Optimal design of structures under dynamic loading

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SUMMARY. The present paper is devoted to the optimal design of structures subjected to static and dynamic loading. The constitutive material is assumed to have an elastic perfectly plastic behaviour. The relevant optimal design problem has been formulated as a minimum volume problem and reference has been made to the so-called statical approach. The minimum volume structure is determined under suitable constraints on the design variables as well as accounting for different resistance limits: it is required that the optimal structure satisfies the elastic shakedown limit and the instantaneous collapse limit, considering acting for each different limit condition a suitably chosen load combination and imposing for each different condition suitably chosen load amplifiers. The adopted load combinations are characterized by the presence of fixed loads, of quasi-statical cyclic loads and dynamic (seismic) loads. In the present context the effects of the dynamic actions are studied on the grounds of the dynamic features of the relevant structure taking into account the natural frequencies. The proposed treatment is referred to the most recent Italian Codes related to the structural analysis and design: therefore, the minimum volume design is developed at first as the search for the optimal structure with simultaneous constraints on the elastic shakedown behaviour (related to serviceability conditions), on the instantaneous collapse (under the combination of fixed and cyclic loads due to the wind effect), on the instantaneous collapse (under the combination of fixed and seismic loads). The Bree diagrams of the optimal structure have been determined in order to characterize the structural behaviour and it has been always found that the structure approaches the instantaneous collapse in conditions of ratchet. With the aim of avoiding such a dangerous condition a further constraint has been introduced into the relevant optimal design problem: in particular, the fictitious plastic activations have been suitably limited. The obtained results show that suitably limiting the referenced quantities the optimal structure exhibits an alternating plasticity behaviour even for load combinations very close to the limit ones. The numerical applications are related to steel frames.

1 INTRODUCTION

Since the first years of the second half of last century, many scientists and engineers faced the problem of the optimal design of elastic plastic structures (see, e.g., [1-6]). Today, many of the fundamental features of the behaviour of an optimal structure under different loading combinations and limit conditions are sufficiently known. These results together with the technological progress led to the gradual introduction of some aspects of optimal design in national and international codes. The main goal of such codes (in the present paper the Italian National Codes [7] is referred to) is to allow the structural engineers for designing structures more adequate for the loading conditions that they will suffer during their design lifetime and, as a consequence, more safe than those given by traditional approaches. In general, the formulation of an optimal design problem, besides the choice of an objective function that very often coincide with the structural volume, substantially requires the definition of appropriate load conditions and the choice of special limiting criteria to impose on the structural behaviour (elasticity, shakedown, instantaneous collapse, sometimes simultaneously imposed) (see, e.g. [8-12]).

As known, the intensity of the loads which the structure must suffer during its lifetime is often a very difficult task due to their randomness characteristic. Furthermore, the definition of a special load model, together to the assignment of reasonable limit values that the load can not overpass, strongly influence the choice of the limit behaviours that the structure must satisfy. So, the optimal design problem might be formulated taking into account quasi static as well as dynamic loads, imposing constrains identifying with different limit conditions related to as many different load conditions characterized by appropriate load multipliers. Therefore, the formulations proposed will make reference to the Italian code, that prescribes that the structural design be effected taking into account different combinations of static and dynamic loadings, amplified by prefixed load multipliers, below which the structure must respect the serviceability conditions and not reach the instantaneous collapse condition.

Unfortunately, imposing the ultimate limit load condition it is not possible to have information on the structural behaviour slightly below the instantaneous collapse: actually, before the collapse the structure could be subjected to ratcheting, i.e. in any case to a fast incremental collapse, or it can be in the field characterized by an oligocyclic fatigue behaviour, i.e. it possesses the capability for resisting to several load cycles before collapsing. Obviously, it could be preferable to have a more safe structure that under the action of high intensity loads doesn't exceed the alternating plasticity limit and exhibits small plastic deformations. Therefore, in this paper the optimal design problem of elastic perfectly plastic frames subjected to fixed, quasi static and dynamic loadings is performed. No constraints on the material ductility have been considered and the hypothesis of small displacements has been assumed. The relevant optimal design problem has been formulated as a minimum volume problem and reference has been made to the so-called statical approach, i.e. the constraints on the limit behaviour of the structure are written in terms of generalized stresses.

The optimal design problem is formulated at first simultaneously imposing an elastic shakedown limit, related to appropriate serviceability conditions, an impending instantaneous collapse limit, related to a combination of fixed and high intensity (dynamic) loads, and an impending instantaneous collapse limit, related to a combination fixed and cyclic loads (wind effect) suitably amplified, as prescribed by the referenced code. The linear elastic response to dynamic loads has been computed by a modal technique [13]. Subsequently, the sensitivity of the structural response has been investigated on the ground of the determination and interpretation of the Bree diagrams of the obtained optimal structure. In all the examined cases it has been observed that the optimal structure exhibits an incremental collapse behaviour even for loads not very close the ones characterizing the instantaneous collapse condition. On the contrary, it is well known that, even suffering same plastic deformations, the structure shows adequate resistance and safety characteristic in conditions of plastic shakedown. Therefore, in order to prevent an incremental collapse condition a suitable further constraint has been introduced into the relevant minimum volume problem consisting in the limitation of the fictitious plastic activations, i.e. same fictitious plastic deformations which characterize the limit behaviour of the relevant design.

The effected numerical applications are related to steel plane frames subjected to fixed and quasi static perfectly cyclic actions as well as dynamic loads. The obtained results are encouraging and they show that suitably limiting the referenced quantities the optimal structure exhibits an alternating plasticity behaviour even for load combinations very close to the limit ones.

Furthermore, the new designs are characterized by just a very modest cost increment with respect to the safety improvement related to the plastic shakedown behaviour.

2 THE MODEL

Let us consider now a shear plane frame just subjected to an horizontal ground acceleration $a_g(t)$. It is modeled as a Multi-Degree-Of-Freedom (MDOF) structure, such that the total number of degrees of freedom is equal to n_f.

The dynamic equilibrium equations can be written in the following form:

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}(t)+\boldsymbol{A}\,\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\,\boldsymbol{u}(t)=\boldsymbol{f}(t) \tag{1}$$

being $\boldsymbol{f}(t)=-\boldsymbol{m}\,a_g(t)$, and are related with the following initial conditions:

$$\boldsymbol{u}(0)=\boldsymbol{0}\,,\ \dot{\boldsymbol{u}}(0)=\boldsymbol{0} \tag{2}$$

In equations (1) and (2) $\boldsymbol{M}$, $\boldsymbol{A}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices (with dimensions $n_f \times n_f$), respectively, which are assumed to be positive ones; $\boldsymbol{m}=\boldsymbol{M}\,\boldsymbol{\tau}$, being $\boldsymbol{\tau}$ the ($n_f \times 1$) influence vector; $\boldsymbol{f}(t)$ is the ($n_f \times 1$) excitation vector, while $\boldsymbol{u}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\ddot{\boldsymbol{u}}(t)$ are the displacement, the velocity and the acceleration ($n_f \times 1$) vectors of the system, respectively, and the over dot means time derivative of the relevant quantity. In equation (2) $\boldsymbol{u}(0)$, $\dot{\boldsymbol{u}}(0)$ represent the initial displacement and velocity vectors.

As it is usual, the dynamic characteristics of the structural behaviour are identified in terms of natural frequencies as well as damping coefficients. In this framework, the following coordinate transformation is usually adopted:

$$\boldsymbol{u}(t)=\boldsymbol{\Phi}\,\boldsymbol{z}(t) \tag{3}$$

being $\boldsymbol{z}(t)$ the modal displacement vector and $\boldsymbol{\Phi}$ the so-called modal matrix of order ($n_f \times n_f$), normalized with respect to the mass matrix and whose columns are the eigenvectors of the undamped structure, given by the solution to the following eigenproblem:

$$\boldsymbol{K}^{-1}\boldsymbol{M}\,\boldsymbol{\Phi}=\boldsymbol{\Phi}\,\boldsymbol{\Omega}^{-2} \tag{4a}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{M}\,\boldsymbol{\Phi}=\boldsymbol{I}_{n_f} \tag{4b}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{K}\,\boldsymbol{\Phi}=\boldsymbol{\Omega}^2 \tag{4c}$$

In equations (4a,c), besides the already known symbols, $\boldsymbol{I}_{n_f}$ represents the $(n_f \times n_f)$ identity matrix while $\boldsymbol{\Omega}^2$ is a diagonal matrix the elements of which are the square of the natural frequencies of the structure, while the over tilde means the transpose of the relevant quantity.

Once the modal matrix $\boldsymbol{\Phi}$ has been determined, the structure can be defined as a classically-damped one if $\tilde{\boldsymbol{\Phi}}\,\boldsymbol{A}\,\boldsymbol{\Phi}=\boldsymbol{\Xi}$ is a diagonal matrix whose j^{th} component is equal to $2\zeta_j\,\omega_j$, being

ω_j and ζ_j the j^{th} natural frequency and the j^{th} damping coefficient, respectively.

According to the Italian Code a study is performed taking into account all structural modes and assuming a constant damping coefficient equal to 0.05. Making reference to the response spectrum $S_d(T)$ defined in the relevant code and known the natural frequencies and the modal matrix, the displacement vector due to the j^{th} mode can be determined as follows:

$$\boldsymbol{u}_j = \boldsymbol{\Phi}_j \frac{\boldsymbol{\Phi}_j^{\mathrm{T}} \boldsymbol{M\tau} \, S_d(T_j)}{\omega_j^2} \tag{5}$$

According to the above referred guidelines the displacements $\boldsymbol{u}$ and the generalized stresses $\boldsymbol{P}$ are combined in a full quadratic way following the equation:

$$E_\ell = \sqrt{\sum_k \sum_j \rho_{jk} \, E_{j\ell} \, E_{k\ell}} \tag{6}$$

being E_ℓ the ℓ^{th} component of the combined effect of the relevant quantity, $E_{j\ell}, E_{k\ell}$ the ℓ^{th} component of the effect due to j^{th} and k^{th} modes, respectively, and ρ_{ij} the correlation coefficients between j^{th} and k^{th} modes expressed by the equation:

$$\rho_{jk} = \frac{8\,\xi^2 \, \beta_{jk}^{3/2}}{\left(1+\beta_{jk}\right)\left[\left(1-\beta_{jk}\right)^2 + 4\,\xi^2 \, \beta_{jk}\right]} \tag{7}$$

in which $\beta_{jk} = \mathrm{T}_k / \mathrm{T}_j$ and T_j, T_k are the periods of the j^{th} and k^{th} modes.

Let now consider the relevant structure as discretized into n finite elements constituted by elastic perfectly plastic material. The typical v^{th} element geometry is fully described by the s components of the vector $\boldsymbol{d}_v \, (v = 1,2,\ldots,n)$ so that $\tilde{\boldsymbol{d}} = \lfloor \tilde{\boldsymbol{d}}_1, \tilde{\boldsymbol{d}}_2, \ldots, \tilde{\boldsymbol{d}}_v, \ldots, \tilde{\boldsymbol{d}}_n \rfloor$ represents the $n \times s$ supervector collecting all the design variables.

According with the guidelines of the great part of international codes, in particular with the Italian one, the design of the relevant structure must be performed taking into account a fixed action mainly related with the gravitational loads, a quasi statical load related to the wind effect, and a dynamic perfect cyclic load, suitably combined. In the present context even the load related to the wind is modeled as a perfect cyclic load; actually, in any case a generic cyclic load can be described through the superposition of a fixed and a perfect cyclic load.

For the aim of the present paper, we now assume that the actions are represented by appropriate combinations of the above referred loads each of which related to different limit conditions; combination C1: fixed load $\boldsymbol{F}_{0h}$ and (reduced) seismic action related to the response spectrum S_d^S, function of the up-crossing probability in the lifetime selected for the structure; combination C2: amplified fixed load $\boldsymbol{F}_0$ and perfect cyclic load related to the wind action $\boldsymbol{F}_{ciw}$;

combination C3: fixed load $\boldsymbol{F}_{0h}$ and seismic action related to the response spectrum S_d^I, function of a different up-crossing probability in the lifetime selected for the structure.

Obviously, the structure must be capable of suffer the above described load combinations according to different limit conditions; in particular, it must respond in an elastic manner (elastic shakedown) when subjected to load combination 1., it must prevent the instantaneous collapse when subjected, alternatively, to combinations 2. or 3.

In the above defined combinations, $\boldsymbol{F}_{0h}$ and $\boldsymbol{F}_0$ are special combinations of gravitational loads as prescribed by the referenced code, S_d^S and S_d^I are the response spectra related to serviceability and instantaneous collapse conditions, respectively, while the reference mechanical cyclic loads related to the wind action are defined as two opposite and independent load conditions $\boldsymbol{F}_{ciw}$, ($i=1,2$), such that $\boldsymbol{F}_{c1w}=\boldsymbol{F}_w$ and $\boldsymbol{F}_{c2w}=-\boldsymbol{F}_w$; therefore, $\boldsymbol{F}_{ciw}$ is a perfect cyclic load.

3 OPTIMAL DESIGN PROBLEM FORMULATION

Let us consider now an elastic plastic structure as above described and, according to the Italian code and to the assumed load model, be subjected to fixed mechanical loads, quasi static perfect cyclic loads (wind effect) and perfect cyclic dynamic seismic loads. The minimum volume design problem formulation, where suitable constraints are imposed on the elastic shakedown behaviour and on the instantaneous collapse, can be written as follows:

$$\min_{\left(\boldsymbol{d},\boldsymbol{u}_0,\boldsymbol{u}_{0h},\boldsymbol{u}_{cw},\boldsymbol{u}_{jch}^S,\boldsymbol{u}_{jch}^I,\boldsymbol{Y}_0^S,\boldsymbol{Y}_{0iw}^I,\boldsymbol{Y}_{0ih}^I\right)} V \tag{8a}$$

$$\boldsymbol{d}-\bar{\boldsymbol{d}}\geq\boldsymbol{0} \tag{8b}$$

$$\boldsymbol{T}\,\boldsymbol{d}-\bar{\boldsymbol{t}}\geq\boldsymbol{0} \tag{8c}$$

$$\boldsymbol{P}_0=\tilde{\boldsymbol{B}}\,\boldsymbol{u}_0\,,\quad \boldsymbol{K}\boldsymbol{u}_0-\boldsymbol{F}_0=\boldsymbol{0} \tag{8d}$$

$$\boldsymbol{P}_{0h}=\tilde{\boldsymbol{B}}\,\boldsymbol{u}_{0h}\,,\quad \boldsymbol{K}\boldsymbol{u}_{0h}-\boldsymbol{F}_{0h}=\boldsymbol{0} \tag{8e}$$

$$\boldsymbol{P}_{cw}=\tilde{\boldsymbol{B}}\,\boldsymbol{u}_{cw}\,,\quad \boldsymbol{K}\,\boldsymbol{u}_{cw}-\boldsymbol{F}_{cw}=\boldsymbol{0} \tag{8f}$$

$$\boldsymbol{P}_{jch}^S=\tilde{\boldsymbol{B}}\,\boldsymbol{u}_{jch}^S\,,\quad \boldsymbol{u}_{jch}^S=\boldsymbol{\Phi}_j\frac{\tilde{\boldsymbol{\Phi}}_j\boldsymbol{M}\boldsymbol{\tau}\,S_d^S\left(T_j\right)}{\omega_j^2}\,,\quad P_{ch\ell}^S=\sqrt{\sum_j\sum_k\rho_{kj}\,P_{kch\ell}^S\,P_{jch\ell}^S} \tag{8g}$$

$$\boldsymbol{P}_{jch}^I=\tilde{\boldsymbol{B}}\,\boldsymbol{u}_{jch}^I\,,\quad \boldsymbol{u}_{jch}^I=\boldsymbol{\Phi}_j\frac{\tilde{\boldsymbol{\Phi}}_j\boldsymbol{M}\boldsymbol{\tau}\,S_d^I\left(T_j\right)}{\omega_j^2}\,,\quad P_{ch\ell}^I=\sqrt{\sum_j\sum_k\rho_{kj}\,P_{kch\ell}^I\,P_{jch\ell}^I} \tag{8h}$$

$$\boldsymbol{\varphi}_i^S\equiv\tilde{\boldsymbol{N}}\boldsymbol{P}_{0h}+\left(-1\right)^i\tilde{\boldsymbol{N}}\boldsymbol{P}_{ch}^S-\boldsymbol{S}\boldsymbol{Y}_0^S-\boldsymbol{R}\leq\boldsymbol{0}\,,\quad \boldsymbol{Y}_0^S\geq\boldsymbol{0} \tag{8i}$$

$$\boldsymbol{\varphi}_{iw}^I\equiv\tilde{\boldsymbol{N}}\boldsymbol{P}_0+\left(-1\right)^i\tilde{\boldsymbol{N}}\boldsymbol{P}_{cw}-\boldsymbol{S}\boldsymbol{Y}_{0iw}^I-\boldsymbol{R}\leq\boldsymbol{0}\,,\quad \boldsymbol{Y}_{0iw}^I\geq\boldsymbol{0} \tag{8j}$$

$$\boldsymbol{\varphi}_{ih}^I\equiv\tilde{\boldsymbol{N}}\boldsymbol{P}_{0h}+\left(-1\right)^i\tilde{\boldsymbol{N}}\boldsymbol{P}_{ch}^I-\boldsymbol{S}\boldsymbol{Y}_{0ih}^I-\boldsymbol{R}\leq\boldsymbol{0}\,,\quad \boldsymbol{Y}_{0ih}^I\geq\boldsymbol{0} \tag{8k}$$

where equations (8a,i,j,k) hold for $i=1,2$ and $\ell=1,2,.......,n_p$, being n_p the total number of plastic nodes.

In equations (8) $\boldsymbol{d}$ is the design variable vector while $\bar{\boldsymbol{d}}$ represents the vector collecting the imposed limit values for $\boldsymbol{d}$, $\boldsymbol{T}$ is the technological constraint matrix with $\bar{\boldsymbol{t}}$ a suitably chosen technological vector, $\boldsymbol{u}_0$ and $\boldsymbol{P}_0$, $\boldsymbol{u}_{0h}$ and $\boldsymbol{P}_{0h}$, $\boldsymbol{u}_{cw}$ and $\boldsymbol{P}_{cw}$, $\boldsymbol{u}_{jch}^S$ and $\boldsymbol{P}_{jch}^S$, $\boldsymbol{u}_{jch}^I$ and $\boldsymbol{P}_{jch}^I$ are the purely elastic response to the assigned fixed loads, the mechanical cyclic load, the reduced dynamic load related to the j^{th} structural mode, the full dynamic load related to the j^{th} structural mode, respectively, in terms of displacements and generalized stresses, $\boldsymbol{P}_{ch}^S$ and $\boldsymbol{P}_{ch}^I$ the combined generalized stress vectors related to reduced and full seismic actions, $\boldsymbol{\varphi}_i^S$, $\boldsymbol{\varphi}_{iw}^I$ and $\boldsymbol{\varphi}_{ih}^I$, ($i = 1,2$), are the plastic potential vectors related to the elastic shakedown limit (apex *S*) and to the instantaneous collapse limit (apex *I*), respectively, $\boldsymbol{Y}_0^S$, $\boldsymbol{Y}_{0iw}^I$ and $\boldsymbol{Y}_{0ih}^I$, ($i = 1,2$), are fictitious plastic activation intensity vectors related to the elastic shakedown limit and to the impending instantaneous collapse, respectively. Finally, $-\boldsymbol{S}$ is a time independent symmetric structural matrix which transforms the plastic activation intensities into the plastic potentials.

4 NUMERICAL RESULTS

In this section the optimal designs of steel frames have been numerically obtained making reference to the formulations proposed into the previous sections. In particular, a multicriterion design (elastic shakedown, instantaneous collapse) has been determined for two six floor frames.

The relevant frames under examination is plotted in Fig. 1a,b; they are constituted by square box section elements (Fig. 1c). The square side measure is assigned for beams and columns, $\ell = 300$ mm, and for cross bracing, $\ell = 200$ mm, while the constant thickness s is assumed as design variable. The cross bracing elements are suitably weakened by holes so that they posses great extensional stiffness and low plastic resistance, do not fear buckling and exhibit plastic dissipation capacity for traction and for compression. Furthermore, $L_1 = 700$ cm, $L_2 = 400$ cm and $H = 400$ cm, Young modulus $E = 21\ \text{MN/cm}^2$, yield stress $\sigma_y = 23.5\ \text{kN/cm}^2$. Two rigid perfectly plastic hinges are located at the extremes of all elements, considered to be purely elastic, and an additional hinge is located in the middle point of the beams. The interaction between bending moment *M* and axial force *N* has been taken into account. In Fig. 1d the dimensionless rigid plastic domain of the typical plastic hinge is plotted in the plane (N/N_y, M/M_y), being N_y and M_y the yield generalized stress corresponding to N and M, respectively.

The structure is subjected to a fixed uniformly distributed vertical load on the beams, $q_0 = 30$ kN/m, to perfect cyclic concentrated horizontal loads (kN) applied on the nodes (wind effect) $\tilde{\boldsymbol{F}}_w = |24 \quad 26.2 \quad 28.4 \quad 30.5 \quad 32.7 \quad 34.9|$, and to seismic actions. We assume that the seismic masses are equal for each floor, $m = 33.64\ \text{kN}\cdot\text{sec}^2/\text{m}$, and located in the intermediate node at each floor, (Fig. 1a,b). The selected response spectra for serviceability conditions (up-crossing probability in the lifetime 81%) and instantaneous collapse (up-crossing probability in the lifetime 5%) are those corresponding to Palermo, with a soil type B, life time 100 years and class IV. The optimal multicriterion design has been computed solving problem (8), assuming $F_{0j}/F_{0hj} = 1.25$, with F_{0j} and F_{0hj} the j^{th} component of the relevant vectors.

The obtained results show that the optimal structures exhibit an incremental collapse behaviour even for loads not very close to the required limit ones, as it is possible to observe through the relevant Bree diagrams (Fig. 2a,c,e,g). In order to prevent such a behaviour, problem (8) has been improved, for structure in Fig. 1a, by introducing the following constraints:

$$\sum_i \widetilde{\boldsymbol{R}}\boldsymbol{Y}^I_{0iw} \leq \overline{\boldsymbol{D}}_w, \tag{8ℓ}$$

$$\sum_i \widetilde{\boldsymbol{R}}\boldsymbol{Y}^I_{0ih} \leq \overline{\boldsymbol{D}}_h \tag{8m}$$

being $\overline{\boldsymbol{D}}_w$ and $\overline{\boldsymbol{D}}_h$ two suitably chosen limits imposed for the fictitious plastic dissipation at the

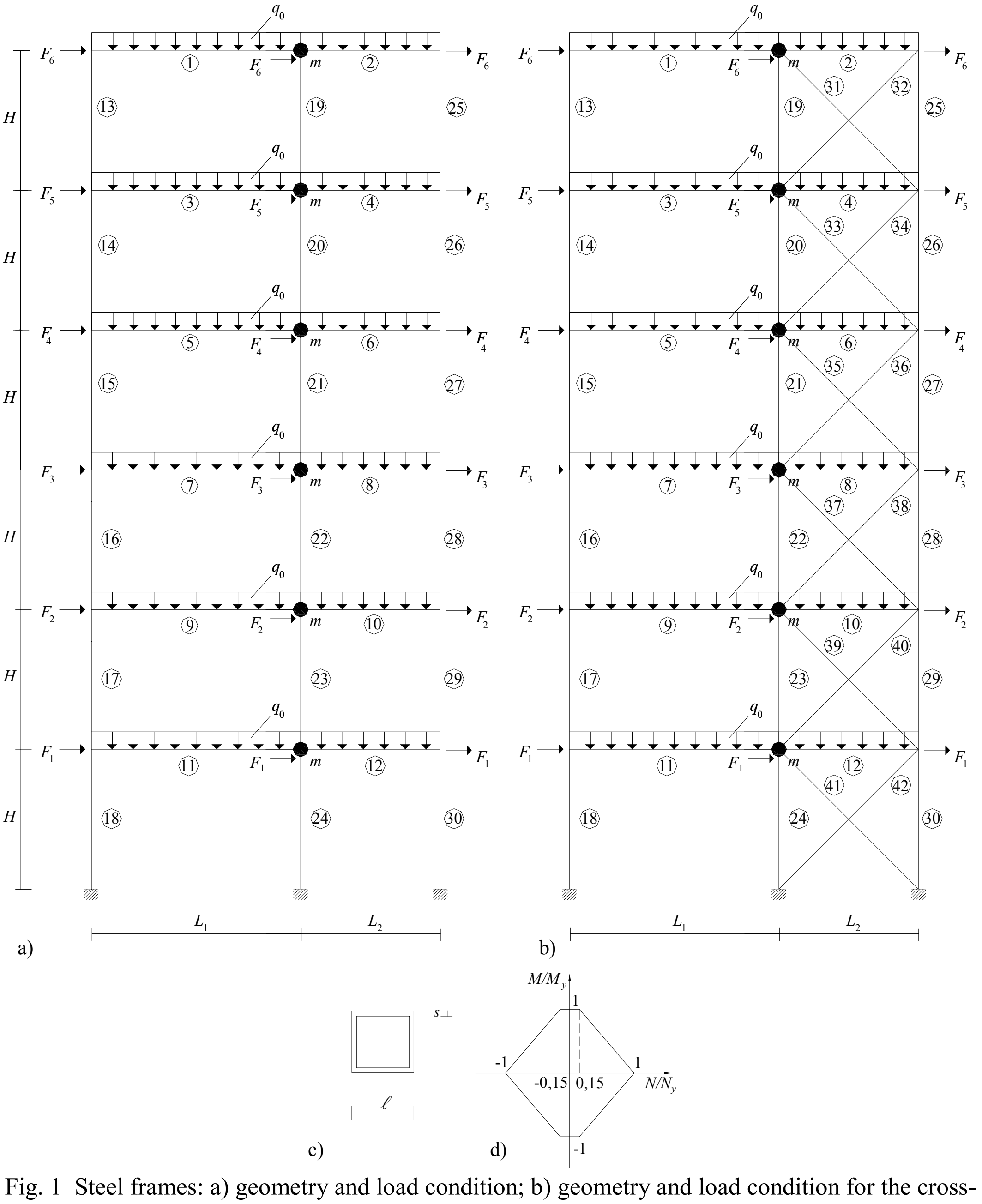

Fig. 1 Steel frames: a) geometry and load condition; b) geometry and load condition for the cross-braced frame; c) typical box cross section; d) rigid plastic domain of the typical plastic hinge.

instantaneous collapse limit related to wind and earthquake effects, and for structure in Fig. 1b simply imposing equal to zero the fictitious plastic activations related to columns and beams. In Fig. (2b,d,f,h) are plotted the Bree diagrams of the improved structures.

An examination of these results shows that for the optimal design reached through the original problem (8) the fixed load condition is quite close to the collapse one, especially for the cross braced frame; the serviceability condition are substantially not influent; the structure collapses for ratcheting even for loads lower than the prescribed ones. In order to deeply investigate the structural response, some useful chosen displacements are determined for the relevant frames performing an elastoplastic analysis with the following load multiplier values: $\xi_{0h}^{I}=1$, $\xi_{ch}^{I}=0.85$ (multipliers of fixed and seismic actions in the seismic limit load combination), $\xi_{0w}^{I}=1.25$, $\xi_{cw}^{I}=1$ (multipliers of fixed and wind actions in the cyclic limit load combination). In Table 1 and 2 the results are summarized, where u^{e} is the horizontal elastic displacement of the upper floor in serviceability conditions, u^{r} is the horizontal residual displacement of the same floor deduced by the above described analysis, w_1^r, w_2^r, w_3^r, w_4^r and w_5^r are the vertical residual displacements of the middle point of the longer beams at floor 1, 2, 3, 4 and 5, respectively. In the following Tables B and C indicate Bending and Cross braced frame results, respectively.

Frame		Volume	u^e	u^r	w_1^r	w_2^r	w_3^r	w_4^r	w_5^r
B	earth.	1.08	68.5	108.6	13.3	22.1	22.9	22.8	21.2
	wind		-----	3.44	4.72	8.89	5.96	4.05	7.15
C	earth.	0.99	34.5	136.1	8.4	14.4	20.1	42.0	136.0
	wind		-----	3.08	58.8	16.7	17.6	17.9	19.2

Table 1. Volumes (m^3) deduced by solving problem (8), elastic and residual displacements (mm).

Frame		Volume	u^e	u^r	w_1^r	w_2^r	w_3^r	w_4^r	w_5^r
B	earth.	1.25	65.8	10.4	1.8	2.3	0.62	14.4	16.7
	wind		-----	1.20	0.002	0.012	0.005	2.63	6.64
C	earth.	1.15	32.4	2.52	0.43	1.12	1.42	0.54	2.72
	wind		-----	0.05	1.46	0.35	0.008	0.002	0.08

Table 2. Volumes (m^3) deduced by solving the improved problem (8), elastic and residual displacements (mm).

5 CONCLUSIONS

The present paper has been devoted to the optimal design of elastic perfectly plastic frames subjected to different load conditions defined as suitable combinations of fixed load, perfectly cyclic loads and dynamic actions. The optimal design problem has been formulated, on the grounds on a statical approach, as the search for the minimum volume structure and two different resistance limits have been simultaneously considered: the elastic shakedown limit and the instantaneous collapse limit, imposing for each one suitably chosen load combinations and appropriate load amplifiers. In the proposed formulation reference has been made to the Italian

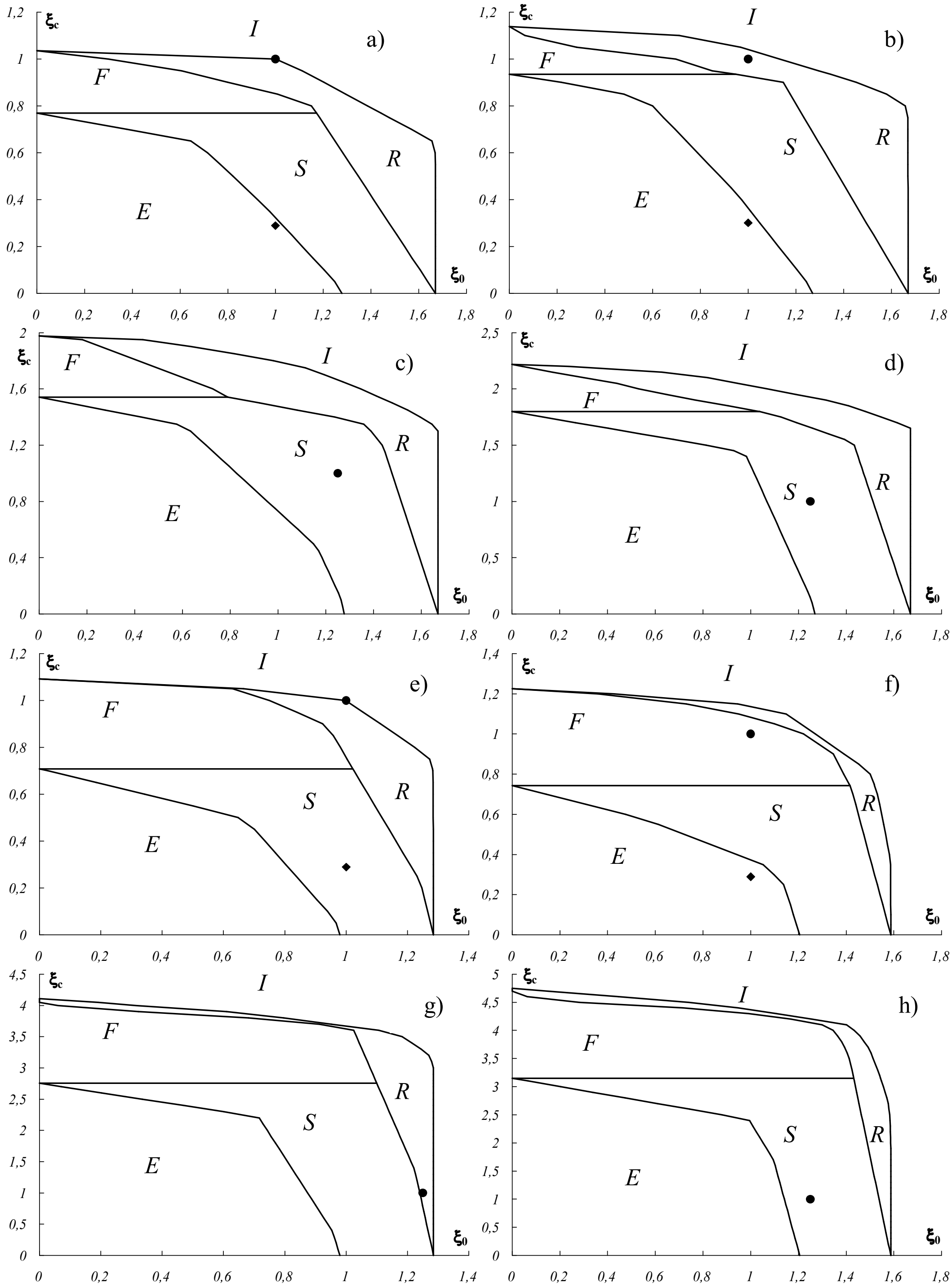

Figure 2: Bree diagrams. a), b) bending frame with seismic loads: original and improved problem; c), d) bending frame subject to wind: original and improved problem; e), f) cross braced frame with seismic loads: original and improved problem; g), h) cross braced frame subject to wind: original and improved problem.

codes related to the structural analysis and design; actually, the serviceability conditions have been defined as the combination of fixed and reduced seismic loads, the ultimate limit loads have been defined alternatively as the combination of fixed and perfect cyclic loads, or as the combination of fixed and dynamic loads. The dynamic features of the given structure have been taken into account in order to identify the seismic loads and to determine the structural response. Two different formulations of the minimum volume design have been proposed: the first one is devoted to the optimal design of the structure with constraints on the elastic shakedown behaviour related to serviceability condition loads and on the instantaneous collapse related to suitably alternative combinations of fixed and perfectly cyclic or dynamic actions, the second one is devoted to the optimal design with the same conditions as before but introducing new constraints related to suitably imposed limits on the fictitious plastic activations. The introduced further constraints guarantee a more safe behaviour of the optimal structure; actually, it has been verified that in such a case the relevant structure exhibits an alternating plasticity behaviour even when the loads reach values very close to the instantaneous collapse ones. The effected numerical applications are related to steel plane frames. In particular, two six plane frames has been investigated. The obtained results are encouraging and furthermore they show that the new designs are characterized by just a very modest cost increment with respect to the safety improvement related to the plastic shakedown behaviour.

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