Non-linear normal modes of a fixed-moored offshore catenary riser

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SUMMARY. Offshore catenary risers are used in the exploitation of deep-water oil and gas fields, such as those along the Brazilian Southeast coast. They are subjected to severe dynamical loads, such as those related to inside flow of fluids under high pressure, external flow of sea currents and imposed motion by the floating production platform, due to sea waves. In this paper the non-linear normal modes (NNM´s) of a fixed-moored catenary riser are addressed, neglecting the motion of the touch down point (TDP) at the touch down zone (TDZ), where the risers meets the seabed, yet taking into account the mobility of the floating production platform at the top end. The NNM´s are useful for the reduced-order modelling of the riser-fluid system, greatly facilitating the forced response analysis under complex dynamic loading, such as that which causes vortex-induced vibration (VIV). NNM´s of catenary risers with moving TDP have already been studied in [2], considering the riser as a curved beam, although the mooring effect has not been taken into account. Here, a finite-element model will be used instead for the moored riser, although the TDP will be kept fixed. In a future work, both the moving TDP and the mooring will be considered. A computational tool [3] based on the invariant manifold procedure proposed by Shaw and Pierre [4] has been used to obtain the NNM´s for 2D frames, using a geometrically non-linear beam theory up to cubic terms.

1 INTRODUCTION

In the past two years Brazil has discovered the largest oil deposits in the country's history and the world's most promising fields since the discoveries made in Kashagan in Kazakhstan in the year 2000. This has put Brazil well on its way to becoming a major producer in the future, but technological and financial hurdles will have to be overcome first. The deposits consist of pre-salt reservoirs. The biggest hydrocarbon accumulations were found in the Santos Basin's Pre-Salt Pole and are located in ultra-deep waters, below a layer of salt that in some areas is thicker than 2,000 meters.

In addition, many onshore and offshore exploration fields have matured notwithstanding the growing global demand for energy and the volatility in oil prices. This scenario has led the operating companies to focus on deep and ultra-deep water exploration bringing forth new concerns on reducing the exploration costs through advanced technological developments.

Consequently, the offshore exploration industry has increased its attention to new realms of research such as non-linear dynamics of risers for deep and ultra-deep water in order to define the most economical and appropriate solution for each reservoir.

If subsea exploration has so far been a complex and demanding activity, from now on, in view of the upcoming developments in deepwater and ultra deepwater, it will be even more challenging.

The offshore industry has already begun to explore in water depths at the limits of the current technology and has plans to access depths over 2,500m. It is developing subsea production systems in preparation for ultra deepwater production, which include more flexible risers and harsh environmental conditions. Under these circumstances, non-linear effects will increase their influence over the riser global dynamics. Moreover, the tendency towards more flexible and lighter subsea structures will boost up the overall non-linear behaviour.

The objective of this paper is to present a numerical study on the non-linear normal modes of a fixed-moored deepwater catenary riser resorting to the invariant manifold approach to determine the free vibration motion. A computational model is sought using the finite element method. The employed finite elements consider geometrical non-linearities. The associated equations of motion take into consideration non-linearities up to third order, leading to third- and forth-order tensors. These non-linear equations allow for both cable and beam behaviour. In this work, the mobility of the floating production unit (FPU) at the top end is taken into consideration by using a spring to model the stiffness of the mooring system and a concentrated mass as the FPU's horizontal inertia. The unilateral contact, a strong source of non-linearities, at the touchdown zone (TDZ) has been neglected here. Instead, the catenary riser has been modelled with a fixed end, just allowing for rotation, at the TDP. Further advances, regarding a numerical approach for the TDZ, are under development, since a considerable programming effort is required to take into consideration the contact problem at TDZ. It is believed that non-linear modes may play an important role in the accurate structural representation of deep-water risers by models with a small number of degrees of freedom. A free-hanging catenary is addressed as a case study considering the first non-linear normal mode of vibration. The ultimate goal is to develop a more comprehensive low-dimensional model of the dynamics of a riser, by introducing non-linear models to capture the coupled dynamics of the fluid-structure system.

2 NONLINEAR MODES

This paper should be regarded as an initial research to evaluate the non-linearl fluid-structure dynamics of risers, which will be modelled as plane frames with geometric non-linearities due to the coupling of tangential and transversal displacements. For the time being, material linearity will be assumed. Torsion and 3D effects, as well as the FPU motion and non-linearities due to the unilateral contact at the TDZ, will be neglected.

Although internal resonance may come into play, thus requiring the consideration of the so-called non-linear multi-modes, the paper will concentrate on the non-linear normal modes.

In future works the non-linear modes, either "normal" or "multi", will be used to project the forced dynamics of risers onto low-dimensional phase spaces, thus generating reliable models with few degrees of freedom, still keeping the essential behaviour of the structure under different sea loading conditions, such as in the cases of high-frequency VIV and low-frequency drifting.

2.1 *The FEM Formulation*

Typically, the equations of motion of a general *n* degree-of-freedom finite-element model of an elastic plane frame with geometric non-linearities under free vibrations read:

$$M_{rs}\ddot{p}_s + D_{rs}\dot{p}_s + U_{,r} = 0 \tag{1}$$

where p_i are the generalised coordinates, and the matrices of mass M_{rs}, equivalent damping D_{rs} and elastic force vector $U_{,r}$ depend on the generalised coordinates and velocities as follows:

$$
\begin{aligned}
M_{rs} &= {}^0M_{rs} + {}^1M_{rs}^{i}\, p_i + {}^2M_{rs}^{ij}\, p_i\, p_j \\
D_{rs} &= {}^0D_{rs} + {}^1D_{rs}^{i}\, \dot{p}_i + {}^2D_{rs}^{ij}\, \dot{p}_i\, p_j \\
U_{,r} &= {}^0K_{rs}\, p_s + {}^1K_{rs}^{i}\, p_i\, p_s + {}^2K_{rs}^{ij}\, p_i\, p_j\, p_s ,
\end{aligned}
\tag{2}
$$

in which ${}^0M_{rs}$, ${}^1M_{rs}^{i}$, ${}^2M_{rs}^{ij}$, ${}^0D_{rs}$, ${}^1D_{rs}^{i}$, ${}^2D_{rs}^{ij}$, ${}^0K_{rs}$, ${}^1K_{rs}^{i}$ and ${}^2K_{rs}^{ij}$, $r,s,i,j = 1,\ldots,n$ are system constants that define the second, third and fourth-order tensors indicated in (2). Einstein's convention for summation is employed.

2.2 *Linear and Non-linear Modes*

During a modal motion, the phase trajectories of a discretised linear system remain confined to two-dimensional eigenplanes, in much the same way as the phase trajectory of a one-degree-of-freedom system with generalised coordinate x remains confined to the plane $x \times \dot{x}$. Due to this invariance property, such eigenplanes are invariant manifolds of the dynamical system.

In non-linear systems the invariant manifolds are no longer planes, and the motions whose trajectories are confined to them are called non-linear normal modes. In general, there are n invariant manifolds, each one corresponding to a different mode; these manifolds contain the equilibrium point and, at this point, are tangent to the corresponding eigenplanes of the linearised system.

Such a topological characterisation of a modal motion suggests the so-called "invariant-manifold procedure" to determine normal modes, which Shaw and Pierre [4] proposed and applied to systems of few degrees of freedom. Soares and Mazzilli [3] extended the procedure to full finite-element models of plane frames.

An alternative technique to evaluate non-linear normal modes of finite-element models was proposed by Mazzilli and Baracho Neto [5]. Such an alternative technique is based on the method of multiple scales.

To handle cases of coupled modal motions of non-linear systems due to internal resonance, the multiple-scales procedure has been successfully extended by Baracho Neto and Mazzilli [6]. In this case, the ensuing forced vibration takes place in an invariant manifold embedded in the phase space, whose dimension is twice the number of the normal modes that interact. This manifold contains a stable equilibrium point, and is tangent there to the sub-eigenspace of the linearised system, which characterises the corresponding linear modes. The multi-mode can be locally described by a linear combination of the linear modes. On this manifold, the system behaves like an M-degree-of-freedom oscillator, where M is the number of coupled normal modes.

2.3 *Invariant Manifold Procedure*

Here, the fundamental steps of the invariant-manifold procedure are followed, having in mind its application to finite-element models of risers.

Introducing the notation $x_i = p_i$ and $y_i = \dot{p}_i = \dot{x}_i$, system (1) can be written in first-order form as

$$
\begin{aligned}
\dot{x}_i &= y_i \\
\dot{y}_i &= f_i(x_1,\ldots,x_n, y_1,\ldots,y_n), \quad i = 1,\ldots,n
\end{aligned}
\tag{3}
$$

Power-series expansions for the functions $f_i(x_1,\ldots,x_n,y_1,\ldots,y_n)$ in the neighbourhood of the equilibrium point are introduced in (4):

$$\begin{aligned} f_i(x_1,\ldots,x_n,y_1,\ldots,y_n) &= B_{ij}x_j + C_{ij}y_j + E_{ijm}x_jx_m \\ &+ F_{ijm}x_jy_m + G_{ijm}y_jy_m + H_{ijmp}x_jx_mx_p + L_{ijmp}x_jx_my_p \ , \\ &+ N_{ijmp}x_jy_my_p + R_{ijmp}y_jy_my_p \end{aligned} \tag{4}$$

where $B_{ij}, C_{ij}, E_{ijm}, F_{ijm}, G_{ijm}, H_{ijmp}, L_{ijmp}, N_{ijmp}$ and R_{ijmp}, $i, j, m, p = 1,\ldots,n$ are known constants.

If, during a modal motion, the trajectory of the solution in the phase-space is restricted to a two-dimensional surface, then it must be possible to express each generalised displacement or velocity as a function of two of them, for instance $u = x_k$ and $v = y_k$, at least in the neighbourhood of the equilibrium point.

By substituting the expressions, named here as *modal relationships*,

$$\begin{aligned} x_i(t) &= X_i(u(t), v(t)), \\ y_i(t) &= Y_i(u(t), v(t)), \quad i = 1,\ldots,n \end{aligned} \tag{5}$$

in (3), we arrive at

$$\begin{aligned} &\frac{\partial X_i}{\partial u}v + \frac{\partial X_i}{\partial v} f_k(X_1,\ldots,X_n,Y_1,\ldots,Y_n) = Y_i \\ &\frac{\partial Y_i}{\partial u}v + \frac{\partial Y_i}{\partial v} f_k(X_1,\ldots,X_n,Y_1,\ldots,Y_n) = \quad , \\ &f_i(X_1,\ldots,X_n,Y_1,\ldots,Y_n) \qquad i = 1,\ldots,n, \end{aligned} \tag{6}$$

which is a non-linear system of partial differential equations having the functions X_i, Y_i as unknowns that may be as difficult to solve as the original equations (1) or (3). However, if we look for an approximate solution, these functions can also be written as power-series expansions

$$\begin{aligned} X_i(u,v) &= a_{1i}u + a_{2i}v + a_{3i}u^2 + a_{4i}uv + a_{5i}v^2 \\ &+ a_{6i}u^3 + a_{7i}u^2v + a_{8i}uv^2 + a_{9i}v^3 \\ Y_i(u,v) &= b_{1i}u + b_{2i}v + b_{3i}u^2 + b_{4i}uv + b_{5i}v^2 \\ &+ b_{6i}u^3 + b_{7i}u^2v + b_{8i}uv^2 + b_{9i}v^3 \end{aligned} \ , \tag{7}$$

where a_{ji}, b_{ji}, $j = 1,\ldots,9$, $i = 1,\ldots,n$ are constants to be determined.

Now, if we substitute (7) and (4) in (6), a system of non-linear polynomial equations having the *a*'s and *b*'s as unknowns is formed. In general, there are *n* solutions to this system, each one corresponding to a different set of modal relationships (5), i.e., a different invariant manifold. Moreover, substituting any of these solutions in (7) and the resulting expressions in (5), the *k*-th equation in (3)-(4) — called the *modal oscillator equation* — characterises the dynamics of the corresponding mode.

Details of the procedure just outlined are avoided here for brevity, but can be found in Soares and Mazzilli [3], where it is also shown that the solution of the non-linear polynomial equation mentioned above can be avoided, provided the eigenvalues and eigenvectors of the linearised system are known.

3 NUMERICAL RESULTS

Figure 1 depicts a planar steel catenary riser (SCR). The geometrical and mechanical properties of the SCR are seen in Table 1. Typical values of natural periods of oscillation of a 88,000 ton semi-submersible platform, moored in 1,800m water depth, are of order 250s, much larger than the first (linear) natural period of the catenary riser, of order of 30s (Figure 2 presents numerical solutions for three different values of axial rigidity and a comparison with WKB analytical approximation; no current; $\theta_L = 70^\circ$; from Pesce and Martins [7]), i.e., the riser dynamics may be considered quasi-static, what turns the hypothesis of fixed-end at the touch down point quite acceptable [7], at least in an initial study. A further local correction at TDZ may be then applied, e.g., via a boundary-layer technique [8].

Young's modulus	$E = 2.1\times10^{11}\,\text{N/m}^2$
Riser length	$L = 1{,}800\text{m}$
Cross-section area	$A = 1.1021\times10^{-2}\,\text{m}^2$
Cross-section moment of inertia	$I = 4.72143\times10^{-5}\,\text{m}^4$
Riser external diameter	$D = 2.032\times10^{-1}\,\text{m}$
Riser thickness	$e = 19.05\text{mm}$
Initial tension (at the top)	$T_{0t} = 2\times10^6\,\text{N}$
Initial tension (at the bottom)	$T_{0b} = 6.914\times10^5\,\text{N}$
Riser mass per unit length (water inside + added mass)	$m = 108\text{kg/m}$
Riser weight per unit length	$p = 727\text{N/m}$

Table 1: Mechanical and geometrical riser properties.

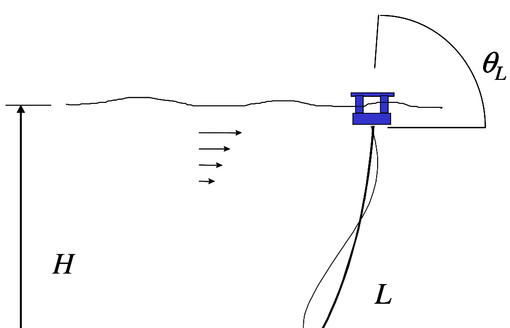

Figure 1: Mechanical and geometrical riser problem (illustration).

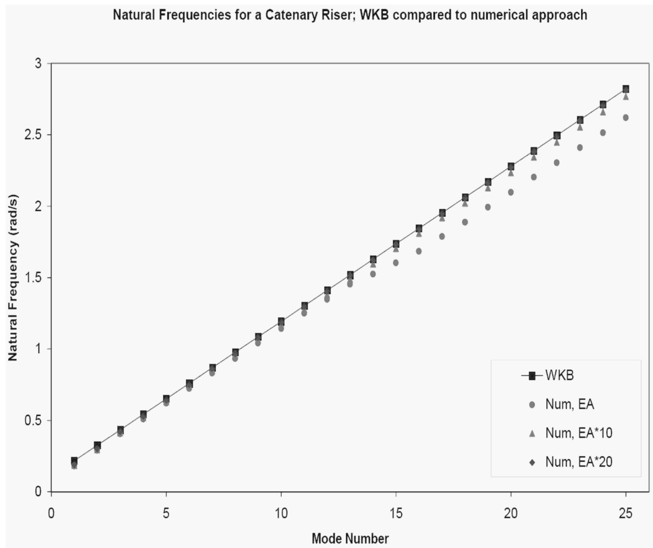

Figure 2: Natural frequencies of a SCR.

Figure 3 shows the employed finite-element model and the boundary conditions for the fixed-moored SCR. A spring is placed at the riser top with the intention of modelling the stiffness of the mooring system. The spring stiffness is *K=90,000 kN/m*. Also, a concentrated mass *M=151,360 ton* is set at the riser top in order to allow for the horizontal inertia of the FPU plus added-mass effects. Figure 4 portrays the SCR finite element model with the numbering of the degrees of freedom. The modal displacement *u* is chosen as the fiftieth degree of freedom. The finite element model consists of 26 non-linear straight beam elements. Although it is only an in-plane analysis, the handling of third- and fourth-order tensors requires a considerable computational effort. To calculate the non-linear modes of this model, it took 17 hours of processing time using a 1.6GHz processor and 2GB of RAM memory.

Using the invariant manifold approach, the non-linear modal oscillator for the first mode is sought as follows:

$$\begin{gathered} \ddot{u} + 0.34646E\text{-}01u + 0.18817E\text{-}13v + 0.11840E\text{-}04u^2 \text{-} 0.20076E\text{-}16uv + 0.33617E\text{-}02v^2 \quad , \\ \text{-}0.31892E\text{-}06u^3 \text{-} 0.59334E\text{-}16u^2v + 0.34710E\text{-}04uv^2 + 0.53243E\text{-}15v^3 = 0 \end{gathered} \tag{8}$$

where u (the fiftieth degree of freedom Figure 4) and v are the modal displacement and the modal velocity ($v = \dot{u}$), respectively.

Equation (8) may be rewritten, by eliminating the almost-null terms, as follows:

$$\ddot{u} + 3{,}46\times10^{-2}u + 1{,}18\times10^{-5}u^2 + 3{,}36\times10^{-3}v^2 - 3{,}19\times10^{-7}u^3 + 3{,}47\times10^{-5}uv^2 = 0\ . \tag{9}$$

Figures 5 e 6 show, respectively, the time-history response and the phase portrait obtained integrating equation (9). The Runge-Kutta Method of fourth order was employed to integrate (9). The initial conditions are *u(0) = 50 m* and *v(0) = 0*.

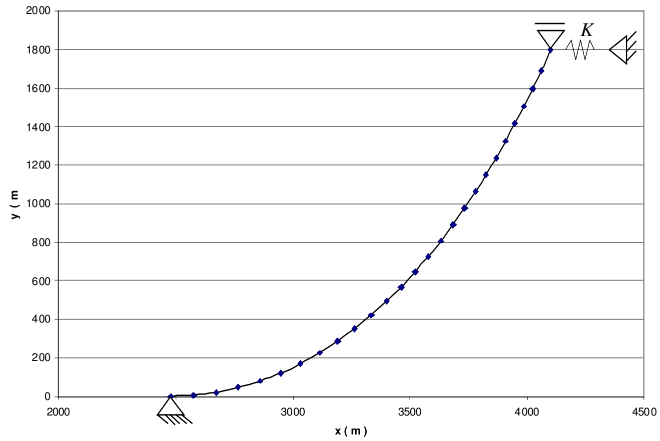

Figure 3: Boundary conditions.

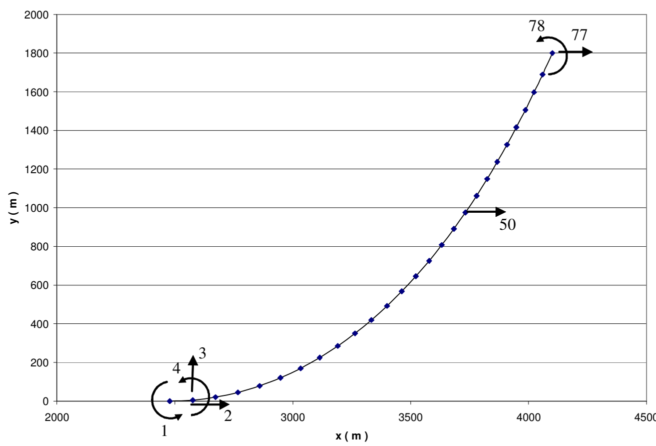

Figure 4: The numbering of the degrees of freedom. The fiftieth degree of freedom is chosen as the modal displacement *u*.

Likewise, Figures 7 e 8 depict, respectively, time-history response and phase portrait for the following initial conditions: *u(0) = 100 m* and *v(0) = 0*. Observing Figures 5 to 8, it is noticeable that, although the linear response partially agrees with the non-linear one, there are remarkable differences between them. It is noteworthy the asymmetric phase trajectories with respect to the modal-velocity axis in the non-linear response (the "minimum" modal displacement is -64m for the non-linear response in opposition to -50m for the linear one in Figure 6). In Figure 8, the minimum non-linear modal displacement is -171m, while the linear one is -100m. Such results are related to the SCR static equilibrium configuration that leads to an asymmetric stiffness. The latter explains the non-symmetric behaviour observed in the Figures 5 to 8.

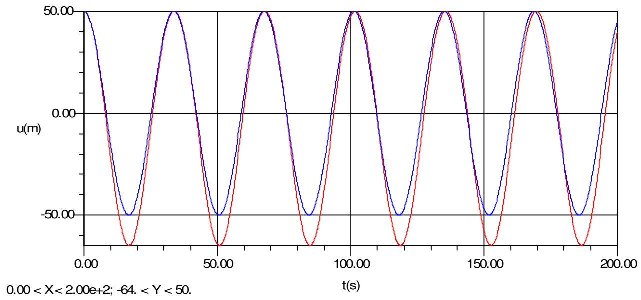

Figure 5: Modal variable time history. Linear[blue] and Non-linear[red].

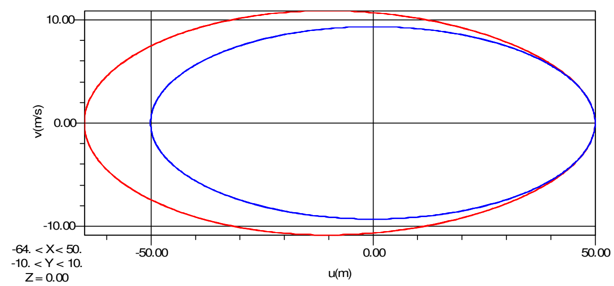

Figure 6: Phase portrait of modal variable. Linear[blue] and Non-linear[red].

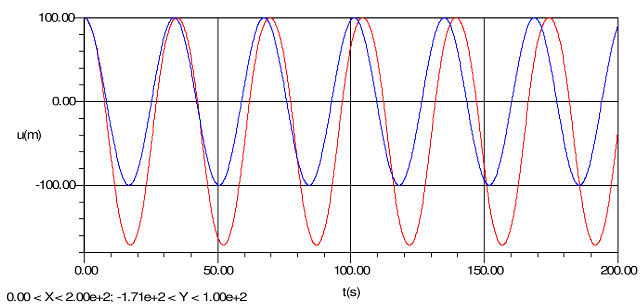

Figure 7: Modal variable time history. Linear[blue] and Non-linear[red].

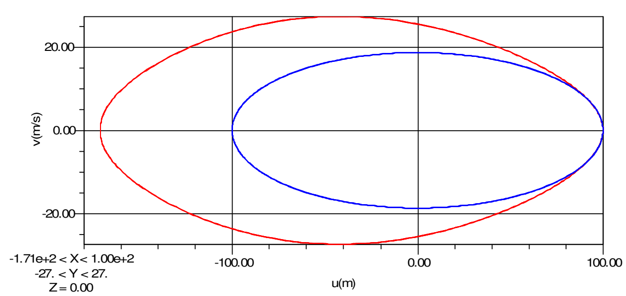

Figure 8: Phase portrait of modal variable. Linear[blue] and Non-linear[red].

Other simulations, considering no mooring system, lead to (algebraically) smaller minimum non-linear modal displacements For the same riser but considering fixed ends, the minimum displacement is -83m for $u(0) = 50m$ and $v(0) = 0$ and -217m for $u(0) = 100m$ and $v(0) = 0$. The physical reasoning behind these results is that the FPU, due to its huge inertia, absorbs a considerable part of the kinetic energy when allowed to move, thus reducing the riser modal response. By studying equation (9), one may notice that the terms in u^3 and uv^2 have opposite signs, i.e., the term in u^3 reduces the system stiffness, whilst the uv^2 term behaves in the opposite way. The winner term depends on the riser parameters/geometry and the given initial conditions. Therefore, it is not trivial to identify the trend to hardening or softening. For initial amplitude of 50m, the non-linear period (33.9s) is slightly larger than the linear one (33.8s). Following this tendency, the non-linear period increases to 34.9s when the initial amplitude is set equal to 100m. The terms in u^2 e v^2 are responsible for the asymmetric behaviour since they are not found in the solutions of symmetrical systems. It is easy to understand their role if one keeps in mind that these terms have the same sign (positive in this study) during the system motion. This means their contribution is non-symmetric with respect to the modal displacement.

It is important to emphasize that unrealistic values of amplitude have been used in this work in order to stress interesting qualitative respects of the results here addressed. For practical purposes, a customisation effort would be needed to adjust the developed procedure to the field requirements.

4 CONCLUSIONS

Results point out that non-linear modelling of riser structural behaviour might be required for large-amplitude vibration. Relevant differences might then appear in the modal shapes and phase trajectories of steel catenary risers with compliant supports. Those latter differences may be important, if drag and inertia fluid loads are further considered.

It is to be further investigated the possibility of non-linear coupling of lower modes, which may require the consideration of multi-modes. For both catenary and straight pre-tensioned risers, for which linear frequencies may be shown to be nearly proportional to the mode number, it would not come as a surprise the appearance of a 1:2 or 1:3 internal resonance of lower modes, leading to strong non-linear couplings. The coupling analysis would also be relevant in the case of multi-modal excitation of higher modes in VIV, despite the fact that, in such a dynamic regime, the amplitude of oscillations is usually small, of the order of one or two diameters only.

It is recalled here that the unilateral contact at TDZ and the motion of the TDP have not been taken into account in this study, so that these effects should be looked at next.

Non-linear modes are believed to play an important role in degree-of-freedom model reduction. To assess how good or bad are the qualitative and quantitative results obtained with the reduced-order model generated from the NNM´s, the riser response to dynamic loads — such as inside flow, VIV and FPU drifting — must be compared with those obtained with the full finite-element model. The first steps in this direction have already been taken in [1].

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