested by international guide lines.

It suggest to also analyze the time-depending behavior of such strengthened elements, when reinforcements are realized by using AFRP or GFRP laminates, due to their potential marked viscous behavior.

The proposed model could represent a valid tool to perform such a verification, giving the exact solution to the problem instead of more approximated ones obtained through numerical approaches available in literature.

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