# Rigid-damaging-frictional constitutive model for delamination analysis of joined bodies via XFEM with tip-element.

Francesco Parrinello, Boris Failla
*DISAG, University of Palermo*
*E-mails*: *francesco.parrinello@unipa.it; bfailla@diseg.unipa.it.*

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SUMMARY: The present work is devoted to analyze the delamination process of joined solids, in the hypothesis of rigid behaviour of sound connecting surfaces (between the joined bodies). The kinematical framework is based on the extended finite element method and the so called tip element is considered in order to analyze the delamination process inside each finite element. The delamination start when the stress threshold is attained on a finite element, then the enriching degree of freedom are released and the jump displacement results to be different than zero. The interfacial constitutive behaviour is modelled as elastic damaging and a residual frictional strength is considered. The constitutive model is developed in a rigorous thermodynamic consistency.

## 1. INTRODUCTION

The present work is devoted to analyze the delamination process of joined solids, generally considered as set of material layers or portions, joined together along surfaces (composite laminates, hybrid structures, friction stir welding) or by means of internal adhesive layers (masonry walls, structural glass). With reference to material mechanical strength, the internal connecting surfaces are often weakness zones, where several nonlinear phenomena reveal and lead to the delamination process. Other nonlinear phenomena can develop inside the solids but they, usually, produce modest effects into the overall mechanical response and, therefore, they are neglected in the present work.

From the kinematical point of view, the delamination process of joined solids is typically characterized by formation of displacement discontinuities, which take place along the internal connecting surfaces. Generally, in order to represent the discontinuous displacement fields, internal connecting surfaces are modelled by means of zero-thickness interface elements. Thus, for a numerical analysis via FEM of composite material structure, in pre-processing phase it is necessary to take great care of meshing the joined bodies and the internal connecting surfaces. In fact, the spatial mesh of joined bodies has to be rigorously confined among the internal adhesion surfaces, while only the last ones have to be meshed by means of interface element, in compliance with the nodal correspondence requirement among the different meshes. This procedure may be too burdensome, especially if compared to computational time necessary for carrying out response of numerical simulation of mechanical problem.

Several contributions proposed in literature [1-3] deal with the constitutive modelling of the nonlinear phenomena at the internal connecting surfaces via interface element formulation. All these models are characterized by a real or fictitious elastic behaviour for the interface initially sound condition. This assumption is realistic when the interface formulation has to model a physical internal adhesive layer with finite thickness. Otherwise, for pure fracture phenomena, with real zero-thickness connecting surfaces, the initially elastic behaviour is uniquely assumed as penalty parameter, necessary for carrying out the numerical analysis. In the latter case elasticity at the interface is just an approximated numerical tool, as it always produces discontinuity in the displacement field, also in the initial state of fully sound interface. Moreover, the elastic behaviour may be the cause of interpenetration between the joined solids under compressive loading conditions.

In the present work the delamination process of joined bodies is approached by means of the eXtended Finite Element Method (XFEM) [5,6], which is an emerging numerical strategy for modelling strong (displacement) as well as weak (strain) discontinuities within a standard finite element framework, without the need of specific remeshing procedure. XFEM has been originally developed in order to catch and to follow free crack propagation inside continuous solids, nevertheless it may be used to model the delamination process along *a priori* known surfaces like the internal connecting ones. XFEM allows to simplify the meshing procedure with respect to the interface element approach, because it is possible to define a unique simple mesh for the whole domain, where the internal connecting surfaces are geometrically described by means of level set method. In the XFEM, the discontinuity of displacement field is governed by specific enriching degrees of freedom, which may be kept constrained, in order to simulate the initially rigid behaviour of the sound connecting surfaces, and released when the relevant activation (or propagation) condition is attained. In this way, it is possible to avoid the above mentioned drawbacks related to the elastic modelling of the sound connecting surfaces.

In the present work, the activation and evolution criteria for delamination is governed by a constitutive model, which is developed in a rigorous thermodynamic framework with the following characteristics: rigid behaviour in sound state up to the traction threshold limit, damage-cohesive behaviour in partially delaminated state with elastic unloading, residual elastic-frictional behaviour in fully delaminated state under compressive traction and sliding.

Finally, the introduction of a tip-element [7-9] adds a further kinematical enrichment to the finite element with internal discontinuity (XFE), which becomes capable of describing a partially cracked element. In the present work, the tip-element is used to model a partially delaminated element, where two regions exist: a rigid sound one with zero jump displacement and a cohesive one with not-zero jump displacement and elastic-damaging-frictional behaviour. The tip-element may be used to simulate a continuous crack propagation process inside each element.

## 2. XFEM FRAMEWORK

In the present work the kinematical formulation developed in [7], which is applicable both to partially cracked finite element (“crack tip element”) and to completely cracked finite element, is considered for modeling the jump displacement along the connecting surface.

The discontinuous displacement field $\mathbf{u}$ of the generic body $\Omega$ is defined as the additive combination of a continuous part $\mathbf{u}_{\text{cont}}$ and a discontinuous one $\mathbf{u}_{\text{disc}}$ :

$$\mathrm{u}(\mathrm{x}) = \mathrm{u}_{\text{cont}}(\mathrm{x}) + \mathrm{u}_{\text{disc}}(\mathrm{x})\,. \tag{1}$$

The continuous displacement field is approximated by standard $C^0$ shape functions $N_I(\mathrm{x})$ , thus

$$\mathrm{u}_{\text{cont}}(\mathrm{x}) = \sum_{I \in N_{\text{tot}}} N_I(\mathrm{x})\mathrm{u}_I \tag{2}$$

where $M_{\text{tot}}$ is the total set of nodes of the meshed domain and $\mathbf{u}_I$ are the nodal displacements. The discontinuous part of the displacement field is defined only for elements intersected by the discontinuity surface $\Gamma$ (sub-domain $\Omega_{\text{enr}}$ in fig. 1 ) and is written in the following form

$$u_{disc}(x) = \sum_{I \in M_{enr}} N_I(x) \Psi_I a_I \tag{3}$$

where $\Psi_I(x)$ are enrichment functions, $a_I$ are nodal enrichment parameters and $M_{enr}$ is the set of the nodes of the elements completely cut by the surface $\Gamma$. For elements in $\Omega_{enr}$ completely intersected by the interfacial surface $\Gamma$, enrichment functions have the following form

$$\Psi_I(x) = \text{sign}(\phi(x)) - \text{sign}(\phi_I), \quad I \in M_{enr}, \quad x \in \Omega_{enr} \tag{4}$$

where $\phi(x) = 0$ is the level set function, that implicitly describes the discontinuity surface locus, and $\phi_I = \phi(x_I)$ is the value of level set function at the $I^{th}$ node. Due to the specific form of the enrichment functions, the enrichment displacement field vanishes outside the enriched elements, and is null in all elements not intersected by the surface $\Gamma$.

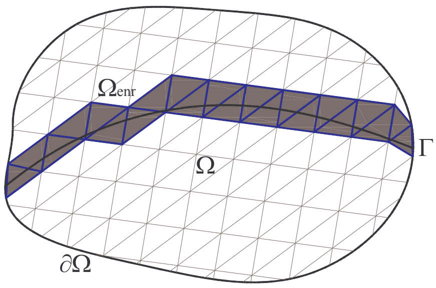


Figura 1: Bodies connected by mean of surface $\Gamma$, the ordinary mesh in the entire domain $\Omega$ and the XFEM elements in the enriched domain $\Omega_{enr}$.

For a three node triangular element the shape functions $N_I(x)\Psi_I$ defined in eq. (3) are null at the side opposite the $I^{th}$ node and they are null on the element subdomain contained by connecting surface $\Gamma$ and the $I^{th}$ node, as shown in fig. 2.

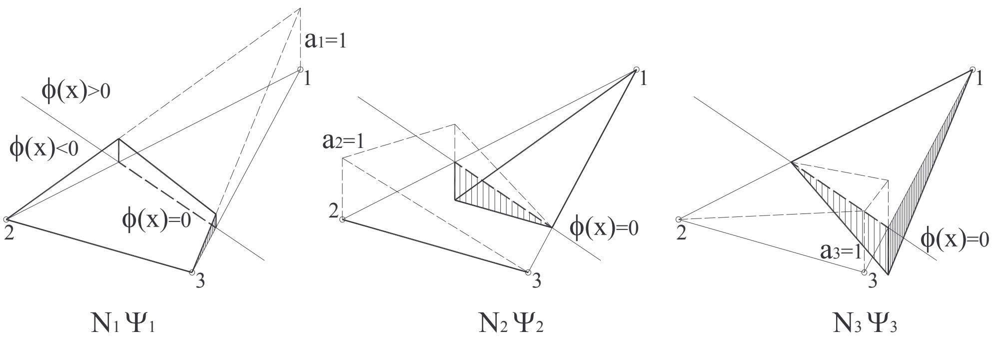


Figure 2: enriching shape functions.

The crack opening displacement at a generic point $P \in \Gamma$ is defined as the jump displacement between the two sides of surface $\Gamma$ and is given by the following relation

$$[\![\mathbf{u}]\!] = \mathbf{u}^+ - \mathbf{u}^- \tag{4}$$

where $\mathbf{u}^+$ and $\mathbf{u}^-$ are displacements respectively of positive edge and negative edge of the crack, as represented in figures (3 a, b), where the jump displacement vector is decomposed in normal and tangential components, with reference to the connecting surface $\Gamma$.

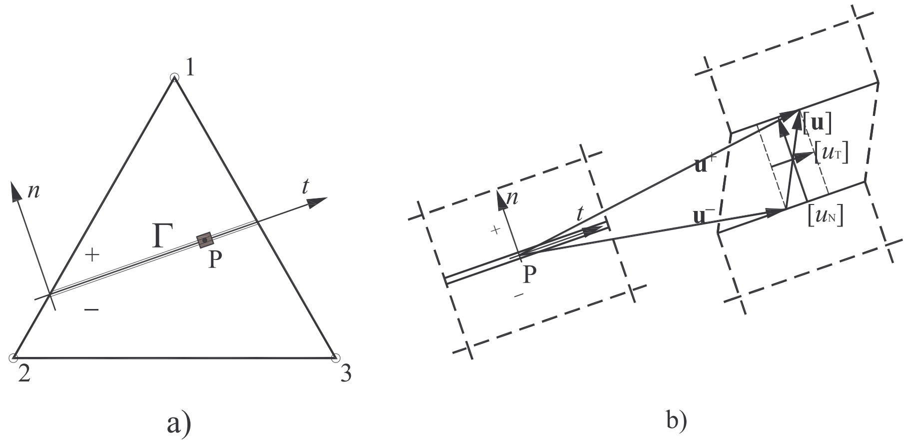


Figure 3: a) extended three node element; b) interface kinematics at the point P.

Considering eqs. (3) and (4), jump displacement is defined in XFEM framework by the following relation

$$[\![\mathbf{u}]\!] = 2\sum_{I=1}^{M_{enr}} N_I \mathbf{a}_I$$

for a completely cracked finite element.

The latter formulation has already been followed by the same authors [10] for the delamination analysis of solids joined by adhesive layer, whose constitutive behaviour is initially elastic. Thus, the interfacial jump displacement represents both a measure of elastic strain at sound points and the crack opening displacement at completely cracked points. For initially sound interfaces, enriching degrees of freedom are active for all finite elements intersected by adhesive layer.

In the present work, XFEM is applied for the delamination analysis of joined solids where the interfacial behaviour is initially rigid, with zero opening displacement, up to the traction threshold is attained and, consequently, cracking phenomenon starts with active jump displacement. In a finite element, the interfacial rigid behaviour is performed by keeping constrained the enriching degrees of freedom, which are released when the cracking condition is reached.

Moreover, in order to catch the cracking propagation phenomenon at the finite element with crack tip inside, the so called tip-element has to be used, for kinematical modelling of partially cracked elements. In this paper, the tip-element formulation developed in [7] is considered; three and six nodes triangular elements with straight interface inside the element have been developed and implemented in a finite element code [12]. In fig. 4a is represented a three nodes triangular element with connecting surface intersecting sides 12 and 13 and partially delaminated from side 12 up to point $P$ (the tip). The discontinuous displacement field of partially delaminated element is obtained by releasing the enriching degrees of freedom of only one node. With reference to the example in figs. 4a, b, enriching degree of freedom of node 2 are released; as shown in fig. 3b the relevant shape function is defined on the triangle 123’ instead of the entire element. Therefore, jump displacement is active only on the delaminated portion AP of the interface and it vanishes at point P.

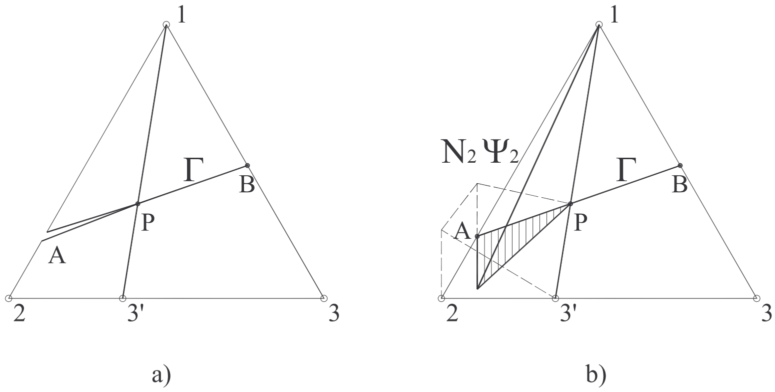


Figure 4: a) tip-element; b) enriching shape function of node 2.

3. CONSTITUTIVE MODEL

The constitutive model proposed in this paper is devoted to model the mechanical behaviour of a connecting surface between two different bodies, which are joined one on each other without the use of any adhesive material. Thus, the interface is physically a zero thickness layer, able to bond the contiguous bodies by means of transmission of tractions between the contact surfaces.

By hypothesis, interface transmits tractions rigidly, without relative displacement (jump displacement) between the two (joining) contact surfaces; moreover, when the interface strength is attained, chemical/mechanical bonds, which keep the adjacent bodies joined, start breaking and, as consequence of that, the interface traction transmission capability progressively reduce up to zero. Once reached the latter mechanical condition, a residual frictional strength is available for the mechanical interaction of the delaminated bodies.

3.1 MESOSCALE LEVEL ANALYSIS

The present constitutive model is developed with the aim of representing, from a phenomenological point of view, the mechanical behaviour of the connecting surfaces; the model is based on the micromechanical interpretation of those phenomena, which govern the traction transmission between the joined bodies surfaces. A representative volume element (RVE), around point $P$ on the connecting surface (fig. 5a), is considered of negligible size with respect to a characteristic dimension of joined bodies and, at the same time, as sufficiently large to contain a representative portion of the materials microstructures (fig.5b).

In the RVE the traction transmission is governed by two different phenomena, namely: the frictional contact, which is active only for compressive traction, and the mechanical/chemical bonds, which are generated in the joining procedure, due to the specific chemical, thermal or mechanical treatment.

In pure tensile traction, only the joining bonds are able to transmit stress and, when the tensile limit strength is attained, they start breaking and producing opening relative displacement between the joining surfaces. Moreover, it is assumed the opening displacement recovery after complete unloading.

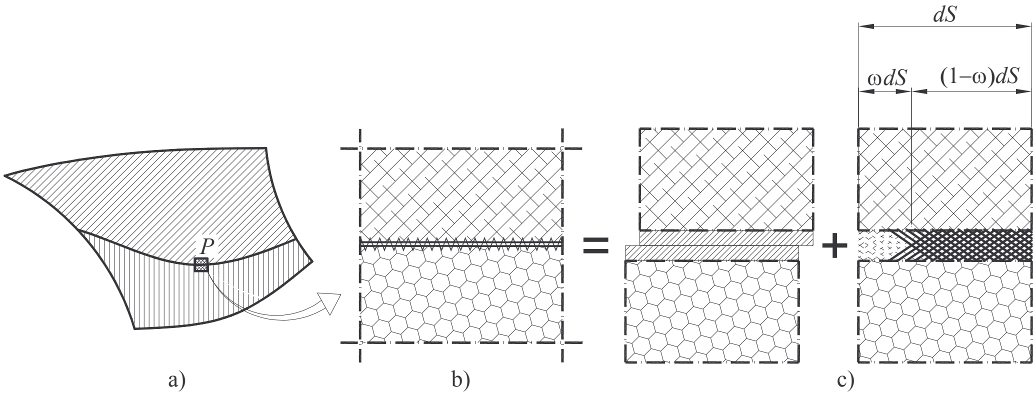


Figura 5: representation at mesoscale level of connecting surface mechanical behaviour, a) point P at the connecting surface; b) RVE of the connected bodies; c) representation of frictional behaviour and of partially damaged joining bonds.

The behaviour of joining bonds is modelled in the ambit of damage theory, as already done by Perego [11] in quasi brittle fracture. The damage parameter $\omega$ is a mesoscale parameter, which measures the ratio of broken bonds area with respect to the joining bonds total area contained in the RVE. The relation between tensile traction $t_b$ and the relative opening displacement $u_b$ is written as in the following:

$$t_b = \frac{1-\omega}{\omega} K_b [\![ u_b ]\!] \qquad (5)$$

where $K$ is a stiffness parameter, introduced only for physical dimension reasons, but its value does not affect significantly the mechanical behaviour. For the sound material condition ($\omega = 0$), eq.(16) states a rigid behaviour, for which displacement must be null. The partially damaged material ($0 < \omega < 1$) is governed by elastic-damaging material and, finally, for $\omega = 1$ the joining bonds do not transmit any traction.

The frictional behaviour should be modelled as rigid-perfectly plastic, but serous computational problems would arise and, therefore, an elastic-perfectly plastic model has to be adopted. So, the frictional traction is defined as

$$\mathbf{t}_f = \mathbf{K}_f [\![ \mathbf{u}_f^e ]\!] . \qquad (6)$$

For sound material ($\omega = 0$) the pure compressive strength is unlimited and the interface is rigid, whereas, for delaminated material, the unlimited compressive strength has to be accomplished by an elastic (penalty) behaviour, which allows mutual penetration of delaminated bodies; it is a necessary approximation.

### 3.2 THERMODYNAMIC FRAMEWORK

The proposed constitutive model is developed in a rigorous thermodynamic setting and it is defined as the superimposition of two different interlaminar phenomena: frictional and mechanical-chemical cohesive bonding. The frictional behaviour is governed by mechanical variables with index "$f$", whereas the behaviour due to interlaminar bonding is governed by mechanical variables with index "$b$". The interfacial constitutive model is based on the following Helmholtz free energy functional:

$$\Psi = \frac{1}{2}\frac{1-\omega}{\omega}\mathbf{u}_b^{e\,T}\mathbf{K}_b\mathbf{u}_b^e + \frac{1}{2}\mathbf{u}_f^{e\,T}\mathbf{K}_f\mathbf{u}_f^e + \Psi_{in} \tag{7}$$

where $\omega$ is the bonds damage parameter, $\mathbf{u}_b^e = [\![\mathbf{u}_b]\!]$ and $\mathbf{u}_f^e = [\![\mathbf{u}_f]\!] - \mathbf{u}_f^p$ are elastic components of jump displacements (with $\mathbf{u}_f^p$ frictional plastic displacement), $\mathbf{K}_b$ and $\mathbf{K}_f$ are, respectively, the bonding stiffness matrix and the elastic (penalty) frictional stiffness one, $\Psi_{in}(\xi)$ is the internal energy, function of the kinematical internal variable $\xi$, which leads the hardening-softening phenomena.

The static variables, conjugated to the kinematical ones defining the Helmholtz free energy functional, are given in the following relations:

$$\frac{\partial\Psi}{\partial\mathbf{u}_b} := \mathbf{t}_b = \frac{1-\omega}{\omega}\mathbf{K}_b\mathbf{u}_b^e \tag{8}$$

which is the traction vector transmitted by interlaminar bonds;

$$\frac{\partial\Psi}{\partial\mathbf{u}_f^e} := \mathbf{t}_f = \frac{1-\omega}{\omega}\mathbf{K}_f\mathbf{u}_f^e \tag{9}$$

which is the frictional traction vector;

$$-\frac{\partial\Psi}{\partial\omega} := Y = \frac{1}{2\omega^2}\mathbf{u}_b^{e\,T}\mathbf{K}_b\mathbf{u}_b^e \tag{10}$$

which is the release rate energy.

All above defined static and kinematic terms have to be considered as mesoscale variables, which measure RVE average values. The jump displacements $[\![\mathbf{u}_b]\!]$ and $[\![\mathbf{u}_f]\!]$ identify the average kinematical configurations of the mechanical elements respectively related to the frictional phenomenon and to the bonding one. In the mesoscale constitutive modelling, the kinematical configuration of the all RVE sub-elements are, generally, assumed be equal to RVE configuration; thus, it means that,

$$[\![\mathbf{u}_f]\!] = [\![\mathbf{u}_b]\!] = [\![\mathbf{u}]\!]. \tag{11}$$

The $Y$ release energy rate variable, conjugated to the damage variable, is not well defined in terms of jump displacements for the sound material ($\omega = 0$). Thus, the energy release rate may be stated in terms of traction, by substituting eq. (19) in eq. (21), obtaining the following relation:

$$Y = \frac{1}{(1-\omega^2)}\frac{1}{2}\mathbf{t}_b^T\mathbf{K}_b^{-1}\mathbf{t}_b \tag{12}$$

which is well defined for $\omega = 0$.

The attainment of the bonds peak strength condition is defined as a damage activation criterion

$$f_d = Y - \chi(\xi) - Y_0 \leq 0 \tag{13}$$

where

$$\dot{\omega} = \frac{\partial f_d}{\partial \omega}\dot{\lambda} = \dot{\lambda}$$
$$\dot{\xi} = -\frac{\partial f_d}{\partial \chi} = \dot{\lambda} \tag{14 a,b}$$

and $\chi(\xi) = \frac{\partial \Psi_{in}}{\partial \xi}$ is the static hardening-softening variable; the relevant loading-unloading laws are defined in the following relations

$$\dot{\lambda} \geq 0 \;\; f_d \dot{\lambda} = 0 \;\; \dot{f}_d \dot{\lambda}\,. \tag{15}$$

The activation condition threshold value is defined as function of the pure tensile traction $\bar{t}_T$ by eq.(23)

$$Y_0 = \frac{1}{2}\frac{\bar{t}_T^2}{K_t} \tag{16}$$

where $K_T$ is the stiffness parameter in pure tensile traction state; as already mentioned, its value does not affect the mechanical response and, therefore, unitary value is assumed ($K_T = 1$). In order to assume independent value of damage activation in pure shear traction $t_2 = \bar{t}_S$, the relevant stiffness parameter assumes the following value

$$K_S = \frac{\bar{t}_T^2}{\bar{t}_S^2} K_T\,. \tag{17}$$

The fracture energy is equal in pure tensile and in pure shear conditions and it depends only on the assumed softening law, which is defined as

$$\chi(\xi) = \frac{1}{2}\frac{\bar{t}_T^2}{K_T}\left[\frac{{K_T}^2 u_f^2}{[K_T(1-\xi)u_f + \bar{t}_T \xi]^2} - 1\right] \tag{18}$$

and it produces a bi-linear response in the traction-displacement diagram, with $u_f$ the ultimate opening displacement.

The frictional strength is classically defined by a Mohr-Coulomb yield surface in a not-associative perfect plasticity, assuming $\alpha$ and $\beta$ respectively as the frictional and the dilatancy coefficients. The yield surface and the plastic potential are

$$f_f \equiv t_{f2} + \alpha t_{f1} \leq 0$$
$$F_f \equiv t_{f2} + \beta t_{f1} \tag{19 a, b}$$

with $\alpha > \beta$ . The flow rule and the loading-unloading conditions are

$$\dot{\mathbf{u}}_f^p = \frac{\partial F_f}{\partial t_f}\dot{\lambda}$$
$$\dot{\lambda} \geq 0, \quad \dot{\lambda} f_f = 0, \quad \dot{\lambda}\dot{f}_f = 0. \qquad (20\text{ a, b})$$

## 4. NUMERICAL ANALYSIS

The above extended finite element with internal tip and the proposed interfacial constitutive model have been implemented in the FEAP finite element code [12]. The results of three numerical analysis are reported in the present paragraph.

### 4.1 Example 1

The first numerical analysis is devoted to validate effectiveness and robustness of the implemented finite element code; this target is pursued comparing the results obtained by Zi *et al* in [7] for a simple double cantilever beam test, where, due to the specimen symmetry, the fracture position is known (fig. 6). The same problem can be regarded as a delamination one, assuming the rigid connecting surface to be coincident to the symmetry line. The results, which are compared in fig. 7 in terms of applied force *vs* displacement of load application point, show that the two responses are practically identical, except for the last descending branch. In fact, the implemented finite element code is not be able to overcome the snap-back problem, even if small, obtained by Zi *et al* in their analysis.

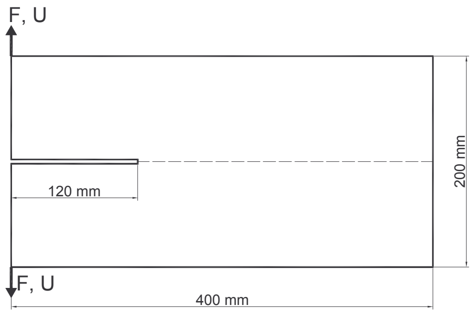


Figure 6: Double cantilever beam test, geometry and loading condition

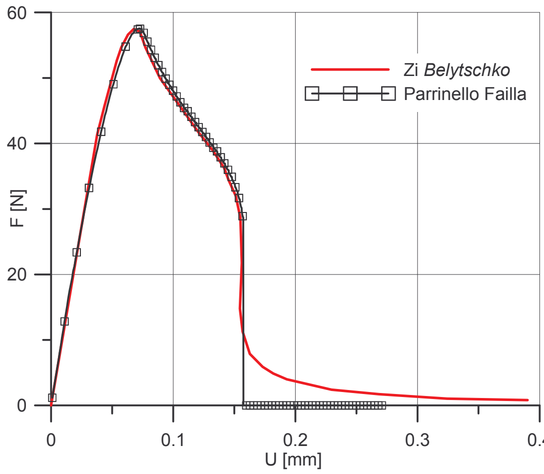


Figure 9: Load-displacement curves obtained for the double cantilever beam test

### 4.2 Example 2

The second numerical analysis regards the structural problem schematically represented in fig. 8. It is a cantilever composed of two glued laminas, partially delaminated, constrained on the right side and loaded on the left side. The load is applied only on the upper lamina and it produce compressive normal traction and shear traction between the joined surfaces. The load is applied by means of a displacement control method and the cyclic loading law is depicted in fig. 9. This example is not supported by experimental data and it is proposed in order to show the effect of friction between the two laminas.

The curve load *vs* displacement is represented in fig. 10. As it can be observed, loading branches and unloading ones are clearly different, because the delamination phenomenon reduce significantly the elastic stiffness. Moreover, the unloading branches are different than the subsequent reloading ones, because of the frictional effects. Finally, the two tangential stress distributions, corresponding to the loading condition depicted in fig. 10, are plotted in figs. 11 and 12. In fig. 11 the cantilever beam is still partially delaminated, whereas in fig. 12 the two laminas are completely separated.

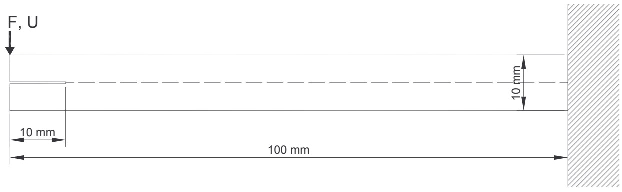


Figure 8: Two laminas cantilever beam, partially delaminated, constrained on the right side and subjected to vertical on the left side.

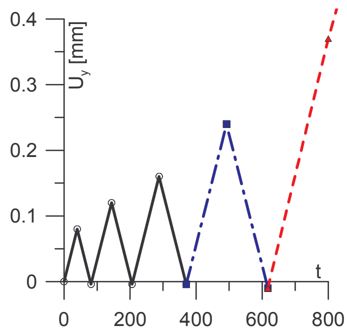


Figure 9: Imposed displacement at the load application point

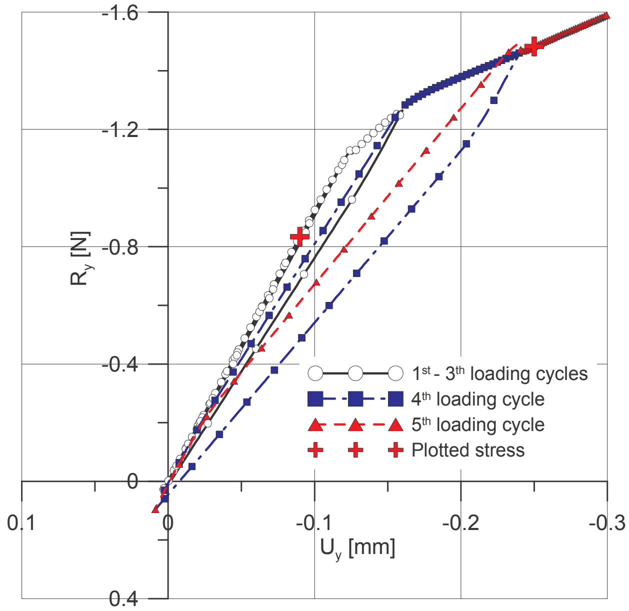


Figure 10: Load-displacement curve of the two laminas cantilever beam subjected to vertical load

## 5. CONCLUSIONS

The proposed XFEM scheme appears suitable for modelling two dimensional problems of joined solids with generally shaped and spatially located connecting surfaces. Once paid the computational cost related to the implementation of the detecting procedure of intersected finite elements, the finite element discretization of joined solids is a trivial problem.

Moreover, XFEM scheme allows to model as rigid the behaviour of the connecting surfaces, which is more realistic than the penalty (elastic) method and it does not require to identify elastic constitutive parameter.

With regard to the cohesive-frictional model proposed, despite validation numerical analyses have not been considered in this context, seems to be able to accurately catch delamination phenomena of mode I and II.

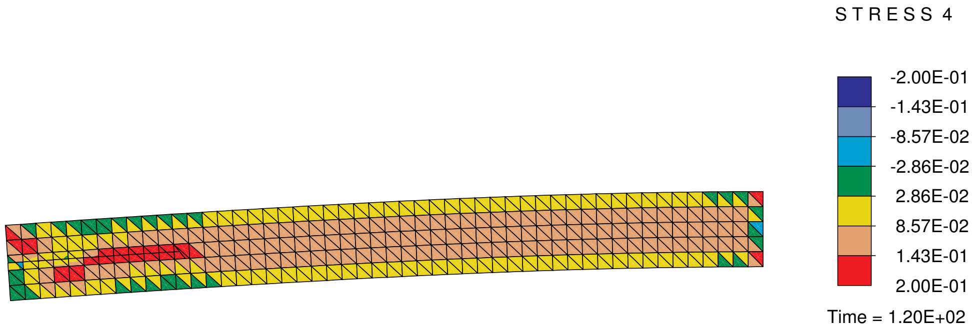


Figure 11: Tangential stress distribution at time t = 750.

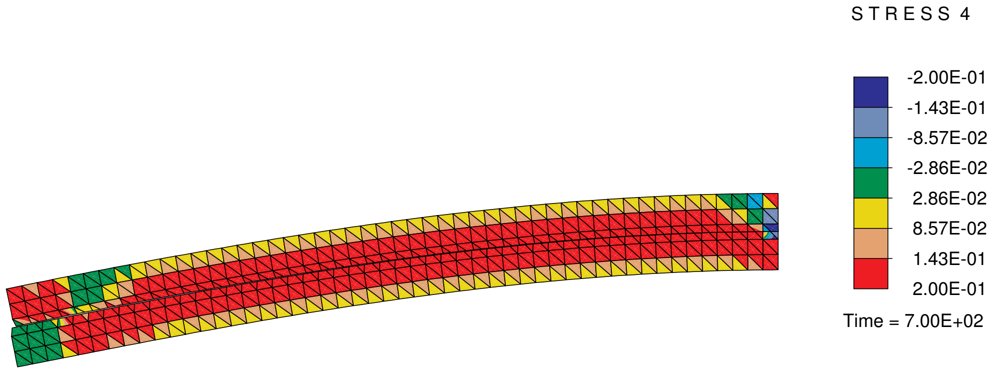


Figure 12: Tangential stress distribution at time t = 700.

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