On the Dynamics of Rigid Block Motion Under Strong Shaking

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Keywords: Rigid block, unilateral model, structural dynamics, worst scenario, optimization

SUMMARY. The paper focuses on the problem of the assessment of the response of structures behaving like rocking rigid blocks acted on by dynamic loading. The response of such a system is poorly robust, and results can be very different in dependence of a number of factors thus making final assessment rather uncertain. After defining the class of possible forcing functions, obeying some suitable constraints, a procedure is developed aimed at searching for the worst possible response; the worst scenario is evaluated by implementing an iterative optimization procedure in an ad-hoc calculus code, involving the calculus of complex block dynamics approached by a distributional approach developed by the authors.

1 INTRODUCTION

The analysis of rocking motion (RM) response of rigid blocks (Fig.1) has attracted the attention of many researchers [1]-[8], especially from a theoretical point of view, starting from the reference analytical frame for the study of RM (Rocking Motion) dynamics based on the formulation introduced by Housner [1] which refers to the equation for the period of the system, depending on the amplitude of rocking, and the equation for the restitution coefficient.

As concerns the vulnerability of systems behaving like rigid blocks, one should first emphasize the poor robustness of analysis of rocking blocks. Even if non-linear dynamics of rigid blocks have been widely investigated in literature, the RM dynamics of rigid blocks are less understood than in many other non-linear vibration problems [3], since, despite the apparent simplicity, the motion of rigid blocks poses difficult problems to solve.

The block is shown to posses extremely complicated dynamics, with many different types of response being revealed [4]. Periodic responses were shown which appear to violate West's formula and some steady state responses of the forced system were shown to be so large as to produce toppling of the block even if the system were unforced [5].

In literature, the sensitivity to initial conditions is widely shown to play a central role and leads to uncertainty in the prediction of the asymptotic dynamics. Therefore safety issues cannot be satisfactorily resolved until an agreed set of conditions is established [4]. Moreover it has been demonstrated that damping has only a qualitative effect on the block motion and that none of the multiple solutions or chaotic responses are radically changed.

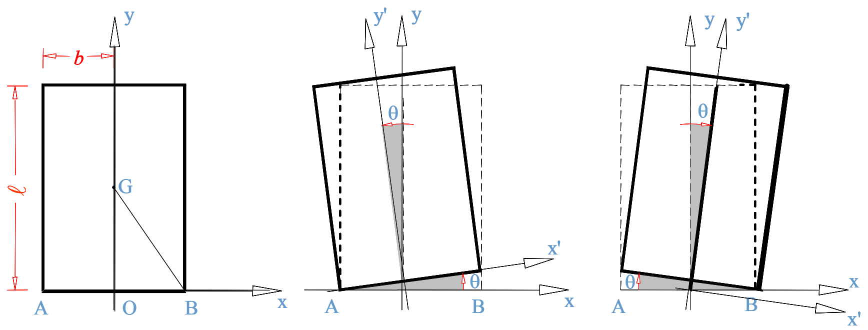

Figure 1: Plane rocking of a rigid block model around its base corners A and B.

Recently the authors have definitively proven the low robustness of the dynamic behaviour of rigid blocks, showing that the introduction of a null distribution such that the time-displacement of the system is not formally altered produces non-null effects on the response of the system, with the impact obeying now to a strongly non-linear equation [6]; the additional null term is able to produce an effect similar to the one of the restitution coefficient, thus pointing out its very aleatory nature [6]. This result is somehow confirmed by the tracing of overturning domains, which have been shown to exhibit significantly irregular shapes [7].

Moreover, one should consider all the problems of practical nature that deeply influence the dynamics of rigid blocks, such as imperfections in the geometry of the base section with non perfect angles of the parallelepiped, imperfections in the level surface of the block basis, and imperfections in the orthogonality of the excitation with respect to the middle plane of the block. Without considering the problems related to the analysis of blocks with non-rectangular basis and other problems possibly occurring during the motion also depending on the material composing the block itself and other phenomena, such as the blunting of the base edges, which has been shown to be able to deeply change the dynamic properties of the block, significantly affecting its response [8].

All of these features characterizing the behaviour of rocking rigid structures, i.e. the low robustness of their response and the high sensitivity of their dynamics to imperfections, make the problem of their vulnerability highly felt in seismic areas, where the forcing function may be very uncertain in its details.

A possible approach to stabilize the forecasting capacity is to assume a “worst scenario” position, that makes the response substantially independent on the details of the excitation. The method was set forth by Drenick in the 70’s [9]-[12] and by Shinozuka in the early 80’s [13] with particular reference to linear structures, and more recently by Elishakoff and Pletner [14] and by Baratta and Zuccaro [15]-[17].

Starting from previous papers [15]-[20] focusing on the cases of indefinitely elastic and elastic-plastic structures subjected to dynamic actions, the paper addresses the possibility of defining the theoretical bounds on the maximum value of the structural response under dynamic loading, for non linear structures behaving like rigid blocks rocking around their base edges under dynamic shaking.

2 THE WORST SCENARIO APPROACH

Very often objects that can be assimilated to rigid blocks are simply supported on some plane surface. They may be subject to some disturbance triggering oscillations that can be modelled as a kind of rocking motion. Most often such disturbance can be viewed at as a base acceleration whose time-history (the *accelerogram*) is known in its general characters but is subject to desultory variations in its details. For instance vibrations induced by some rotating machine can be known with respect to its basic frequency range, but a number of other effects may be quite random, like the filtering through the supporting structural organism, the influence of ageing, of imperfect alignment of the shafts and so on. The problem is mostly felt in the area of seismic excitation, where objects or walls simply resting on tables or on foundations are exposed to be overturned down by earthquake-induced base shaking.

Generally speaking, basic characters of the excitation can be identified in the average power spectrum, in its maximum value, its duration and so on. So a suitable *class* of forcing functions can be reliably defined, by building up a functional space where it is assumed it must be contained. All the accelerograms in the class differ from each other by random quantities that cannot be predicted in details. In other words the accelerograms in the class form a *stochastic process* that could be treated by means of the methods and procedures of the relevant theory to produce the probability distribution of the maximum response. Apart from the difficulty to treat the response of a highly nonlinear system in a stochastic context, probabilistic results may be quite illusory depending on the acquisition of statistical data for the parameters of the stochastic process.

A different approach is to search for the worst response produced by accelerograms fitting the basic properties, that can be assumed to be collected in a vector ℓ, but subject to random realizations of the details, that can be collected into a vector **x** containing the n realizations of the random variable generating the ordinates of the forcing function at different time instants.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be the sample of n realizations of a basic random variable $\tilde{x}$ and ℓ the assigned local characteristics reflecting the statistics of the admissible accelerograms: the component of the structure response can be evaluated by iterated sampling of the random vector **x** in a very large range (n=1000 or more).

Since the modalities of their generation, such numbers should verify the following relationships, the closer the larger n is

$$\sum_{i=1}^{n} x_i = n\mu_x \tag{1}$$

$$\sum_{i=1}^{n} (x_i - \mu_x)^2 = n\sigma_x^2 \tag{2}$$

$$n(a,b) = n[P(b) - P(a)] \quad \forall (a,b) \in X^2 \tag{3}$$

where μ_x and σ_x^2 are respectively the expected value and variance of $\tilde{x}$, n(a,b) is a number of components of **x** falling in the interval (a,b), X is the range of the values of **x**, and P(x) is the distribution function of $\tilde{x}$.

In the case when one is interested, as it is the case here, in the worst situation that can occur for a structure when it is acted on by badly defined forcing time-histories, apart from the decision concerned with gross shaking parameters like the maximum instantaneous accelerations, or total energy and so on.

Having assumed the above scenario, for a given structure any component R of the structural response will be a function of the sample vector **x**, conditioned on the assumed value of ℓ and the problem reduces then to maximize the function $R(\mathbf{x}|\ell)$,

$$R^*(\mathbf{x}|\ell) = \operatorname*{Sup}_{\mathbf{x}} R = \operatorname*{Sup}_{\mathbf{x}} R(\mathbf{x}|\ell) \tag{4}$$

with the components of **x** that obey the above Eqs (1)- (3).

3 THE RIGID BLOCK DYNAMICS UNDER PURE ROCKING

3.1 *The RM dynamics of rigid blocks by a distributional approach*

As concerns the dynamics of the rigid block in Figure 1, with mass m and dimensions $2b\times\ell$, subject to its own weight **W** and to the excitation **F**, with components horizontal and vertical components W_x, W_y, and F_x, F_y respectively, rocking around its base edges A and B, one refers to the original distributional approach introduced for the simplified model by the authors in [6].
The unilateral behaviour of the model involves the adoption of two unilateral hinges at the bottom at the points A and B, which only admit counter-clockwise (positive) rotations $[\theta(t)>0]$ around A, involving the detachment from the hinge B, and clockwise (negative) rotations θ around B $[\theta(t)<0]$, involving the detachment from the hinge A.
The motion of the block is described by the displacement vector $\mathbf{s}_P(t)$, with components $s_{px}(t)$, $s_{py}(t)$ in the horizontal and vertical directions .
After denoting by $\dot{\theta}(t), \ddot{\theta}(t)$, the velocity and the acceleration of the block respectively, as the first and the second time derivatives of its rotation, the final dynamic equilibrium condition is expressed in the form

$$\begin{aligned}
&\ddot{\theta}(t) = H_1[\theta(t)]\ddot{\theta}_A(t) + H_2[\theta(t)]\ddot{\theta}_B(t) = \frac{1}{2}\left(\{H_1[\theta(t)] - H_2[\theta(t)]\}[\ddot{\theta}_A(t) - \ddot{\theta}_B(t)] + [\ddot{\theta}_A(t) + \ddot{\theta}_B(t)]\right) \\
&\Rightarrow \ddot{\theta}(t) = \frac{1}{2}\left\{\overline{H}[\theta(t)][\ddot{\theta}_A(t) - \ddot{\theta}_B(t)] + [\ddot{\theta}_A(t) + \ddot{\theta}_B(t)]\right\}
\end{aligned} \tag{5}$$

where the introduced step functions and their derivatives with respect to the rotation denoted by the apexes $(\cdot)', (\cdot)''$, etc., depend on the Dirac function and its derivatives as follows

$$\begin{aligned}
&\overline{H}(x) = H_1(x) - H_2(x) \\
&H_1(x) = \int_{-\infty}^{x} \delta(t)dt = \begin{cases} 0 & \text{if } x<0 \\ 0.5 & \text{if } x=0 \\ 1 & \text{if } x>0 \end{cases}, H_2(x) = 1 - \int_{-\infty}^{x} \delta(t)dt = 1 - H_1(x) = \begin{cases} 1 & \text{if } x<0 \\ 0.5 & \text{if } x=0 \\ 0 & \text{if } x>0 \end{cases} \\
&H_1'[\theta(t)] = \delta[\theta(t)] \ ; \ H_2'[\theta(t)] = -\delta[\theta(t)] \ ; \ H_1''[\theta(t)] = \delta'[\theta(t)] \ ; \ H_2''[\theta(t)] = -\delta'[\theta(t)], \quad \text{etc.}
\end{aligned} \tag{6}$$

and $\ddot{\theta}_A(t)$ and $\ddot{\theta}_A(t)$ are given in the form

$$\ddot{\theta}_A(t) = -2\frac{G[\theta(t),\dot{\theta}(t)]\cdot[V_y(t)\cdot s_{P2x} - V_x(t)\cdot s_{P2y}] + [a_y(t)\cdot V_x(t) - a_x(t)\cdot V_y(t) + g_y V_x(t) - g_x V_y(t)]}{V_y(t)\cdot\{2F[\theta(t)]\cdot s_{P2x} + \cdot s_{P1x}\} - V_x(t)\cdot\{2F[\theta(t)]\cdot s_{P2y} + \cdot s_{P1y}\}}$$

$$\ddot{\theta}_B(t) = -2\frac{G[\theta(t),\dot{\theta}(t)]\cdot[S_y(t)\cdot s_{P2x} - S_x(t)\cdot s_{P2y}] + [a_y(t)\cdot S_x(t) - a_x(t)\cdot S_y(t) + g_y S_x(t) - g_x S_y(t)]}{S_y(t)\cdot\{2F[\theta(t)]\cdot s_{P2x} + \cdot s_{P1x}\} - S_x(t)\cdot\{2F[\theta(t)]\cdot s_{P2y} + \cdot s_{P1y}\}} \tag{7}$$

$$\ddot{\theta}_A(t) = -2\frac{G[\theta(t),\dot{\theta}(t)]\cdot[V_y(t)\cdot s_{P2x} - V_x(t)\cdot s_{P2y}] + [a_y(t)\cdot V_x(t) - a_x(t)\cdot V_y(t) + g_y V_x(t) - g_x V_y(t)]}{V_y(t)\cdot\{2F[\theta(t)]\cdot s_{P2x} + \cdot s_{P1x}\} - V_x(t)\cdot\{2F[\theta(t)]\cdot s_{P2y} + \cdot s_{P1y}\}}$$

$$\ddot{\theta}_B(t) = -2\frac{G[\theta(t),\dot{\theta}(t)]\cdot[S_y(t)\cdot s_{P2x} - S_x(t)\cdot s_{P2y}] + [a_y(t)\cdot S_x(t) - a_x(t)\cdot S_y(t) + g_y S_x(t) - g_x S_y(t)]}{S_y(t)\cdot\{2F[\theta(t)]\cdot s_{P2x} + \cdot s_{P1x}\} - S_x(t)\cdot\{2F[\theta(t)]\cdot s_{P2y} + \cdot s_{P1y}\}}$$

with

$$a_x(t) = F_x(t)/m, \quad a_y(t) = F_y(t)/m, \quad g_x(t) = W_x(t)/m, \quad g_y(t) = W_y(t)/m; \tag{8}$$

$$F[\theta(t)] = \frac{1}{2}\{\overline{H}[\theta(t)] + \overline{H}'[\theta(t)]\theta(t)\}; \; G[\theta(t),\dot{\theta}(t)] = \frac{1}{2}\{2\overline{H}'[\theta(t)] + \overline{H}''[\theta(t)]\theta(t)\}\dot{\theta}^2(t) \tag{9}$$

$$\mathbf{V}(t) = \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} s_{Px}(t) + b \\ \ell + s_{Py}(t) \end{pmatrix}; \; \mathbf{S}(t) = \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} s_{Px}(t) - b \\ \ell + s_{Py}(t) \end{pmatrix}; \tag{10}$$

During the motion, because of the unilateral nature of the constraints, the following inequalities should be satisfied concerning the vertical unknown components of the reactions at A and B, $\mathbf{R}_{Ay}(t)$ and $\mathbf{R}_{By}(t)$,

$$\mathbf{R_{Ay} \geq 0} \quad ; \quad \mathbf{R_{By} \geq 0} \tag{11}$$

3.2 *Discussion about the low robustness of the RM dynamics of rigid blocks*

It is interesting to observe some anomaly in the description of the dynamic behaviour of the above introduced unilateral rigid model [6]. Let consider the expression of the displacement $\mathbf{s}(\mathbf{x},t)$ of any point of the model characterized by the position vector $\mathbf{x}$ with respect to the origin O of the reference axes, given by the superposition of the two motions around A and B by means of the displacement vectors $\mathbf{s}_A(\mathbf{x},t)$ and $\mathbf{s}_B(\mathbf{x},t)$, as follows

$$\mathbf{s}(\mathbf{x},t) = H_1[\theta(t)]\cdot\mathbf{s}_A(\mathbf{x},t) + H_2[\theta(t)]\cdot\mathbf{s}_B(\mathbf{x},t) \tag{12}$$

and superpose an additional term corresponding to a null distribution, as follows

$$\mathbf{s}(\mathbf{x},t) = H_1[\theta(t)]\cdot\mathbf{s}_A(\mathbf{x},t) + H_2[\theta(t)]\cdot\mathbf{s}_B(\mathbf{x},t) + r\,\delta(\theta)\theta^n\cdot[\mathbf{s}_A(\mathbf{x},t) - \mathbf{s}_B(\mathbf{x},t)] \tag{13}$$

Thereafter, one realizes that it is possible to change Eq. (12) in way that the time-displacement of the system is not formally altered, by adding a null distribution, like in Eq. (13). This produces some formal change in the coefficients in Eqs (9) of the acceleration and of the square angular velocity in the dynamical equilibrium equation Eq.(5), which turn into

$$F[\theta(t)] = \frac{1}{2}\left\{\overline{H}[\theta(t)] + \overline{H}'[\theta(t)]\theta(t)\right\} + r\left[\delta'[\theta(t)]\cdot\theta^{n+1}(t) + (n+1)\delta[\theta(t)]\cdot\theta^{n}(t)\right]$$

$$\begin{aligned} G\left[\theta(t),\dot{\theta}(t)\right] = \dot{\theta}^2(t)\cdot\Bigg\{&\frac{1}{2}\left\{2\overline{H}'[\theta(t)] + \overline{H}''[\theta(t)]\theta(t)\right\} + \\ &+ r\left[\delta''[\theta(t)]\cdot\theta^2(t) + 2(n+1)\delta'[\theta(t)]\cdot\theta(t) + n(n+1)\delta[\theta(t)]\right]\cdot\theta^{n-1}(t)\Bigg\} \end{aligned} \tag{14}$$

The impact obeys now to a strongly non-linear equation, and numerical solutions must be sought. To this aim, the distribution $\delta(x)$ and its derivatives can be substituted by functions $\alpha_k(x)$ and their derivatives, that in the limit uniformly converge to $\delta(x)$ and its derivatives.
The results prove [6] that increasing the coefficient r of the null distribution produces an increasing damping and that this effect is rather uniform when k tends to become larger and larger and functions $\alpha_k(x)$ tend to $\delta(x)$ and henceforth it is expected that the numerical solutions of the equilibrium for the displacement, velocity and acceleration fields converge to the solutions of the theoretical case.
So it is expected that the superposition of a null term to the displacement of the model produces a non-null consequence on its motion, an effect that is one more reason to assess that rocking motion can be strongly affected by the way one approaches the problem [5].

4 THE IMPLEMENTATION OF THE PROCEDURE. NUMERICAL RESULTS

4.1 *The basic constants*

The basic constants ℓ are defined on the basis of the investigation of the source of the disturbance. Instances for the definition of the forcing functional space are:

a) n-dimensional linear spaces:

$$f(t) = \sum_{i=1}^{n} c_i f_i(t) \tag{15}$$

Where the coefficients c_i are subject to some constraints aimed at fitting the macro-characters of the excitation collected in the vector ℓ. The base functions $f_i(t)$ express the particular nature of the disturbance, and are in general found in some sets of mutually orthogonal functions (e.g. trigonometric functions, Hermite, Laguerre and other polynomial sets and so on).

b) delta-correlated processes:

In this class are contained white and shot-noise processes. After filtering, these particular functions can give raise to elaborated functional classes that are anyway generated starting from delta-correlated processes. Applications are found in the synthesis of artificial site-compatible earthquakes (see e.g. [18]), especially aimed at checking seismic vulnerability of non-linear structures.

In all the cases, the relevant macro-parameters ℓ are those defining the maximum value of the forcing function, the duration of the disturbance, the spectral energy distribution, the grow-up process and so on depending on the problem at hand.

4.2 *The response maximization procedure*

For simplifying the operative procedure in the constraints Eqs (1)-(3), **x** is assumed as a Random Standard Gaussian variable (RSG), ($\mu_x = 0$, $\sigma_x^2 = 1$) and the objective function in Eq. (4) is maximized.
The optimization process is performed by random-search procedures that verify the above constraints almost spontaneously (see [21]); alike the classical maximization methods which turn out to be inadequate for the constraints described.
Basically, with reference to Figure 2, where the level curves of the objective function are reported together with a possible optimization path, the response maximization procedure consists of an iterative process, based on the generation at each step of a new accelerogram compatible with the assigned properties of the disturbance and on the evaluation of the response of the structural model.

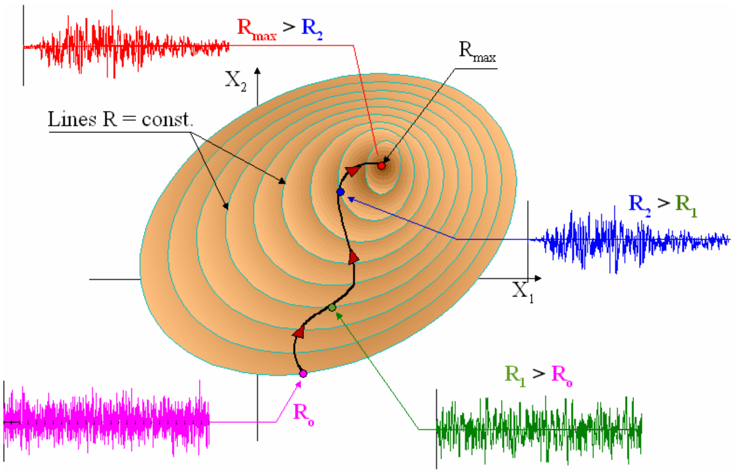

Figure 2: The response maximization procedure.

Starting from an initial point R_o deriving from an initial random vector $\mathbf{x}_o$ (Fig.2), a second random vector $\mathbf{x}_1$ in the neighbourhood of $\mathbf{x}_o$ is generated and the relevant block response is evaluated $R_1 = R(\mathbf{x}_1|\ell)$. If the attempt made by the sample vector $\mathbf{x}_1$ is not successful, i.e. if $R_1 < R_o$, a new perturbation vector $\mathbf{x}_1$ is generated and the steps are repeated until the attempt is successful, i.e. $R_1 > R_o$. In this case $\mathbf{x}_o$ can be substituted by $\mathbf{x}_1$, R_o by R_1, and the procedure is iterated in way to produce a sequence of vectors $\mathbf{x}_o, \mathbf{x}_1, \ldots \mathbf{x}_r, \ldots$ yielding an increasing sequence of response values $R_o, R_1, \ldots R_r, \ldots$; the procedure ends when, after a reasonable number of trials, say k_f, no more successful vectors **x** can be randomly produced.

4.3 *Numerical results*

As an example, it is assumed that the disturbance is a segment of a white-noise sample with given duration and maximum acceleration. The force acts on a simply-supported rigid block (Fig.1) with dimensions b = 0.4 m, h = 5 m, s =1 m, with the restitution coefficient ρ set equal to 0.8 and 1. The response is analyzed by means of the numerical solution of the classic equation [1] of the block dynamics under large displacements.

In Figs 3 and 5 a sample function (the red continuous line), randomly generated, is plotted yielding the following maximum response values R:

$\rho = 0.8$, $R = 0.005$; $\rho = 1.0$, $R = 0.128$

After the optimization process, the following worst-scenario results are obtained in terms of worst response R_{max}:

$\rho = 0.8$, $R_{max} = 0.285$; $\rho = 1.0$, $R_{max} = 0.424$

corresponding to the relevant worst disturbances plotted in Figs 3, 5 (the blu dotted lines) for the restitution coefficients $\rho = 0.8$ and 1, respectively. Figs 4, 6 report the results of advancements of the optimization procedure with the iterations, with the final worst responses attained. for the two cases.

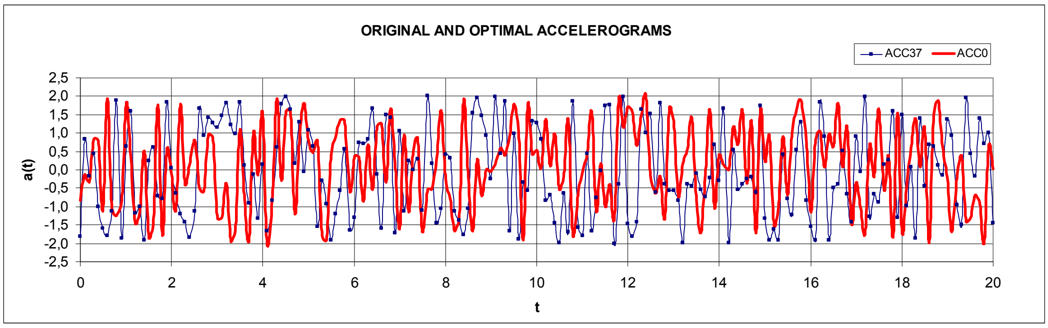

Figure 3: Original (red) and optimal (dotted blu) accelerograms for $\rho = 0.8$.

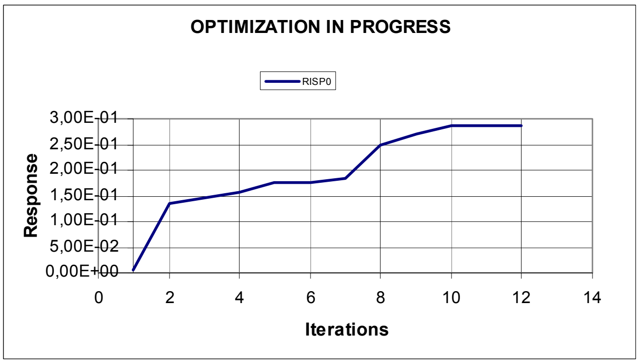

Figure 4: Maximum Response attained during the optimization iterative procedure for $\rho = 0.8$.

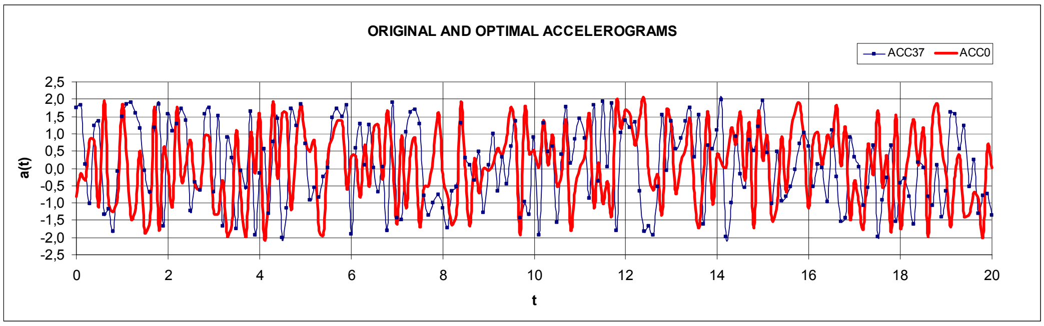

Figure 5: Original (red) and optimal (dotted blu) accelerograms for ρ = 1.

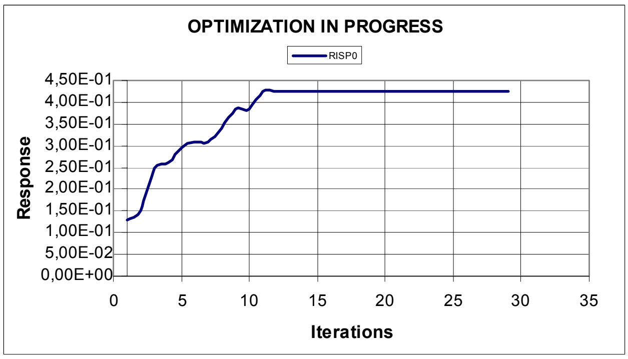

Figure 6: Maximum Response attained during the optimization iterative procedure for ρ = 1.

5 DISCUSSION OF THE RESULTS AND CONCLUSIONS

The poor robustness of RM response of rigid blocks resting on a rigid support surface pushes to approach the problem by a "worst scenario approach", thus stabilizing the results through a procedure that turns out to be effective especially when the forcing function is not sharply defined, as it may happen for instance in seismic assessment problems.

The results prove that worst-scenario yields results that may be much more severe than sample evaluations based on random choice of the forcing time-history. Worst-scenario results are likewise more severe than probabilistic evaluations based on the statistics of the response process, possibly evaluated by MonteCarlo methods, since the probability measure of the worst-scenario may most of the times be zero, despite of the fact that it is anyway one possible realization and also includes possible instability of the mathematical-numerical model.

6 ACKNOWLEDGEMENTS

This work has been financially supported by funds of MIUR within a PRIN project.

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