A global analysis of tall buildings subjected to horizontal loads

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SUMMARY. A general method is proposed for the analysis of the lateral loading distribution in three-dimensional structures composed of any kind of bracings (frames, framed walls, shear-walls, closed and/or open thin-walled cores and tubes), under the customary assumption of floor slabs being undeformable in their planes. This general formulation allows the analysis of high-rise structures by taking into account the torsional rigidity of the elements composing the building, without gross simplifications even in the case of very complex shapes and with the contemporary presence of different kinds of bracing. The method is aimed at gaining an insight into the force flow within the structure, in order to understand how the building response may be governed.

1 INTRODUCTION

From the structural viewpoint, tall building means a multi-storey construction in which the effects of horizontal actions and the need to limit the relative displacements take on primary importance [1,2]. A profound understanding of the force flow in these complex structural systems is often very difficult, and a huge commitment in terms of design, technology and economic resources is required. While in the design of low-rise structures the strength requirement is the dominant factor, with increasing height, the importance of the rigidity and stability requirements to be met to counter wind and earthquake actions grows until they become the prevailing design factors (note that the latest skyscrapers rise to over 400 m, the tallest of all being the Taipei Financial Center, 508 m high, while in Dubai they are currently constructing the "Burj Dubai" tower, to exceed 800 m). For this reason, the traditional solutions providing for load-bearing main and secondary parts tend to be forsaken in favour of a global approach, whereby the structure is conceived in a unitary fashion, i.e., as a single cantilever beam projecting out from the foundations. At any rate, the key issue in structural design continues to be the choice of an appropriate design model, that is able to reproduce faithfully the actual conditions of a structure.

Over the last three decades, it seems that a large part of the engineering community has followed a path towards the use of Finite Element models also in the early stage of design, but in the last few years the discussion and debate on the advantages and disadvantages of abandoning the use of analytical models seems to set a reverse trend. These models cannot be renounced to fully understand the complex behaviour of high-rise structures.

With the aim at acquiring insight into the effects of the stability element typologies and arrangements in tall buildings within a unified framework, and the capability of modelling complex structures and different typologies, in this paper we propose a three-dimensional formulation based on the work by Carpinteri [3]. The formulation is extended to encompass any combination of bracings, including bracings with open thin-walled cross-sections, which are analyzed in the framework of Timoshenko-Vlasov's theory [4,5] of sectorial areas, and according to the approach by Capurso [6], as recently proposed by Carpinteri *et al.* [7]. Numerical examples, investigating the structural response of tall buildings characterized by bracings with different cross-sections and height, show the effectiveness and flexibility of the proposed approach.

2 ANALYTICAL FORMULATION

The general formulation of the problem of the external lateral loading distribution between the bracings of a three-dimensional civil structure, originally presented in [3], will be revisited in this section. The structure is idealized as consisting of M bracings interconnected through floors undeformable in their planes and the axial deformations of bracings are not considered. With these hypotheses, the floor movement can be expressed by three generalized coordinates: the two translations, ξ and η, in X- and Y-direction of the global coordinate system origin (Fig. 1) and the floor rotation, ϑ. If N is the number of stories, the external load will be represented by a $3N$-vector F, whose elements are three elementary loads for each floor and, more exactly, two shear forces, p_x and p_y, and the torsion moment, m_z. In the same way, the internal loading transmitted to the i-th element will be represented by a $3N$-vector F_i and obtained from the preceding F through a premultiplication by a distribution matrix. Let p_i be the $2N$-vector representing the shear-loadings p_x and p_y, on the i-th element in the global coordinate system XY (Figs. 1 and 2), and m_i the N-vector representing the torsion moments, so that:

$$F_i = \begin{Bmatrix} p_i \\ m_i \end{Bmatrix} \tag{1}$$

The internal loadings F_i transmitted to the i-th bracing and related to the global coordinate system XY are connected with the same loadings F_i^* related to the local coordinate system $X_i^*Y_i^*$ (the origin of this system is in the center of twist C_i and the $X_i^*Y_i^*$ axes are parallel to the central ones, see Figs. 1 and 2b):

$$p_i^* = N_i \, p_i \, , \tag{2}$$

$$m_i^* = m_i - \psi_i \times p_i \cdot u_z \, , \tag{3}$$

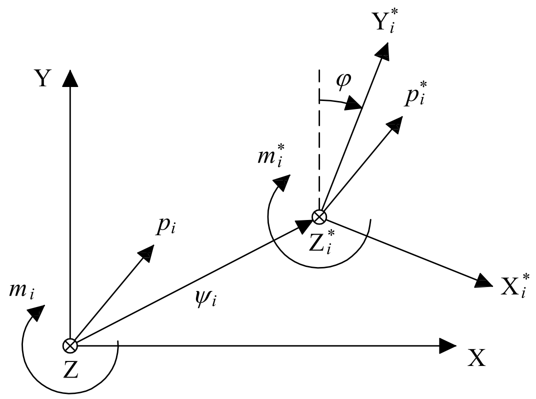


Figure 1: Global and local coordinate systems. The Z-axis completes the right-handed global system XYZ and Z_i* completes the right-handed local system X_i^* Y_i^* Z_i^*.

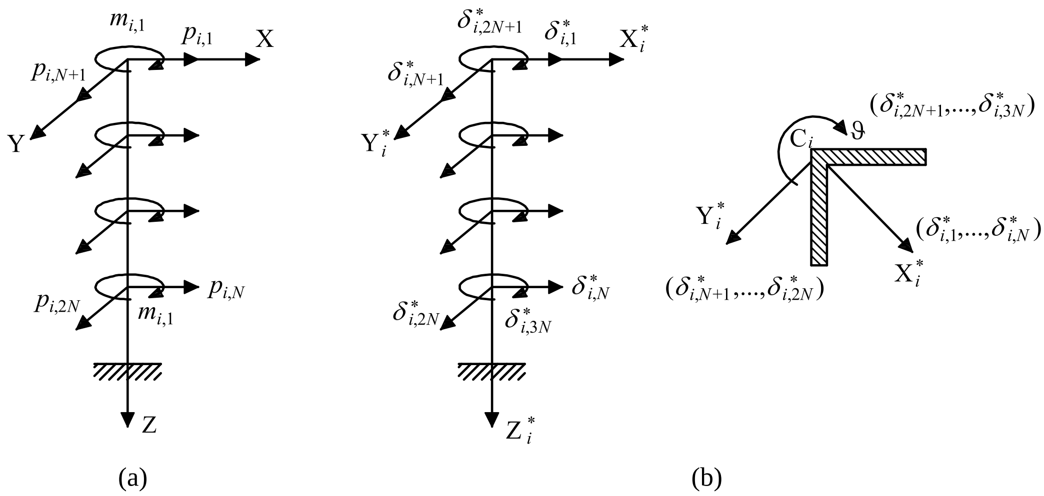


Figure 2: Internal loadings F_i (transmitted to the i-th bracing) in the global coordinate system (a); degrees of freedom of the i-th bracing in the local coordinate system $X_i^*Y_i^*Z_i^*$, axonometry and top view (b). Note that the highest floor is indicated with 1 and the lowest with N.

where the superscript * is used to indicate the loadings in the local coordinate system $X_i^*Y_i^*$, N_i is the orthogonal matrix of transformation from the system XY to the system $X_i^*Y_i^*$, ψ_i is the coordinate-vector of the origin of the local system $X_i^*Y_i^*$ in the global one XY, u_z is the unit vector in the Z-direction (note that ψ_i x $p_i u_z$ is a scalar triple product). The orthogonal matrix N_i is represented as:

$$N_i = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}, \tag{4}$$

where each element represents a diagonal N x N-matrix and φ is the angle between the X-axis and the X_i^*-axis (Fig. 1).

Equations (2) and (3) may be represented in the matrix form:

$$F_i^* = A_i F_i, \tag{5}$$

where:

$$A_i = \begin{bmatrix} N_i & 0 \\ C_i^T & I \end{bmatrix}, \tag{6}$$

I is the identity matrix, 0 the null matrix and the N x $2N$-matrix C_i^T is defined as

$$C_i^T = \begin{bmatrix} -y_i & x_i \end{bmatrix}, \tag{7}$$

where each element is a diagonal N x N-matrix and (x_i, y_i) are the components of vector ψ_i.

The displacements δ_i in the global coordinate system XY are then connected with the displacements δ_i^* in the local system $X_i^*Y_i^*$:

$$\delta_i^* = B_i \delta_i , \tag{8}$$

where:

$$B_i = \begin{bmatrix} N_i & 0 \\ 0 & I \end{bmatrix} . \tag{9}$$

The internal loadings F_i^* are connected with the displacements δ_i^* through the relation:

$$F_i^* = K_i^* \delta_i^* , \tag{10}$$

where K_i^* is the stiffness matrix in the local system. Recalling Eqs. (5) and (8), we get

$$A_i F_i = K_i^* B_i \delta_i . \tag{11}$$

Pre-multiplying both the members by the inverse A_i^{-1}:

$$F_i = \left(A_i^{-1} K_i^* B_i \right) \delta_i , \tag{12}$$

it follows that the stiffness matrix in the global system for the i-th bracing is

$$K_i = A_i^{-1} K_i^* B_i , \tag{13}$$

where:

$$K_i^* = \begin{bmatrix} K_{pi}^* & 0 \\ 0 & K_{mi}^* \end{bmatrix} . \tag{14}$$

The displacement 3N-vector δ_i of the i-th element is connected with the displacement 3N-vector δ of the rigid floors by the relation:

$$\delta_i = T_i \delta , \tag{15}$$

where the transformation 3N x 3N-matrix T_i is:

$$T_i = \begin{bmatrix} I & C_i \\ 0 & I \end{bmatrix} , \tag{16}$$

and the 2N x N-matrix C_i is defined in Eq. (7).
Equation (12) can be rewritten:

$$F_i = K_i T_i \delta = \overline{K_i} \delta , \tag{17}$$

where $\overline{K}_i = K_i T_i$ is the stiffness of the i-th element with respect to the floor displacements. For the global equilibrium we have:

$$\sum_{i=1}^{M} F_i = \sum_{i=1}^{M} \overline{K_i}\,\delta, \tag{18}$$

$$F = \overline{K}\,\delta, \tag{19}$$

where $\overline{K}$ is the global stiffness matrix of the rigid floors. Recalling Eqs. (17) and (19), we get:

$$\delta = \overline{K_i}^{-1} F_i = \overline{K}^{-1} F \tag{20}$$

and then:

$$F_i = \overline{K_i}\,\overline{K}^{-1} F\ . \tag{21}$$

Equation (21) solves the problem of the external loading distribution between the resistant elements of a building. It is formally analogous to the equation for the distribution of a force between different in-parallel resistant elements in a plane problem. In fact, the distribution matrix $\overline{K}_i\ \overline{K}^{-1}$ is the product of the partial stiffness matrix by the inverse of the total stiffness matrix, as well as in the plane problem the distribution factor is the product of the partial stiffness by the inverse of the total stiffness. The sum of the distribution matrices is equal to the unit matrix. Details on the condensation procedure for the computation of the stiffness matrices according to the de Saint Venant theory of beams with closed cross-section can be found in Humar and Kandhoker [8]. On the other hand, it can be observed that the above presented formulation is general and allows one to treat any kind of structural elements, such as frames, shear-walls and thin-walled sections either open or closed. To this aim, a procedure to compute the stiffness matrices for open thin-walled cross-section is proposed in the next section.

3 STIFFNESS MATRIX OF OPEN THIN-WALLED BRACINGS

The behaviour of open thin-walled cross-section is treated according to the Timoshenko-Vlasov [4,5] torsion theory of beams with thin open cross-section, characterized by the presence of the warping stiffness. With reference to Figs. 1 and 2, let us consider a generic beam in its local system $X_i^*\ Y_i^*\ Z_i^*$; if the beam sections are undeformable in their planes, the section movements can be expressed by three displacements: the two translations ξ_i^* and η_i^* (in X- and Y-direction of the local coordinate system origin, see Fig. 2) and the floor rotation ϑ_i^*. Under the customary assumption that the external loads are concentrated transverse flexural loads and torsional moments only, so that the longitudinal force N at any value of z should be zero, according to the analytical approach by Capurso [6], it can be shown that the following relations, written in synthetic matrix form, hold:

$$M_i^* = -EJ_i\delta_i^{*''} \tag{22}$$

$$T_i^* = -EJ_i\delta_i^{*'''} \tag{23}$$

if we introduce the following vectors:

$$\delta_i^* = \begin{Bmatrix} \xi_i^* \\ \eta_i^* \\ \vartheta_i^* \end{Bmatrix},\ M_i^* = \begin{Bmatrix} M_{i,y}^* \\ M_{i,x}^* \\ B_i^* \end{Bmatrix},\ T_i^* = \begin{Bmatrix} T_{i,x}^* \\ T_{i,y}^* \\ M_{i,z}^* \end{Bmatrix} \tag{24}$$

and the matrix of inertia:

$$J_i = \begin{bmatrix} J_{yy} & J_{xy} & J_{y\omega} \\ J_{xy} & J_{xx} & J_{x\omega} \\ J_{y\omega} & J_{x\omega} & J_{\omega\omega} \end{bmatrix} \tag{25}$$

where M_x and M_y are the bending moments, B is the bimoment [4,5], T_x and T_y, are the shear forces, M_z is the torsional moment, E is the Young modulus and the apex $'$ corresponds to derivation with respect to the axial coordinate z. J_{xx}, J_{xy} and J_{yy} are the (second order) moments of inertia, $J_{\omega\omega}$ is the sectorial moment of inertia and $J_{x\omega}$ and $J_{y\omega}$ are the sectorial products of inertia.

By further deriving Eq. (23) we get:

$$F_i^* = EJ_i\delta_i^{*\mathrm{IV}} \,. \tag{26}$$

Since the matrix of inertia J is symmetric and positive definite, except some anomalous cases detailed in [6], Eq. (26) can be inverted and the components of vector $\delta_i^{*\mathrm{IV}}$ can be computed:

$$\delta_i^{*\mathrm{IV}} = \frac{1}{E} J_i^{-1} F_i^* \,. \tag{27}$$

Considering Eq. (27) and integrating it with the appropriate boundary conditions:

$$\delta = 0 \,, \qquad \delta' = 0 \,, \tag{28}$$

at the clamped base of the beam ($z = L$), and:

$$\delta'' = 0 \,, \qquad \delta''' = 0 \,, \tag{29}$$

at the free-end ($z = L$), we obtain the relation between the displacements δ_i^* and the loads F_i^*, therefore the compliance matrix C_i of the i-th bracing:

$$\delta_i^* = \frac{1}{E} \overline{J_i}^{-1} L^3 \overline{Q} F_i^* = C_i\, F_i^* \,, \tag{30}$$

where the matrix $\overline{Q}$ is a $3N$ x $3N$-matrix of non-dimensional influence coefficients determined through the integration. Its structure is block diagonal, with three equal (full) submatrices Q:

$$\overline{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & Q \end{bmatrix} . \tag{31}$$

In case of a bracing with constant cross-section, the computation of the terms of the upper triangular part of the N x N sub-matrix Q provides the generic term q_{ij}

$$q_{ij} = \frac{1}{6}(N - j + 1)^2 (2N + j - 3i + 2) \cdot \quad (32)$$

The lower triangular part is obtained exploiting the symmetry of Q. In the case of bracings with variable cross-section, the computation can be performed by means of the Mohr's theorem.

By inverting the compliance matrix C_i we finally get the expression of the stiffness matrix in the local system K_i^* of the i-th bracing with open thin-walled cross-section:

$$K_i^* = (C_i)^{-1} = \frac{E}{L^3} \overline{Q}^{-1} \overline{J}_i \cdot \quad (33)$$

4 NUMERICAL EXAMPLE

Numerical examples chosen to show the flexibility and effectiveness of the proposed approach are presented in this section. The structure is an asymmetric 20-story tube-in-tube system with square plane layout. The asymmetry is chosen in order to investigate both the flexural and the torsional behaviour of the building when subjected to horizontal wind loads. As shown in Fig. 3, the internal core is closed, whereas the external tube is made by two bracings with 'C'-shaped open cross-sections. The Young's modulus is E = 2.4×10^4 MPa, the Poisson ratio ν = 0.18. The story height is H = 4.0 m, corresponding to a total height L = 80 m. The member cross-section properties are given in Table 1.

Concerning the loads, in this example we consider the actions of wind only. For the sake of simplicity, we assume constant wind pressure over the height of the structure. Due to the hypothesis of infinite rigidity of the floors in their plane, the wind actions can be applied as a system of concentrated horizontal loads passing through the barycentre of the pressure distribution.

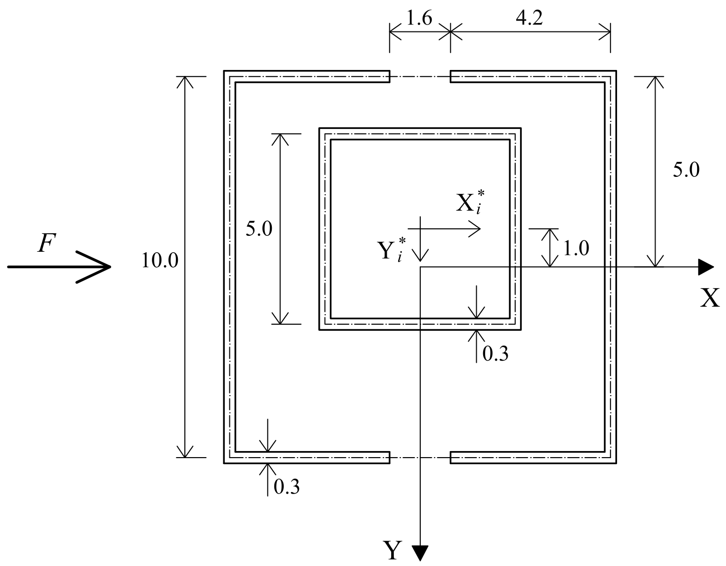


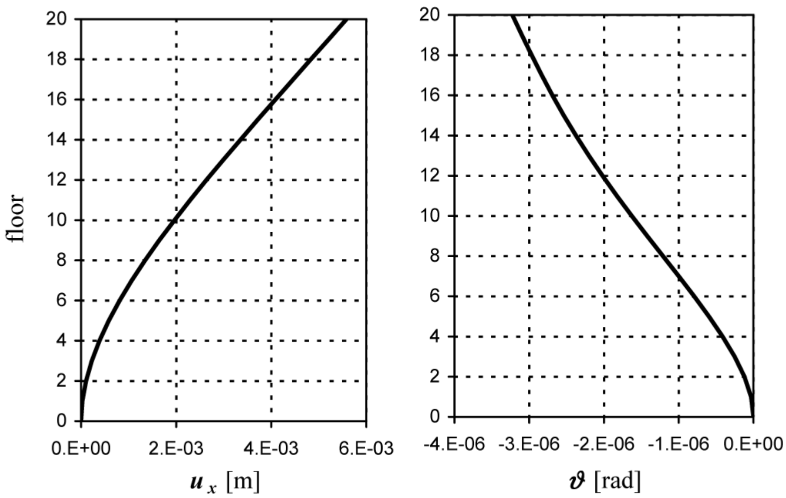
Figure 3: Floor plane of the example building. Tube-in-tube structure with asymmetric core.

Table 1: Cross-section properties of the close thin-walled core and of the external open bracings.

	Core	**'C'-shaped bracings**
Second moment J_{xx} [m^4]	25.09	88.00
Second moment J_{yy} [m^4]	25.09	43.21
Warping moment $J_{\omega\omega}$ [m^6]	0.00	171.54
Torsional rigidity (à la de Saint Venant [m^4])	37.50	0.166
Global coordinate x_c of the shear centre [m]	0.00	13.27
Global coordinate y_c of the shear centre [m]	–1.00	±6.50
Angle ω [rad]	0.00	0.00

The intensity of wind actions is computed in a simplified way by considering the reference kinetic pressure provided by the Italian Technical Regulations [9], in which the wind actions are supposed to be static and directed according to the principal axes of the structure. Supposing that the building is located in Turin (Italy), and, for the sake of simplicity, not considering the exposition-, shape- and dynamic-coefficients [9], the obtained concentrated loads have the following values: F=13.67 kN. In the computations, the wind force is applied in the X-direction only.

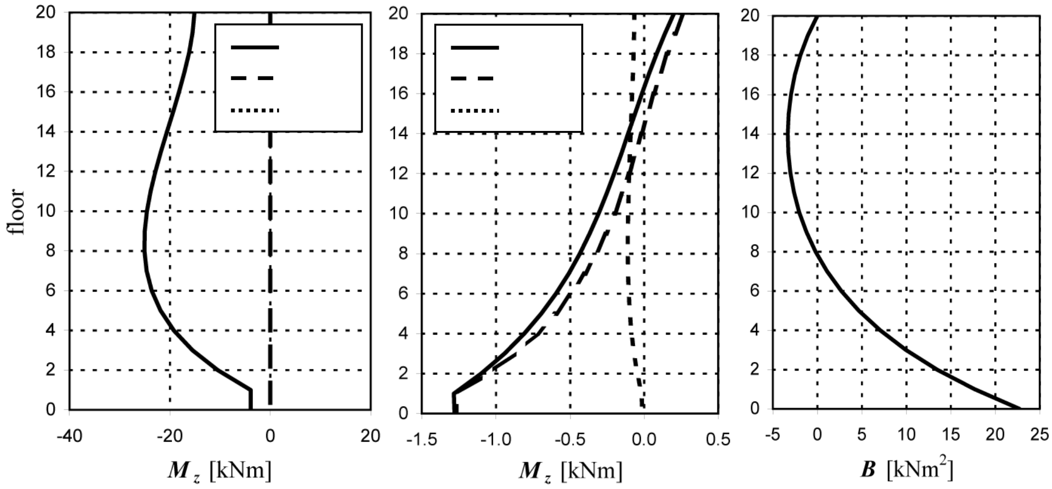
Results are summarized in Figs. 4 to 6. In Fig. 4(a) and (b) displacements in the X-direction and rotations at the floor levels are reported respectively; as can be seen, the flexural deformed shape does not display change of sign of the curvature, whereas rotations do. An inflection point is clearly visible at the level of floor 8 in Fig. 4(b).



(a) (b)

Figure 4: Displacements of the floors in the global coordinate system. Translation in the X-direction (a); and rotation (b).

This fact is in tight connection with the diagrams of the torsion moment M_z, reported in Fig. 5(a) and (b) for the internal core and the external 'C'-shaped bracings, respectively. In both plots it can be clearly seen that the primary part M_{dSV} of the torsional moment displays a maximum at the same height. Regarding the internal closed core, it obviously supports a larger part of the total moment and presents null warping moment M_ω. On the other hand, the 'C'-shaped cross-sections display smaller primary moment and larger warping moment M_ω. The latter changes its sign from the top of the building (where it is positive) to the ground and shows a faster rate of increase below the floor 8. The primary moment is negligible with respect to the warping one in the external bracings. This is a consequence of the fact that the internal core is closed and can sustain large torsional loads, thus influencing the behaviour of the other bracings. All this can be confirmed if we look at the bimoment *B* in the external ‘C’-shaped bracings (plotted in Fig. 5(c)): we can see that the sign changes at the level of floor 8, being negative above it. As expected, the maximum value is at the ground floor, and the bimoment is null at the top of the building.



(a) (b) (c)

Figure 5: Torsion of the internal core (a); and torsion (b) and bimoment (c) of the external ‘C’-shaped bracings along the building height.

The interaction between bracings with different height has been investigated by assuming the internal core of the floor plane shown in Fig. 3, 20-storeys high and the ‘C’-shaped bracings 10-storeys high. The most interesting result concerns the shear force distribution. Linear increasing functions from the top to the bottom of the building are obtained in case of bracings with equal height, as shown in Fig. 6a (note that the continuous line represent the sum of the contributions of the two external bracings). On the contrary, a strong discontinuity with a cuspidal point at the top level of the lowest bracing is predicted when different heights are considered (see Fig. 6b). Such a jump in the shear distribution is due to the assumption of floor slabs undeformable in their planes. It is worth noting that this result, also confirmed by finite element analyses, is in disagreement with those obtained in [10] for infinitely stiff floors. Finally, the ratio between the maximum value of the shear force and that acting at the ground level as a function of the difference in height among the considered bracings is plotted in Fig. 6c. As it can be seen, for difference in height greater than 25% the maximum shear force is not at the ground level, as usually expected.

5 CONCLUSIONS

The numerical algorithm for the lateral loading distribution between the elements of a three dimensional civil structure [3], extended in this paper by introducing thin-walled bracing elements with open cross-section, can be employed to predict the gross structural deformations of tall buildings with different structural typologies, i.e. composed of any kind of bracings (frames, framed walls, shear-walls and closed and/or open thin-walled cores and tubes). The general formulation pesented in this paper offers, compared to a detailed FE simulation, ease of use and reduced effort in preparing the model, as well as in the result interpretation, with sufficient accuracy in the preliminary and conceptual design stage. In addition, such a global approach provides a clear picture of the key structural parameters governing the tall building behaviour.

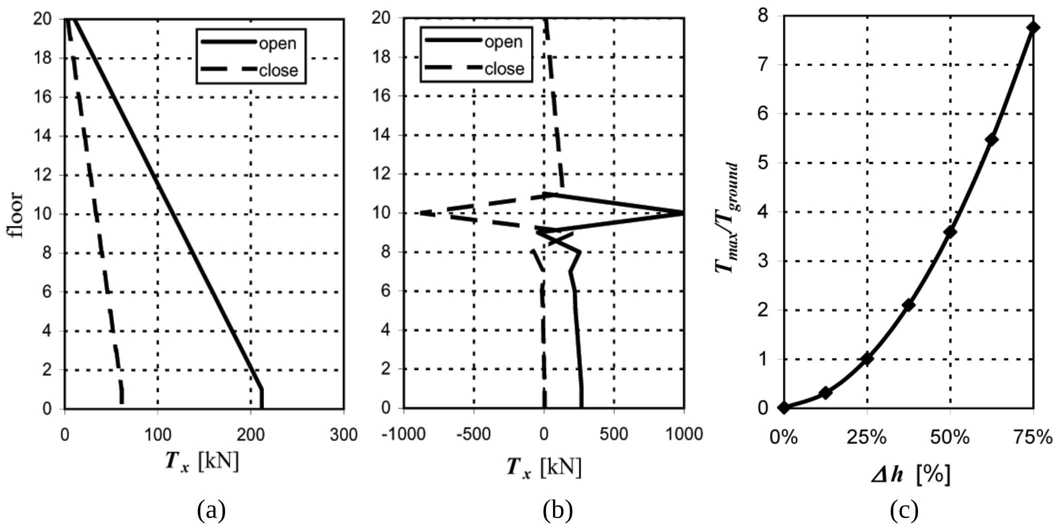


Figure 6: Shear force distributions in case of bracings with equal height (a); and different height (b). Ratio between maximum and ground shear force by varying the difference in height among bracings (c).

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