Finite thickness interface model for un-strengthened and strengthened bed joint of masonry walls in plane loaded

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SUMMARY. In this paper a homogenisation procedure to model un-strengthened and strengthened masonry walls, under service actions, in plane loaded is adopted. Two homogenisation models are proposed: an analytical 2D model and a numerical 2D model. Both of them allow to determine values of homogenised membrane moduli, for running bond texture, starting from the effective micro-structure of masonry (blocks and joints) as the FRP strengthening. A numerical analysis, both at constitutive level and structural level, has been carried out to evaluate the sensitivity of masonry behaviour to FRP repointing strengthening.

1 INTRODUCTION

Masonry walls - made of clay bricks and mortar joints - are, usually, subject to in plane and/or out of plane loading. Hence - due to the low tensile strength of mortar joint - strengthening procedure is needed. An interesting technique is FRP repointing, which consists of embedding continuous FRP strips in the bed joint by suitable paste. Few studies have been carried out on this technique [1, 2] in the last years. Generally, this technique is proposed in masonry structures to control cracking phenomena, because these cracks appear on the point of the structure failure.

Here, the term masonry is used to define periodic brickwork connected through thin mortar joints and arranged according to a periodic texture.

The analytical model starts from the model already proposed by Cecchi and Sab [3], in which masonry is assumed composed by rigid or elastic blocks connected by zero thickness mortar joints, modeled as linear cohesive zero thickness interfaces. This assumption allows to obtain the solution of field problem on the REV, in analytical form. Hence, equations of elastic in plane moduli are obtained. Here an implementation of Cecchi and Sab [3] - zero thickness joint model - is proposed, considering the effective joint thickness - linear cohesive finite thickness interface -; and the case of strengthened bed joint is proposed.

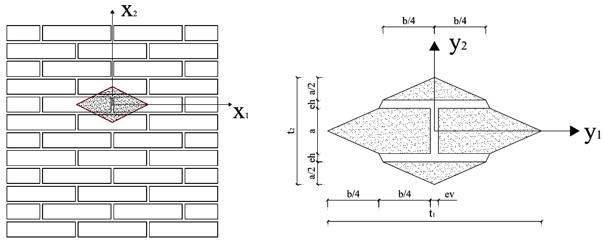
The results carried out by the analytical model for finite joint thickness un-strengthened and strengthened are compared with a 2D numerical F.E. model.

2 HOMOGENISATION FIELD PROBLEM FOR STRENGTHENED MASONRY

Periodic brickwork like masonry is investigated by means of homogenisation procedures. According to homogenisation procedure a REV (Representative Elementary Volume) is defined. The body as a whole is obtained by regular repetition of REV that presents small dimensions if compared to the overall size of the body itself and contains in a small scale all the geometric and mechanic properties to describe the body as a whole. Let be ($\boldsymbol{x}$) a reference system for the global description of the masonry wall in the macroscopic scale and let be ($\boldsymbol{y}$) a reference system for the elementary module *Y-REV* in the microscopic scale. Here a 2D plane model is developed, hence the Y module, as shown in figure 1 may be defined as:

$$Y = \left]-\frac{t_1}{2}, \frac{t_1}{2}\right[\times \left]-\frac{t_2}{2}, \frac{t_2}{2}\right[\tag{1}$$

where t_α (greeck index α=1, 2) are the dimensions of *Y-REV*.



a) b)

Figure 1: *Y-REV* of running bond texture in 2D analytical model.

Then, the following auxiliary problem is solved on the *Y-REV*:

$$\begin{cases} \mathrm{div}\boldsymbol{\sigma} = 0 \\ \boldsymbol{\sigma} = \mathbf{a}(\boldsymbol{y})\,\boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} = \mathbf{E} + \mathrm{sym}(\mathrm{grad}\,\mathbf{u}^{\mathrm{per}}) \\ \boldsymbol{\sigma}\,\mathbf{e}_\alpha \quad \text{anti-periodic on } \partial Y \\ \mathbf{u}^{\mathrm{per}} \text{periodic on } \partial Y \end{cases} \tag{2}$$

$\boldsymbol{\sigma}$ is the Cauchy stress tensor; $\boldsymbol{\varepsilon}$ is the microscopic strain tensor, $\mathbf{E}$ is the macroscopic in plane strain tensor; $\mathbf{e}_\alpha$ is the unit vector in the in plane directions; $\mathbf{u}^{per}$ is a periodic displacement field, $\mathbf{a}$ is the constitutive function defined as: $\mathbf{a}^{\mathrm{B}}$ for $\boldsymbol{y} \in$ block and $\mathbf{a}^{\mathrm{J}}$ for $\boldsymbol{y} \in$ joint.

The constitutive function of panel subjected to in plane loading may be written as:

$$\mathbf{N} = \langle \boldsymbol{\sigma} \rangle = \mathbf{A}^H \mathbf{E} \tag{3}$$

where $\mathbf{N}$ is the in plane membrane tensor, $\mathbf{A}^{\mathrm{H}}$ is the constitutive homogenised moduli and $\langle\cdot\rangle$ is the average operator. On the basis of these considerations, the macroscopic field problem at the structural level may be built.

3 BASIC ASSUMPTIONS IN 2D ANALYTICAL HOMOGENISATION MODEL

Previous results may be found in Cecchi and Rizzi [4] and Cecchi and Sab [3]. This latter analytical model considers three perturbative parameters:

- $\varepsilon=l/L$ ratio between the l size of the cell and the L dimension of the overall panel - typical of homogeneisation procedure;
- $\beta=e/l$ ratio between the e joint thickness and the l size of the cell.
- $\xi=E^m/E^b$, ratio between the E^m mortar Young modulus and the E^b block Young modulus;

The multiparameter model allows to obtain in a symbolic form the expressions of $\mathbf{A}^H$ homogenised moduli. In [3, 4] the mortar joint is considered as a zero thickness interface with an isotropic constitutive function that is directly obtained as a linear function of the displacement jump across the joint. The hypothesis of finite joint thickness is considered in [5]. Here the finite joint thickness is still considered and the bed joint is strengthened by FRP strips.

Hence, the first step envisages the definition of strengthened bed joint constitutive function. Following Cecchi and Sab [3] the constitutive function between the traction and the **[u]** jump at the interface is:

$$\boldsymbol{\sigma}\,\mathbf{n} = \mathbf{K}[\mathbf{u}] \tag{4}$$

where

$$\mathbf{K}_{\alpha\beta} = \frac{1}{e}\mathbf{a}_{\alpha\beta\gamma\delta}\mathbf{n}_\gamma\mathbf{n}_\delta \tag{5}$$

n is the normal to the interface and e is the mortar joint thickness.
In the case of head joint, which is made only of mortar - isotropic head joint - the constitutive function (5) becomes:

$$\mathbf{K}_v = \frac{1}{e_v}(\mu^m\mathbf{I} + (\mu^m + \lambda^m)(\mathbf{n}\otimes\mathbf{n})) \tag{6}$$

where e_v is the head joint thickness, μ^m and λ^m are the Lamé constants of mortar.

In the bed joint, constitutive equivalent function, $(K')^H$ bulk modulus and $(K'')^H$ shear modulus may be evaluated by reference to Voigt hypothesis (upper bound- $(\mathbf{K})^H = (\mathbf{K})^V$ -) or Reuss hypothesis (lower bound- $(\mathbf{K})^H = (\mathbf{K})^R$ -)). Under Voigt hypothesis the bed joint constitutive function is:

$$(K'_h)^V = \frac{K'^m e^m + K'^{FRP} e^{FRP}}{e^m + e^{FRP}},\ (K''_h)^V = \frac{K''^m e^m + K''^{FRP} e^{FRP}}{e^m + e^{FRP}} \tag{7}$$

whereas under Reuss hypothesis, the bed joint constitutive function is:

$$(K'_h)^R = \frac{K'^m K'^{FRP}(e^m + e^{FRP})}{K'^m e^{FRP} + K'^{FRP} e^m},\ (K''_h)^R = \frac{K''^m K''^{FRP}(e^m + e^{FRP})}{K''^m e^{FRP} + K''^{FRP} e^m} \tag{8}$$

where e^m is the mortar joint thickness, e^{FRP} is the FRP strip thickness, $K'^m=\lambda^m+2\mu^m$, $K''^m=\mu^m$ and $K'^{FRP}=\lambda^{FRP}+2\mu^{FRP}$, $K''^{FRP}=\mu^{FRP}$.

The second step requires the solution of auxiliary field problem (2). In the case of blocks connected by interfaces, as already explained by Cecchi and Sab [3] the field problem (2) becomes:

$$\begin{cases} \operatorname{div}\boldsymbol{\sigma} = 0 \\ \boldsymbol{\sigma} = \mathbf{a}^{\mathrm{B}}\boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} = \mathbf{E} + \operatorname{sym}(\operatorname{grad}\mathbf{u}^{per}) \\ \boldsymbol{\sigma}\,\mathbf{e}_\alpha \ \text{ anti-perodic on } \partial Y \\ \mathbf{u}^{\mathrm{per}} \text{periodic on } \partial Y \\ [\mathbf{u}] \neq 0 \text{ discontinuous for } \boldsymbol{y} \in \Sigma \\ [\boldsymbol{\sigma}\,\mathbf{n}] = 0 \text{ continuous for } \boldsymbol{y} \in \Sigma \\ \boldsymbol{\sigma}\,\mathbf{n} = \mathbf{K}[\mathbf{u}] \ \text{ interface constitutive function for } \boldsymbol{y} \in \Sigma \end{cases} \tag{9}$$

where ∂Y is the boundary of the *Y-REV*, **[u]** is the jump of displacement at the finite thickness interface and $\sum$ is the interface; **K** is the constitutive function of the finite thickness joint.

Obviously the solution is sensitive to the following assumptions:

- periodicity of boundary conditions;
- mortar joints modelled as interfaces; hence, due to small size of *e* if compared with other dimensions of the joint, the transversal contraction is negligible and it is not taken in account.

Respect to the previous paper of Cecchi and Sab [3] - zero thickness joint -, here the solution of the field problem (9) is obtained on the effective *Y-REV* area and not on Y^b*-REV* area - relative to the only block area.

3.1 *Influence of strengthened joint*

The homogenised moduli for elastic brick with a=width; b=height; t=thickness and elastic joint with e_h=horizontal joint thickness and e_v=vertical joint thickness are obtained in *Y-REV* following the procedure of Cecchi and Sab [3] but the model is implemented for finite joint dimensions. The obtained homogenised constitutive functions, if compared with those in [6], show exactly the same structure: the differences lie in the ratios e_v/b and e_h/a which are substituted respectively by $e_v/(b+e_v)$ and $e_h/(a+e_h)$ and for bed joint the expressions (7) and (8) are used.

The diagram of figure 2 shows in ordinate, at the left side, the ratio between the bed joint constitutive function for strengthened joint under Voigt hypothesis $(\mathbf{K}_h)^V$ and for un-strengthened one; whereas at the right side, the ratio between the bed joint constitutive function for strengthened joint under Reuss hypothesis $(\mathbf{K}_h)^R$ and for un-strengthened one. In abscissa the ζ^{-1} $(=E^{FRP}/E^m)$ ratio between the E^{FRP} FRP Young modulus and the E^m mortar Young modulus (E^m=1000 MPa) is reported.

The $\mathbf{K}_h$ values under Voigt hypothesis increase linearly increasing the ratio ζ^{-1}. The FRP Young modulus is assumed between 145 GPa and 300 GPa; the right side of the diagram in figure 2 has to be considered, respect the dashed vertical line. The $\mathbf{K}_h$ values under Reuss hypothesis increase quickly for low value of ζ^{-1}; whereas the value is almost constant for value of ζ^{-1} bigger than 50. That means the variation of FRP Young modulus does not influence enough the constitutive function of bed joint strengthened by FRP repointing technique, under Reuss hypothesis.

A numerical analysis has been developed to evaluate the sensitivity of each $A_{\alpha\beta\gamma\delta}{}^{H}$ in plane moduli - under plane strain hypothesis - to strengthened bed joint, considering finite joint thickness. The case of un-strengthened joint ($A_{\alpha\beta\gamma\delta}^{H}$*_US*) and strengthened joint ($A_{\alpha\beta\gamma\delta}^{H}$*_FRP*) normalised versus the $A_{\alpha\beta\gamma\delta}{}^{b}$ in plane modulus of homogeneous masonry made of block (E^b=90 GPa) has been investigated.

The increment of modulus $A^H{}_{1111}$ in the strengthened masonry, for $\xi^{-1}>20$, respect to the un-strengthened one, is smaller than that evaluated for the other moduli. The increment is maximum for $\xi^{-1}=25$; it increases for low value of $\xi^{-1}<25$, whereas it decreases for low value of $\xi^{-1}>25$. The increment is between 3% and 4%. Different considerations should be done for in plane $A^H{}_{2222}$, $A^H{}_{1122}$ and $A^H{}_{1212}$ moduli: the strengthened joint increases the values of moduli, sensitively. The increment of ξ^{-1} induces an increment of these moduli, in particular: the modulus $A^H{}_{2222}$ increases from 2% to 9%; the modulus $A^H{}_{1122}$ increases from 6% to 13.5%; the modulus $A^H{}_{1212}$ increases from 0.4% to 8%. The contribution of FRP repointing technique is more relevant for high value of ξ^{-1} for these homogenised moduli. The increment of membrane moduli, in the 2D analytical model, considering strengthened and un-strengthened masonry may be evaluated by:

$$\Delta A^H_{\alpha\beta\gamma\delta} = \frac{A^H_{\alpha\beta\gamma\delta}\_FRP - A^H_{\alpha\beta\gamma\delta}\_US}{A^H_{\alpha\beta\gamma\delta}\_FRP} \cdot 100 \tag{10}$$

where $A^H{}_{\alpha\beta\gamma\delta}\_FRP$ is the modulus evaluated considering FRP strengthened bed joint and $A^H{}_{\alpha\beta\gamma\delta}\_US$ is the modulus evaluated considering mortar bed joint. The results are plotted in figure 3 where in abscissa the ξ^{-1} ($=E^b/E^m$) ratio between the E^b brick Young modulus and the E^m mortar Young modulus is reported.

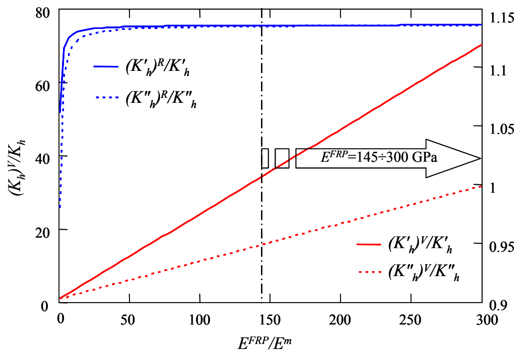


Figure 2: Ratio between the constitutive function of strengthened - $(\boldsymbol{K}_h)^V$ or $(\boldsymbol{K}_h)^R$ - and un-strengthened bed joint $\boldsymbol{K}_h$ versus ξ^{-1}.

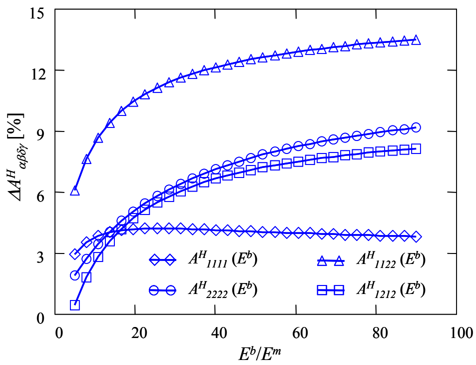


Figure 3: The increment of membrane moduli $A^H{}_{\alpha\beta\delta\gamma}$ versus ξ^{-1}.

3.2 *Justification*

As shown in Figure 3 the sensitivity of $A^H{}_{\alpha\beta\delta\gamma}$ versus ξ^{-1} is consistent only when $\xi^{-1}<20$. A justification of this phenomenon is connected to the asymptotic multi-parameter analytical model. By assuming $\beta=e/l$ ratio between the e joint thickness and the l size of the cell and $\xi=E^m/E^b$, ratio between the E^m mortar Young modulus and the E^b block Young modulus, the asymptotic problem is focused on the case: $\beta\to 0$; $\xi\to 0$. If β tends to zero the mortar joint becomes an interface, if ξ tends to zero the joint becomes infinitely deformable. The asymptotic problem depends on how the two parameters tend to zero. Considering $\xi=\xi(\beta)$, if β tends to zero more quickly than ξ. Hence the case of perfectly cohesive joint is obtained, there is a perfect continuity between the blocks which may be assumed as one single homogeneous material; if β and ξ tend to zero with the same velocity. Hence the case of cohesive joint is obtained, with possible jump of the displacement at the interface.

In the case of strengthened bed joint, in the asymptotic model, the ξ ratio value is substituted by $\varsigma^{-1}=\mathbf{a}^J/\mathbf{a}^B$, where $\mathbf{a}^J$ is the constitutive function of the reinforced joint. Hence due to the high value of FRP constitutive function the case of perfectly cohesive joint - perfect continuity between the blocks - is obtained and the homogenised moduli in this analytical model result not sensitive to the strengthening.

4 BASIC ASSUMPTIONS ON F.E. HOMOGENISATION MODEL

A numerical model has been formulated to evaluate the homogenised in plane moduli $A_{\alpha\beta\gamma\delta}{}^H$ of masonry wall un-strengthened and strengthened by FRP repointing technique. The auxiliary field problem on the elementary cell, plotted by dashed line - due to the symmetry - may be reported to the only *Y/4-REV* –the origin of axes is centred in the *REV* centre - (Fig. 4).

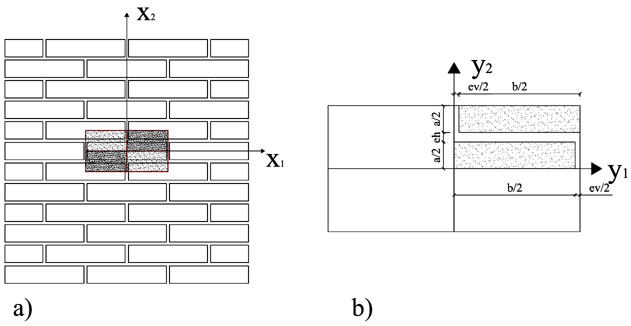


Figure 4: *Y-REV* of running bond texture in 2D F.E. model.

The field problem that must be solved is (2) with the following boundary conditions:

$$\mathbf{u}(\mathbf{y}) = \begin{pmatrix} E_{11}y_1 + E_{12}y_2 \\ E_{21}y_1 + E_{22}y_2 \end{pmatrix} + \begin{pmatrix} u_1^{per}(\mathbf{y}) \\ u_2^{per}(\mathbf{y}) \end{pmatrix} \tag{11}$$

with $E_{12}=E_{21}$.

The homogenised in plane moduli are obtained solving field problem (2). Hence according to equations (3) the homogenised constitutive functions are:

$$A^H_{\alpha\beta\gamma\delta} = \frac{2}{S'}\int \sigma_{\alpha\beta}dY \tag{12}$$

where S′ is the area of *Y/4-REV* .

5 NUMERICAL ANALYSIS

The analytical results carried out considering finite joint thickness and strengthened bed joint, have been compared with the numerical ones, such as to defines the application limit of analytical model. This latter, as already explained, may be managed very easy - due to analytical symbolic expression of elastic moduli. The moduli of un-strengthened and strengthened masonry have been evaluated to verify the sensitivity of masonry macroscopic behaviour to CFRP repointing technique. The analysis has been carried out for FRP with different longitudinal elastic modulus: E^{FRP} = 145·10^3; 210·10^3; 300·10^3 MPa; and ν^{FRP} = 0.4.

The analysed masonry is made of UNI clay bricks (250×55×120 mm^3 width × height × thickness) with longitudinal elastic modulus: E^b = 5000÷90000 MPa, and ν^b = 0.2; whereas mortar properties are: E^m = 1000 MPa; ν^m = 0.2 and mortar joint thickness is e_v=10 mm for head joint and e_h=10 mm for bed joint in the case of un-strengthened masonry. When the strengthened masonry is considered, the horizontal joint thickness is composed by two mortar layer 4.4 mm thick and one central layer of CFRP material 1.2 mm thick. The whole thickness of bed joint is still 10 mm. The FRP material considered in the present research is made of carbon fibres (CFRP).

The F.E. models are built for each masonry - un-strengthened or strengthened -, meshing the *Y/4 -REV*. Three elements are used in the meshing of the mortar joint across its e_h thickness: mortar, FRP and mortar. In the case of un-strengthened masonry the mortar bed joint is discretized by 3 elements across e_h thickness. Both joint, block and FRP elements are modelled by 4 nodes bi-dimensional elements.

The elementary cell - *Y/4-REV* - is meshed by 182 2D 4 nodes elements (Fig. 5):

1- un-strengthened masonry *Y/4-REV*: 120 F.E. elements for modelling brick and 62 F.E. elements for modelling mortar joint;

2- strengthened masonry *Y/4-REV*; 120 F.E. elements for modelling brick, 48 F.E. elements for modelling mortar joint and 14 F.E. elements for modelling CFRP.

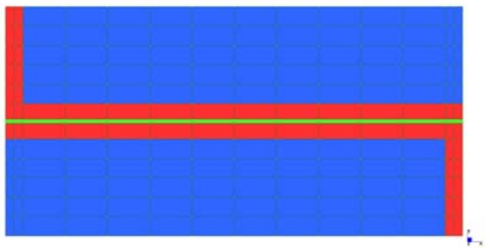


Figure 5: 2D F.E. model mesh.

A numerical analysis has been performed for each $A_{\alpha\beta\gamma\delta}{}^{H}$ in plane modulus under plane strain hypothesis. For different ξ^{-1} (=E^b/E^m) ratio between the E^b brick Young modulus and the E^m mortar Young modulus the $A_{\alpha\beta\gamma\delta}{}^{H}$_NUM homogenised numerical modulus and the $A_{\alpha\beta\gamma\delta}{}^{H}$_AN homogenised analytical are reported.

The following remarks may be pointed out:

- $A^{H}{}_{1111}$ in plane modulus for strengthened masonry evaluated numerically (NUM) is bigger than the one evaluated analytically (AN) and the difference decreases when ξ^{-1} increases. The analytical results are not influenced by FRP Young modulus as explained in paragraph 3.1 and 3.2, in fact the case of perfectly cohesive joint is obtained due to FRP.
- $A^{H}{}_{2222}$, $A^{H}{}_{1122}$, $A^{H}{}_{1212}$ membrane moduli are not sensitive to FRP Young modulus both in the analytical model and the numerical model.
- $A^{H}{}_{2222}$, $A^{H}{}_{1122}$ and $A^{H}{}_{1212}$ in plane modulus evaluated analytically is lower than the one evaluated numerically, and the difference decreases when ξ^{-1} increases.

The sensitivity to CFRP repointing technique is analysed. The numerical analysis has been carried out comparing the un-strengthened masonry with the strengthened one - with different longitudinal Young modulus -.

The increment of in plane stiffness is function of both the E^{FRP} - FRP longitudinal modulus- and the ratio between block modulus and mortar modulus. In fact increasing the value of ξ^{-1} the difference between the un-strengthened value and the strengthened one increases too. The increment of in plane stiffness in masonry panel is more evident along y_1 axis direction.

In particular the $A^H{}_{1111}$ in plane modulus is sensitive to the E^{FRP} , whereas the others in plane moduli are not sensitive. In fact, the FRP axial stiffness has to be taken into account only to increase the strengthened masonry stiffness along horizontal joint. The in plane stiffness increment is about 20÷60 % in function of masonry parameters (block and mortar stiffness) and FRP mechanical properties. This latter depends on the nature of the fibre used in the FRP strengthening.

The sensitivity of FRP repointing technique is evaluated according to the equation (10), used for the analytical case.
The $A^H{}_{2222}$, $A^H{}_{1122}$, and $A^H{}_{1212}$ in plane moduli are sensitive to ξ^{-1} ratio.

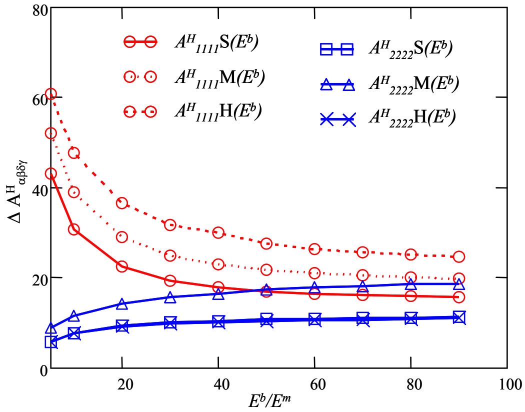


Figure 6: $\Delta A^H{}_{\alpha\beta\gamma\delta}$ increment of in plane stiffness for strengthened masonry compared with un-strengthened one.

In figure 6, the $A^H{}_{\alpha\beta\gamma\delta}$ in plane modulus is plotted versus the ξ^{-1} ratio, considering slow (S), medium (M) and high (H) longitudinal Young modulus of FRP for $A^H{}_{1111}$, whereas only the slow modulus is considered for the other moduli that are not sensitive to different longitudinal Young modulus of FRP . The $A^H{}_{1111}$ in plane stiffness reduces when the ξ^{-1} ratio increases in function of the E^{FRP} parameter, whereas the $A^H{}_{2222}$, $A^H{}_{1122}$ and $A^H{}_{1212}$ in plane stiffnesses increase slowly in the full range, showing more evident increment for small ξ^{-1} ratio.

6 STRUCTURAL ANALYSIS: SHEAR MASONRY PANEL

A comparison, at a structural level, between 2D heterogeneous and homogenised model has been performed. A panel with 1160 mm height (H), 1030 mm width (W) and 120 mm thickness (T) is analysed. The blocks are 250×55×120 mm^3 and the mortar joint is 10 mm thickness. The repointing technique is applied at each bed joint. The masonry panel is fixed at the base and an unit horizontal displacement (u_1=1 mm) is applied at the top, whereas the vertical displacement are zero. The 2D homogenised and heterogeneous models are meshed by bi-dimensional elements (4 node) (Fig. 7).

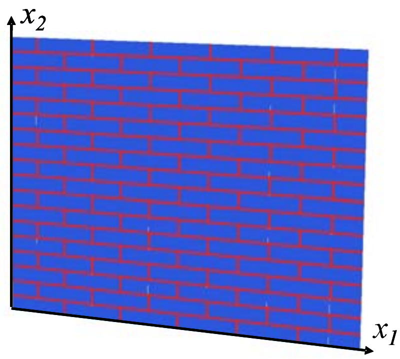


Figure 7: 2D full scale F.E. model.

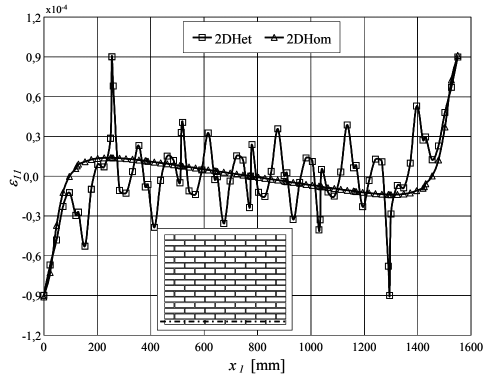


Figure 8: Comparison between 2D heterogeneous F.E. model and 2D homogenised model in terms of ε_{11} for ξ^{-1}=10.

The comparison between the 2D heterogeneous F.E. model and 2D homogenised F.E. model is carried out for 10≤ ξ^{-1} ≤ 90. The effectiveness of homogenisation procedure is tested comparing the strain ε_{11}, ε_{22}, ε_{12} at the base of masonry panel (first course of bricks). The 2D homogenised model as expected fits enough at the mean values of strain distribution. The jump in the diagram of heterogeneous model is related to head joint along the horizontal cross section (Fig. 8, 9, 10). The same analysis may be conducted for strengthened masonry panel, evaluating the effect of slow, medium and high FRP longitudinal Young modulus.

The numerical results carried out show that a good agreement between 2D heterogeneous model and 2D homogenised model for both un-strengthened and strengthened cases. The homogenised model is included between the strain distribution at brick layer (*B*) and bed joint layer (*J*); the homogenised solution is more close to *B* layer distribution because the area fraction of block is bigger than that of joint and this is more evident increasing the ratio ξ^{-1} [5]. Obviously, the 2D homogenised model has the same strain distribution along *B* layer and *J* layer because of its homogeneity.

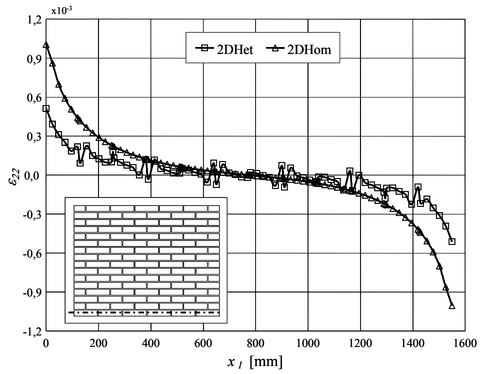


Figure 9: Comparison between 2D heterogeneous F.E. model and 2D homogenised model in terms of ε_{22} for ξ^{-1}=10.

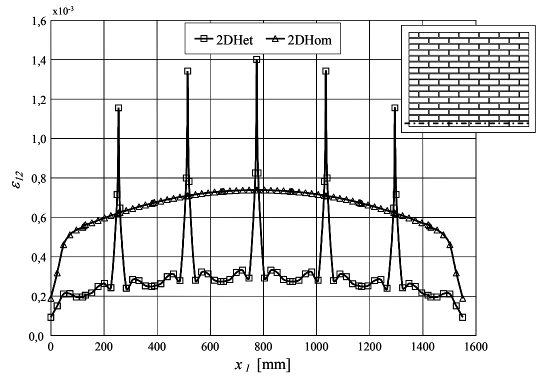


Figure 10: Comparison between 2D heterogeneous F.E. model and 2D homogenised model in terms of ε_{12} for ξ^{-1}=10.

7 CONCLUSIONS

The proposed 2D homogenisation procedures allows to define the membrane moduli $A^{H}{}_{\alpha\beta\gamma\delta}$ of un-strengthened and strengthened masonry, considering a wider set of internal parameters (i.e. relative size of the joints, relative deformability of the joints) and the case of strengthened joint by FRP repointing technique. In fact, the investigation is focused on FRP repointing technique that is spreading in the restoration design of masonry heritage.

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