

Dynamic phenomena of interfacial cracks in laminated structures

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SUMMARY. A general model to predict dynamic behavior of interfacial cracks in laminated structures is proposed. The model is developed on a combined approach based on Fracture Mechanics and moving mesh methodology. The former is utilized to predict the crack growth, whereas the latter defines the way to take in account for the geometry changes on the basis of the invoked fracture parameters. Consistently to the Fracture Mechanics, the crack propagation depends from the energy release rate and its mode components, which are calculated by means of the decomposition methodology of the J-integral expression. The geometry variation, produced by the crack advance, is taken into account by means of a moving mesh strategy based on the Arbitrary Lagrangian-Eulerian (ALE) formulation. The coupling characters of the governing equations of the proposed model arising from the Fracture Mechanics and the moving mesh methodology are discussed. Comparisons with experimental results are reported to validate the proposed modeling. Moreover, a parametric study is developed to investigate some features regarding of the crack tip behavior, such as crack arrest phenomena and rate dependence effects of the interfacial crack growth.

1 INTRODUCTION

Composite materials are frequently affected by interface cracking damage modes, which strongly reduce the structural integrity of the laminated structures, leading to premature and catastrophic failure modes. From experimental point of view, many observations on laminated composite structures have shown that typically the evolution of such interfacial cracks is strongly time dependent. The measured crack tip speeds during the crack propagation are relatively high, ranging also close to the Rayleigh wave speed of the material [1]. Moreover, the crack tip behavior is strongly affected by the time rate of the external loading, which typically produces high amplifications of the fracture parameters, identifying the crack tip behavior. Therefore, in order to predict accurately the interfacial crack growth, models developed in dynamic framework are strongly required. Moreover, the analysis of delamination phenomena need a detailed description of the growth area since, typically, the interfacial cracks propagate very rapidly, i.e. at such speeds close to those of the material waves.

In order to predict the crack growth conditions, several approaches have been proposed in the literature. For instance, the node release technique [2] is based on the assumption that the crack growth is described by uncoupling nodes at the crack faces, whose acting tractions are reduced as far as the crack opens. In this approach, the evolution of the crack is strictly dependent from the size of the element mesh around the crack tip, since it governs the amount of the crack advance. Moreover, the advancing process is not really continuous since a proper iteration scheme is necessary to evaluate accurately the dynamic crack growth during the time integration. Models based on the virtual crack closure method (VCCM) calculate the energy release rate as the work

performed by the internal traction forces at the crack faces during a virtual crack advance of the tip. In dynamic fracture mechanics, the VCCM is applied by using the modified form, in which the ERR, during the time integration, is evaluated as a function of the product between the reaction forces and the relative displacements at the crack tip and at nodes close to the crack tip front, respectively [3]. The prediction of the energy release rate is strictly dependent from the mesh discretization of the crack tip. Analogously to the node release technique, the crack growth is not continuous, since the crack is able to advance only of a length equal to that adopted in the definition of the mesh element size at the tip zone. Moreover, in order to evaluate correctly the crack evolution during the time integration, an accurate discretization of the crack tip is needed and thus the computational cost of the analysis may increase notably. Crack growth phenomena can be predicted also by means of damage formulation making the use of interface cohesive elements. In this framework, strain softening interface elements with a damaged constitutive relationship are introduced between crack faces. In order to predict the crack growth phenomena, accurately, a detailed finite element mesh at the crack tip front is often required [4].

Crack growth phenomena should be analyzed by means of the moving mesh methodology, which is able to simulate the motion of an initial defect by changing the position of the mesh points by means of prescribed displacements or velocities. Typically, moving mesh techniques are utilized in the field of fluid mechanics, since large displacements are involved in the deformed configuration. However, some applications can be recovered in solid mechanics also in the context of crack propagation. In particular, moving element procedures for non-singular finite element methods can be recovered in [5]; in which the entire mesh is moved with the crack tip. Contrarily, local mesh update procedures based on the motion of a small portion of the crack tip are developed by Nishioka and Atluri [6], in which the singular fields are reproduced as far as the finite elements move with the crack.

In the present paper, a combination of Fracture Mechanics and moving mesh methodology. The former predicts the crack growth, by the use of the ERR concept and a proper crack advance criterion, whereas the latter is introduced to account for the changes of the geometry during the crack evolution. The moving mesh strategy is developed in the framework of Arbitrary Lagrangian-Eulerian (ALE) methods. The mesh elements configuration is decoupled by the material motion introducing a fictitious reference coordinate system, in which the position of the mesh points does not introduce any mesh distortion during the geometry variation. This choice is motivated by the fact that in composite laminated structures, the crack path is constrained to advance in the interfaces and thus the moving mesh technique becomes a suitable tool to modify the geometry of the model and to account for the evolution of preexisting interfacial cracks. In order to ensure an accurate prediction of the crack growth, the dynamic energy release rate mode components are calculated by means of the decomposition methodology of the J-integral [7], which is proposed in the framework of an unsteady crack growth. It is worth noting the integration path utilized to calculate the J-integral and its mode components moves with the crack tip, ensuring a high discretization of the mesh only on a small region around the crack tip. However, as far as distorted mesh is achieved due to mesh motion of the tip, a standard remeshing algorithm is utilized to ensure accuracy in the finite element results. The crack tip motion is based on an explicit dynamic criterion, in which the relation between ERR mode components and velocity of the crack tip is assumed to be a material property of the composite laminate. Comparisons with experimental results are reported to validate the proposed modeling. Moreover, a parametric study is developed to show some characteristic phenomena of the dynamic crack growth.

2 FRACTURE MECHANICS AND MOVING MESH METHOD: A COMBINED APPROACH

The general formulation of the structural model is consistent to a 2D plane stress approach, in which the behavior of each lamina is homogeneous, linear and elastic. The crack growth is predicted by the use of the moving mesh methodology based on an ALE formulation. This technique despite the conventional Lagrangian or Eulerian approaches provides a better way to take into account mesh movements [5-6]. In order to account the behavior of laminated structures, a multilayer formulation is adopted. In particular, the composite structure is modeled as an assembly of orthotropic elastic layers, which are connected each other by perfect and imperfect interfaces, reproducing the continuity between each lamina or the presence of initial interfacial defects (Fig.1).

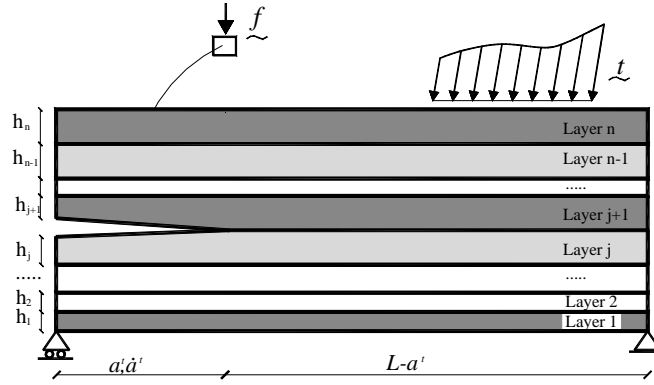


Figure 1: Schematic illustration of the delaminated model subjected to external loads

The governing equations in the material or Lagrangian configuration can be written by means of the principle of d'Alembert, taking into account, for each lamina, virtual works of inertial, external and internal forces:

$$\sum_{i=1}^n \int_{V_i} \underline{\sigma} \delta \underline{\nabla} \underline{u} dV + \sum_{i=1}^n \int_{V_i} \rho \ddot{\underline{u}} \delta \underline{u} dV = \sum_{i=1}^n \int_{\Omega_i} \underline{t} \delta \underline{u} dA + \sum_{i=1}^n \int_{V_i} \underline{f} \delta \underline{u} dV \quad (1)$$

where n is the total number of layers, \underline{u} are the actual displacement vector of the laminate, $\underline{\sigma}$ is the Cauchy stress tensor, \underline{t} is the traction forces vector on the free surface, \underline{f} is the volume forces vector, dV and dA are the volume and the loaded area in the material configuration. Consistently to ALE formulation, the motion of the body is described in the referential configuration B_L , in which the corresponding coordinates (\underline{r}) do not coincide with the material points, but proceed arbitrarily in such a way to reduce the mesh distortion during the geometry changes produced by the crack movements (Fig.2). In the referential coordinates, the motion of the continuum is described by means of a family of mapping $\chi: B_R \rightarrow B_L$ between the material and referential configurations, as:

$$\underline{X} = \chi(\underline{r}, t), \quad \underline{u}(\underline{X}, t) = \underline{u}_R(\chi(\underline{r}, t), t) = \underline{u}_R(\underline{r}, t), \quad (2)$$

where \underline{u}_R is the displacements vector in the referential configuration. Moreover, the gradient operators are related each other by introducing the Jacobian \underline{J} of the transformation between the

material and the referential configurations as:

$$\nabla_{\underline{X}} \underline{u} = \nabla_{\underline{r}} \underline{u} \underline{J}^{-1}, \quad \underline{J} = \frac{d\underline{X}}{d\underline{r}} = \nabla_{\underline{r}} \chi(\underline{r}, t), \quad (3)$$

According to ALE formulation, the time derivatives of a generic physical field in referential and material configurations, represented here and in the following with the dot and the prime, respectively, can be related by the following relationship [6]:

$$\dot{f} = f' - \underline{X}' \frac{d}{d\underline{X}} f(\underline{X}, t), \quad (4)$$

where \underline{X}' represents the relative velocity of the grid points in the material reference system.

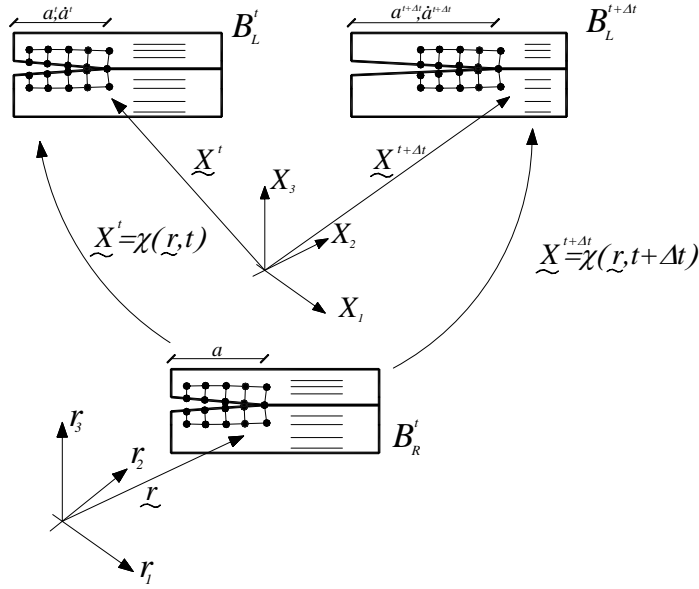


Figure 2: Referential and Material configurations

Therefore, substituting Eq.s (3)-(4) into Eq.(1), the governing equations for the laminated structure are written in the referential configuration as:

$$\begin{aligned} & \sum_{i=1}^n \int_{V_{r_i}} \underline{C} (\nabla_{\underline{r}} \underline{u}_R \underline{J}^{-1}) \delta (\nabla_{\underline{r}} \underline{u}_R \underline{J}^{-1}) \det(\underline{J}) dV_r + \sum_{i=1}^n \int_{V_{r_i}} \rho [\underline{u}_R'' - 2 \nabla_{\underline{r}} \underline{u}_R' \underline{J}^{-1} \cdot \underline{X}' - (\nabla_{\underline{r}} \underline{u}_R \underline{J}^{-1}) \cdot \underline{X}'' + \\ & + \nabla_{\underline{r}} (\nabla_{\underline{r}} \underline{u}_R \underline{J}^{-1}) \underline{J}^{-1} \underline{X}' \underline{X}' + \nabla_{\underline{r}} \underline{u}_R \underline{J}^{-1} \cdot (\nabla_{\underline{r}} \underline{X}' \underline{J}^{-1}) \underline{X}'] \delta \underline{u}_R \det(\underline{J}) dV_r = \quad , \quad (5) \\ & = \sum_{i=1}^n \int_{\Omega_{r_i}} \underline{t} \delta \underline{u}_R \det(\bar{\underline{J}}) d\Omega_r + \sum_{i=1}^n \int_{V_{r_i}} \underline{f} \delta \underline{u}_R \det(\underline{J}) dV_r \end{aligned}$$

where \underline{C} is the elasticity matrix, $\det(\bar{\underline{J}})$ is the determinant of a scalar metric representing the ratio of differential areas and Ω_r is the loaded area in the referential configuration. It is worth noting that in Eq.(5) the unknown quantities are the kinematic variables for both plane stress and mesh points position. Therefore, the mesh movements should be addressed to reduce distortions of the mesh elements and to handle for changes of the geometry produced by the crack growth. In order

to account mesh movements of the domain, a rezoning mesh method is required. The aim of the mesh regularization technique is to provide an easy way to move the position of the crack tip front, keeping the computational mesh undistorted during the whole calculation. In the proposed formulation, a smoothing variational method based on Winslow approach is utilized, in which the horizontal and vertical mesh displacements, namely $\Delta X_1 = X_1 - r_1$ and $\Delta X_2 = X_2 - r_2$, are evaluated by solving the following expressions :

$$\nabla_x^2 \Delta X_1 = 0, \quad \nabla_x^2 \Delta X_2 = 0. \quad (6)$$

Internal and external boundary conditions need to be introduced to reproduce the crack growth. With reference to the laminate scheme reported in Fig.3, the boundary conditions must reproduce the geometry changes produced by the crack tip motion. Without loss of generality and for clearness of exposition, a single delamination model in a double cantilever beam (DCB) scheme is considered (Fig.3). In particular, the external boundary conditions must be written to reproduce a mesh motion, which is fixed in the vertical direction for all the contour lines and for the horizontal lines, the evolution of the mesh points should avoid any displacements along the longitudinal direction.

The crack tip motion is expressed by means of an explicit speed dependent fracture criterion in terms of the mode components and the corresponding critical values. In the proposed modeling it is assumed that, the dynamic fracture toughness of the ERR mode components are based on the following expressions:

$$G_{ID}(c_t) = \frac{G_{0I}}{1 - \left(\frac{c_t}{V_R}\right)^m}, \quad G_{IID}(c_t) = \frac{G_{0II}}{1 - \left(\frac{c_t}{V_R}\right)^m}, \quad (7)$$

in which (G_{0I}, G_{0II}) correspond to the initial toughnesses of the ERR mode components. Therefore, the toughness criterion under mixed mode loading conditions is based on the following additive expression, which predicts the crack growth as far as it becomes positive:

$$g_f = \frac{G_I}{G_{ID}(c_t)} + \frac{G_{II}}{G_{IID}(c_t)} - 1 \leq 0 \quad (8)$$

where (G_I, G_{II}) are the actual mode components at a generic time. Therefore, assuming initial conditions to be homogeneous, the boundary conditions are defined by the following relationships:

$$\begin{aligned} (\Delta X_1 = 0, \Delta X_2 = 0) & \quad \text{on } \Omega_1 \cup \Omega_2, \\ \Delta X_2 = 0 & \quad \text{on } \Omega_3 \cup \Omega_4 \\ \Delta X_1' = 0 & \quad \Leftrightarrow \text{if } g_f < 0 \quad \text{on } \Omega_t, \\ \Delta X_1' = c_t & \quad \Leftrightarrow \text{if } g_f \geq 0 \quad \text{on } \Omega_t, \\ \Delta X_2' = 0 & \quad \text{on } \Omega_t, \\ \Delta X_1(0) = 0, \Delta X_2(0) = 0, \Delta \dot{X}_1(0) = 0, \Delta \dot{X}_2(0) = 0 \end{aligned} \quad (9)$$

Weak forms of smoothing differential equations are derived, by multiplying Eq.s (23) by a weight functions $w_1(X_1, X_2)$ and $w_2(X_1, X_2)$ then integrating by part. Moreover, the boundary conditions regarding the prescribed crack tip speed, i.e. Eq.(9), is taken into account for by means of non-ideal weak constraint based on the Lagrangian multiplier method [8]. Therefore, the resulting equations regarding the ALE formulation are:

$$\begin{aligned} \int_V \underline{\underline{v}}_{\underline{\underline{x}}} \Delta X_1 \underline{\underline{v}}_{\underline{\underline{x}}} w_1 dA + \oint_{\Omega} [\delta \lambda (X'_1 - c_t) + \lambda \delta \dot{X}_1] ds = 0, \\ \int_V \underline{\underline{v}}_{\underline{\underline{x}}} \Delta X_2 \underline{\underline{v}}_{\underline{\underline{x}}} w_2 dA = 0, \end{aligned} \quad (10)$$

where λ is the Lagrangian multiplier. Finally, introducing the vector $\Delta \underline{\underline{X}}^T = [\Delta X_1, \Delta X_2]$, Eq.(10) is written in the referential coordinate system by using transformation rules given by Eq.(3),(4) leading to the following relationship:

$$\int_{V_r} (\nabla_{\underline{\underline{r}}} \Delta \underline{\underline{X}} J^{-1}) \cdot (\nabla_{\underline{\underline{r}}} w J^{-1}) \det(J) dV_r + \oint_{\Omega_r} [\delta \lambda (\underline{\underline{X}}' - \underline{\underline{c}}_t) \underline{\underline{i}} + \lambda \delta \dot{\underline{\underline{X}}}_1] (\bar{J}) ds = 0, \quad (11)$$

with $w^T = [w_1, w_2]$ the weight function vector and $\underline{\underline{i}}^T = [1, 0]$ the propagation direction vector of the interfacial crack and $\underline{\underline{c}}_t^T = [c_t, 0]$ is the crack tip speed vector.

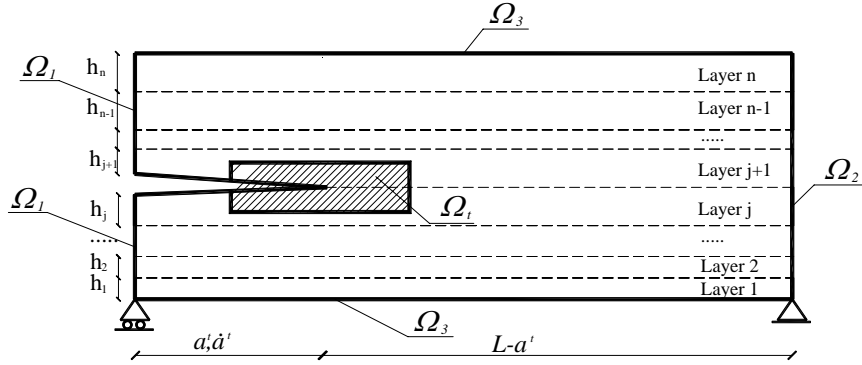


Figure 3: Referential and Material configurations

3 EVALUATION OF THE DYNAMIC ENERGY RELEASE RATE

The dynamic ERR is evaluated by using the J-integral concept [6]. In the literature several expressions exist to evaluate dynamic energy release rate based on J-integral formulation. In particular, the ERR is defined as the rate of mechanical energy flow out of the body and into the crack tip per unit crack advance and it should be recovered by the limit of the following balance equation

$$J = \lim_{\epsilon \rightarrow 0} \oint_{\Omega_\epsilon} \left[(W + K) n_1 - \underline{\underline{t}} \frac{\partial \underline{\underline{u}}}{\partial \underline{\underline{X}}} \right] ds \quad (12)$$

where n_1 is the component with respect to X_1 of the normal along integration contour, $\underline{\underline{u}}$ is the displacement, $\underline{\underline{t}}$ the traction vector, f is the body force, ρ is the volume density, Ω_ϵ is a closed path surrounding the crack tip front, W is the strain energy density and K is the kinetic energy. However, since the analysis is developed in dynamic framework, the integral function in Eq. (12) loses its path independent property. Moreover, the evaluation of the limit procedure reported in Eq.(12), involves numerical complexities in the integral calculation due to the high gradients of the crack tip fields. To overcome such difficulties, an alternative expression of the J-integral, which is path independent, is utilized [6]:

$$J = \oint_{\partial\Omega_c} \left[(W + K)n_1 - t \frac{\partial u}{\partial X} \right] ds + \int_{\Omega_c} \left[\rho(\ddot{u} - \underline{f}) \nabla \underline{u} - \rho \dot{\underline{u}} \nabla \dot{\underline{u}} \right] dA \quad (13)$$

In those cases where mixed mode loading conditions are involved, the total value of the ERR and the corresponding modal components are evaluated by the use of the decomposition methodology of the J -integral expression [7]. In this framework, the mode components of the energy release rate are written as:

$$J_I = G_I = \oint_{\partial\Omega_c} \left[(W^S + K^S)n_1 - \sigma_{ij}^S n_j \frac{\partial u^S}{\partial x} \right] ds + \int_{\Omega_c} \left[\rho(\ddot{u}^S - \underline{f}^S) \nabla \underline{u}^S - \rho \dot{\underline{u}}^S \nabla \dot{\underline{u}}^S \right] dA, \quad (14)$$

$$J_{II} = G_{II} = \oint_{\partial\Omega_c} \left[(W^{AS} + K^{AS})n_1 - \sigma_{ij}^{AS} n_j \frac{\partial u^{AS}}{\partial x} \right] ds + \int_{\Omega_c} \left[\rho(\ddot{u}^{AS} - \underline{f}^{AS}) \nabla \underline{u}^{AS} - \rho \dot{\underline{u}}^{AS} \nabla \dot{\underline{u}}^{AS} \right] dA,$$

with $J = J_I + J_{II}$, (G_I, G_{II}) are the mode I and mode II ERR components. It is worth noting that the superscripts (S) and (AS) in Eq.(14), represent the symmetric and antisymmetric components with respect to a plane containing the crack tip. Expressions reported in Eq.(14) are validated numerically through comparisons arising from FE results based on VCCM.

4 FINITE ELEMENT IMPLEMENTATION AND NUMERICAL RESULTS

Governing equations given by Eq.s (5), (11) introduce a set of non linear equations, which are solved numerically, using a user customized finite element program, i.e. COMSOL Multiphysics TM version 3.5 [8]. Finite element expressions are written for 2D plane stress modeling, utilizing Lagrangian interpolation shape functions. The governing equations regarding the plane stress and ALE formulations are written in the weak form in the referential configuration and are solved by using a finite element approach. In order to compute accurately the ERR with the aid of J-integral formula, a fine discretization mesh and a standard numerical integration method (quadrature) is adopted on the contour line and on the area surrounding the crack tip.

At first, comparisons with experimental results are performed to validate the proposed modeling for a loading scheme involving mixed mode loading condition. Comparisons with experimental results available from the literature [9] on a carbon/epoxy AS4/3501-6 unidirectional composite are proposed. The structural scheme refers to a modified end-notched flexural (ENF) scheme. The material properties and parameters regarding the crack criterion are reported in Tab.n.1. The specimen is 274 mm long, 14 mm wide and 6.8 mm thick with a 59 mm precrack. The crack is located at 1/3 of the total laminate thickness, producing mixed mode deformations. The loading rate of the applied displacements at midpoint of the laminate is equal to 0.025 mm/s. The crack propagation is produced introducing a strip of adhesive film at the crack tip, which enforces the crack to grow at high speeds. In order to reproduce correctly the experimental results, at first the laminate is displaced statically to an initial value of the ERR, leaving the crack tip fixed in the initial position. During this stage, the specimen stores strain energy, which is released once the crack tip is allowed to grow. It is worth noting that since the relationship between crack tip speed and ERR toughness is not provided in the experimental results, the material parameters, involved in the definition of the crack criterion, are taken as adjustable variables. In Fig. 4 comparisons with experimental results, in terms of time history of the crack tip displacements are reported. The agreement between proposed and experimental results is noted.

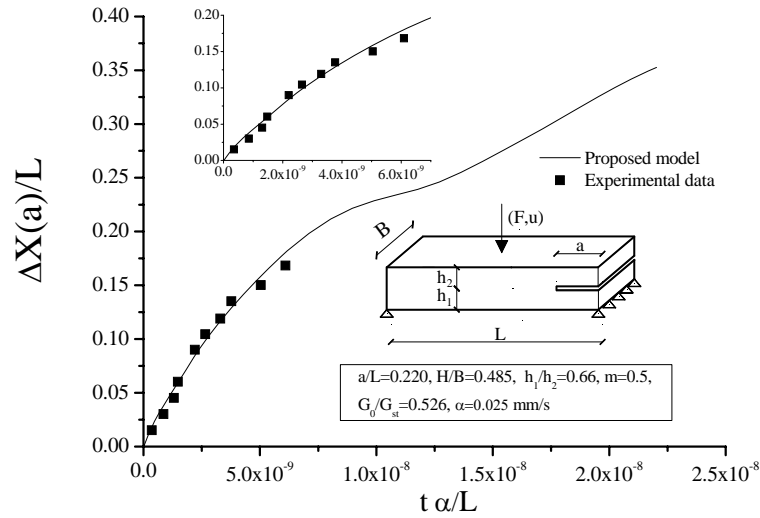


Fig. 4 Mixed mode dynamic crack growth in a three points bending test scheme. Comparison between experimental data [8] and proposed results: time history of the crack tip displacement $\Delta X(a)$ normalized on the total length of the laminate (L).

E_1 [Mpa]	$E_2=E_3$ [Mpa]	$G_{12}=G_{23}$ [Mpa]	ρ [Kg/m ³]	$\nu_{12}=\nu_{13}$	m	G_0
142E3	10.3E3	7.2E3	1580	0.27	0.5	300

Tab.1. Mechanical properties of unidirectional fiber-reinforced AS4/3501-6 Carbon/Epoxy [8].

Once, the model is validated through comparisons with experimental data, sensitivity results are developed. In particular, in Fig. 5 the influence of the loading rate on the ERR for a laminated structure subjected to two equally spaced delaminations over the thickness not shifted in space in a ENF is reported. The results are presented in terms of the time history of the dynamic amplification factors of the ERR. High amplifications with respect to the static case are noted, whereas when the loading curve is proportional to the first period of vibration of the structure (T_1), the ERR tend to the statical value..

Finally, an analysis on DCB scheme with two delaminations, different initial lengths and thicknesses involving a mixed mode loading condition is analyzed. The laminate is subjected to a loading condition with equal applied end displacements. The main aim of the proposed investigation is to evaluate the interaction behavior of propagating delaminations on a laminate subjected to opening end displacements. The time evolution of the external loading is based on a linear-constant function, whereas the amplitude of the rate effects is chosen in such a way to produce crack growth at different intensities of loading rates. In Fig. 6 the time history of the crack tip speeds of the lower and upper delaminations is reported for different loading curves. Moreover, in Fig.7, time histories of the tip displacements for lower and upper cracks, as a function of the loading rate are proposed. The results show that initially the upper crack advances alone. Subsequently, after an amount of time, the lower sublaminates satisfies the crack criterion and a simultaneous growth of both cracks is observed. During the initiation phase, i.e. when the crack

growth is enforced by the loading curve, the displacements and the cracks grow very rapidly. In this phase, the kinetic energy of the system is increased, producing high speeds in the crack growth. Subsequently, as far as the constant value in the loading curve is reached, the speeds of the cracks are reduced and then arrest phenomena of the cracks are observed.

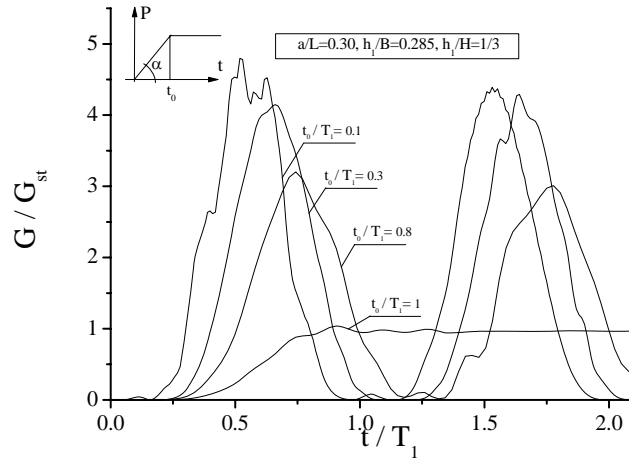


Fig. 5 ENF scheme with two delaminations equally spaced laminate on the thickness. Time history of the dynamic amplification factors (G/G_{st}) for different rates of the external loading, with G_{st} representing the static value of the ERR.

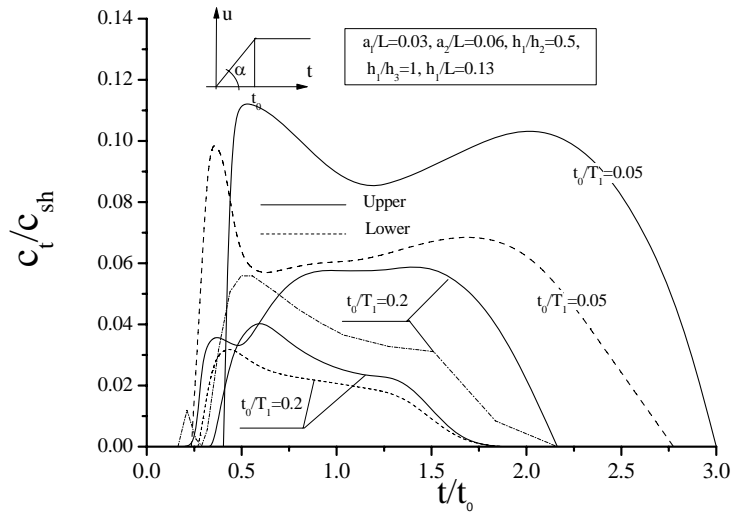


Fig. 6 ENF scheme with two delaminations on the thickness and shifted in space. Time history of the lower and upper crack tip speeds normalized on the shear wave speed of the material (c/c_{sh}) for different rates of the external loading.

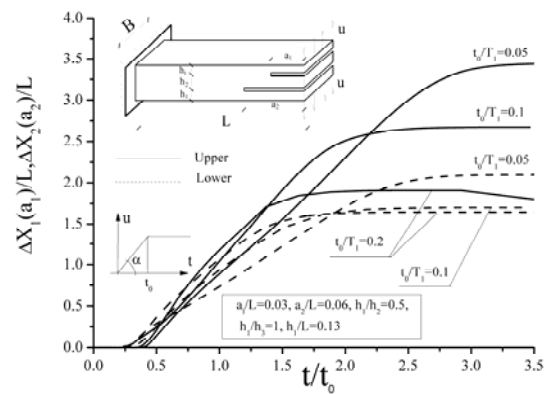


Fig. 7 ENF scheme with two delaminations on the thickness and shifted in space. Time history of the lower and upper tip displacements normalized on the total length of the laminate for different rates of the external loading.

CONCLUSIONS

A FE model to predict dynamic crack growth in composite structures is proposed. The moving mesh strategy combined with a Fracture Mechanics approach is able to predict properly the time dependent behavior of delamination phenomena. The proposed modeling is based on a generalized mixed mode dynamic fracture toughness criterion, which depends on a limited number of adjustable variables. Comparisons with experimental results for loading conditions involving mode mix at high speeds of the crack tip are proposed. The parametric study shows the influences of the loading rates effects on the dynamic ERR, which determine high amplifications of the fracture variables characterizing the crack tip evolution.

References

- [1] Ravi-Chandar K. Dynamic Fracture, Amsterdam: London : Elsevier, 2004
- [2] Yagawa,G.,Sakai,Y.andAndo,Y .Fast Fracture and Crack Arrest, G.T. Hahnand, M.F.Kanninen (eds), ASTMSTP 627,109–122, (1977).
- [3] Xie D, Biggers SB, Calculation of transient strain energy release rates under impact loading based on the virtual crack closure technique, International Journal of Impact Engineering 2007; 34 (6):1047-1060.
- [4] Corigliano A, Mariani S, Pandolfi A. Numerical analysis of rate-dependent dynamic composite delamination. Composites Science and Technology 2006; 66: 766–775.
- [5] Koh H.M. and Haber R.B., Elastodynamic formulation of the Eulerian Lagrangian kinematic description, J. Appl. Mech. 53, 839 845, 1986.
- [6] Nishioka,T.and Atluri,S.N. (1986). Computational Methods in the Mechanics of Fracture, S.N. Atluri (ed.) ,Elsevier Science Publishers,335–383.
- [7] Greco F, Lonetti P. Mixed mode dynamic delamination in fiber reinforced composites. Volume 40, Issue 5, Pages 379-392, Composites Part B: Engineering, 2009.
- [8] Comsol, Reference Manual, Comsol AB, Stockholm, 2007.
- [9] Tsai JL, Guo C, Sun CT. Dynamic delamination fracture toughness in unidirectional polymeric composites, Composites Science and Technology 2001; 61(1):87-94.