A nonlinear Cosserat-Cauchy homogenization procedure for regular masonry based on transformation field analysis

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SUMMARY. A nonlinear homogenization procedure is presented for deriving macroscopic mechanical behavior of periodic masonry material. In particular, a Cosserat medium is adopted at the macroscopic level, while a classical Cauchy model is used at the microscopic level, where the constitutive behavior of bricks and mortar is known in detail. The nonlinear homogenization is performed developing a Transformation Field Analysis (TFA). A numerical procedure is implemented to solve the evolutive nonlinear homogenization problem based on a prediction-correction technique. Some numerical applications are performed on the masonry Unit Cell (UC).

1 INTRODUCTION

Regular masonry is a composite material made of bricks and mortar characterized by an anisotropic nonlinear behavior. Most of the approaches proposed in literature to model the complex nonlinear mechanical response of masonry materials may be grouped into three families: micromechanical procedures, which model in detail the mechanical behavior of each constituent and their geometrical properties, i.e. size and texture; macroscopic models considering masonry material as an equivalent homogenized medium characterized by a properly identified phenomenological constitutive law; multi-scale approaches, where both the microscopic and macroscopic levels are modeled, deriving the constitutive behavior of the homogenized equivalent medium by means of an up-scaling procedure.

Recently, multi-scale approaches have been developed in the field of modeling heterogeneous materials and, in particular, they have been satisfactorily employed in the structural analysis of periodic masonry [1]. To date most of the proposed multi-scale models are based on the so-called first order homogenization technique, making use at both macro- and micro-scale of the classical Cauchy continuum. On the other hand, it has been shown that such techniques fail in reproducing structural responses characterized by the presence of high deformation gradients, damage localization regions and relevant ratio between the macrostructural and microstructural characteristic sizes. In order to overcome such drawbacks, enhanced homogenization methods have been proposed; they are based on the use of second gradient continuum models [2], as well as enriched models, e.g. micropolar Cosserat continuum [3].

In this paper a nonlinear homogenization procedure is presented for deriving macroscopic mechanical behavior of masonry material. In particular, a Cosserat medium is adopted at the macroscopic level, while a classical Cauchy model is employed at the microscopic level, where the constitutive behavior of bricks and mortar is known in detail. Then, the constitutive response at a macroscopic point of the equivalent medium is evaluated by solving a nonlinear micromechanical problem on a Representative Volume Element (RVE), properly selected at the microscopic level. To this end a non standard boundary value problem is stated for the RVE, where periodic kinematic conditions are assigned at the boundary. In particular, the displacement field in the RVE is expressed as the superposition of a prescribed field, depending on the macroscopic Cosserat strain components, and a periodic one. As for the prescribed field, a suitable kinematic map bridging the macro-Cosserat and the micro-Cauchy scales is formulated for rectangular RVE. A linear elastic constitutive law is assumed for bricks, while a coupled damage-friction cohesive model is used for mortar joints. The nonlinear homogenization is performed developing a Transformation Field Analysis (TFA) procedure [4, 5], where the microscopic inelastic strain field is decomposed into a finite set of uniform transformation fields. In particular, for each mortar joint, a uniform inelastic strain is assumed and, as a consequence, a uniform transformation field is considered.

A numerical procedure is implemented to solve the evolutive nonlinear homogenization problem based on a prediction-correction technique. Some numerical applications are performed on a RVE, properly selected to represent a commonly used texture for masonry, comparing the results obtained with the proposed TFA procedure with the ones carried out by a nonlinear micromechanical finite element analysis, and showing the effectiveness of the presented approach.

2 MICROPOLAR MASONRY MODEL WITH PERIODIC MICROSTRUCTURE

Heterogeneous masonry material with periodic microstructure can be replaced by an equivalent Cosserat homogeneous medium at the macro-level. A 2D Cosserat continuum involves an additional rotational kinematical degree of freedom with respect to the classical Cauchy model. In fact, the displacement vector contains three independent kinematic fields, representing the translations U_1 and U_2 and the rotation Φ, respectively, at each point $\mathbf{X} = (X_1, X_2)^T$ of the body volume Ω. In the following the analysis is developed in the framework of small strain and displacement assumption. Adopting Voigt's notation, the deformation field is described by the infinitesimal strain vector $\mathbf{E} = \{E_1 \;\; E_2 \;\; E_{12} \;\; E_{21} \;\; K_1 \;\; K_2\}^T$, whose components are defined as:

$$E_1 = \frac{\partial U_1}{\partial X_1},\; E_2 = \frac{\partial U_2}{\partial X_2},\; E_{12} = \frac{\partial U_2}{\partial X_1} - \Phi,\; E_{21} = \frac{\partial U_1}{\partial X_2} + \Phi,\; K_1 = \frac{\partial \Phi}{\partial X_1},\; K_2 = \frac{\partial \Phi}{\partial X_2} \tag{1}$$

where E_1 and E_2 are the extensional strains, E_{12} and E_{21} are the non symmetric shear strains, K_1 and K_2 the micro-curvatures.

In the following it appears useful to introduce a different description of the macroscopic Cosserat shear deformation components, with the aim of easily recognizing the in-plane Cauchy deformation components in the vector $\mathbf{E}$. Then, the macroscopic Cosserat strain $\mathbf{E}$ is now expressed as:

$$\mathbf{E} = \{E_1 \quad E_2 \quad \Gamma_{12} \quad \Theta \quad K_1 \quad K_2\}^T \tag{2}$$

where $\Gamma_{12} = E_{12} + E_{21}$ is the classical in-plane Cauchy shear strain and $\Theta = E_{12} - E_{21} = 2(W - \Phi)$ is the rotational deformation, representing twice the difference between the rigid rotation W and the Cosserat rotation field Φ, being:

$$W = \frac{1}{2}\left(\frac{\partial U_2}{\partial X_1} - \frac{\partial U_1}{\partial X_2}\right) \tag{3}$$

The dual quantities in the virtual work sense associated with the deformation components (2) are collected in the stress vector $\mathbf{\Sigma}$, which may be expressed as:

$$\mathbf{\Sigma} = \{\Sigma_1 \quad \Sigma_2 \quad \Sigma_{12} \quad Z \quad M_1 \quad M_2\}^T \tag{4}$$

Hereafter, the analysis is limited to the case of regular periodic masonry. In particular, a 2D plane strain analysis is developed. The chosen UC is characterized by rectangular shape with dimensions $2a_1$ and $2a_2$, parallel to the coordinate axes x_1 and x_2, as shown in Figure 1a. The mortar thickness is denoted by s and the brick sizes by b and h.

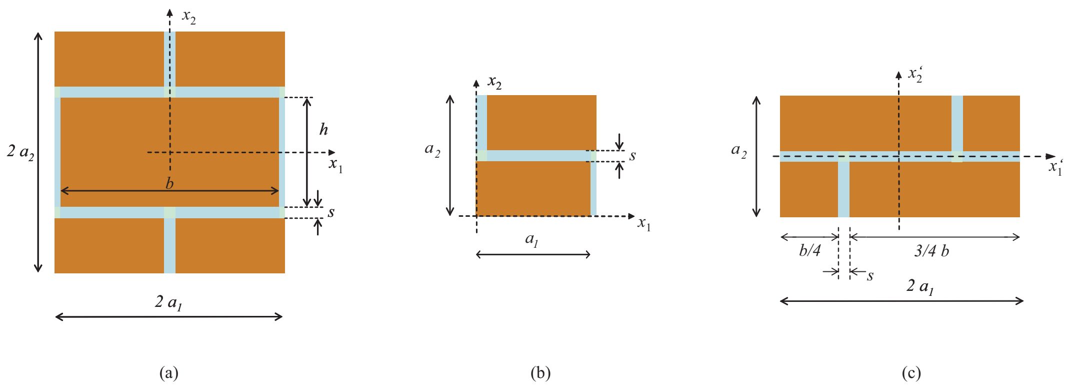

Figure 1: (a) UC; (b) quarter of UC; (c) half of UC.

The linear elastic stress-strain relationship is adopted for the brick:

$$\boldsymbol{\sigma}^B = \mathbf{C}^B \boldsymbol{\varepsilon} \tag{5}$$

where $\mathbf{C}^B$ is the elastic matrix of the masonry brick, $\boldsymbol{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \gamma_{12}\}^T$ is the strain vector and $\boldsymbol{\sigma}^B = \{\sigma_1^B, \sigma_2^B, \tau_{12}^B\}^T$ is the stress vector.

A damage-friction constitutive law for the mortar material is adopted [5]. A local coordinate system is introduced in the typical mortar joint, with T and N denoting directions parallel and orthogonal to the mortar joint, respectively. The stress $\boldsymbol{\sigma}^M$ occurring at the typical point of the mortar, is obtained as a suitable combination of two stresses, $\boldsymbol{\sigma}^u$ and $\boldsymbol{\sigma}^d$, according to the formula:

$$\boldsymbol{\sigma}^M = (1-D)\boldsymbol{\sigma}^u + D\boldsymbol{\sigma}^d \tag{6}$$

where D is the damage parameter. The two stress vectors $\boldsymbol{\sigma}^u$ and $\boldsymbol{\sigma}^d$ are related to the strain vector in the mortar, $\boldsymbol{\varepsilon}$, by the constitutive equations:

$$\boldsymbol{\sigma}^u = \mathbf{C}^M \boldsymbol{\varepsilon}, \qquad \boldsymbol{\sigma}^d = \mathbf{C}^M \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right) \qquad \mathbf{C}^M = \begin{bmatrix} C_{TT}^M & C_{NT}^M & 0 \\ C_{NT}^M & C_{NN}^M & 0 \\ 0 & 0 & G^M \end{bmatrix} \tag{7}$$

where $\boldsymbol{\varepsilon}^p$ is the vector of the inelastic strain due to the possible unilateral opening effect and to the friction sliding. Taking into account the constitutive Eqs. (7), Eq. (6) becomes:

$$\boldsymbol{\sigma}^M = \mathbf{C}^M \left(\boldsymbol{\varepsilon} - \boldsymbol{\pi}\right) \tag{8}$$

where $\boldsymbol{\pi}$ is the vector of the inelastic strain accounting for damage, unilateral contact and friction.

The evolution of the inelastic slip strain component γ_{NT}^p is governed by the classical Coulomb yield function:

$$\varphi\left(\boldsymbol{\sigma}^d\right) = \mu\,\sigma_N^d + \left|\tau_{NT}^d\right| \tag{9}$$

where μ is the friction parameter and σ_N^d and τ_{NT}^d are the normal and tangential components of $\boldsymbol{\sigma}^d$. The non-associated flow rule is considered as:

$$\dot{\gamma}_{NT}^p = \dot{\lambda}\frac{\tau_{NT}^d}{\left|\tau_{NT}^d\right|} \qquad \dot{\lambda} \geq 0 \quad \varphi\left(\boldsymbol{\sigma}^d\right) \leq 0, \quad \dot{\lambda}\,\varphi\left(\boldsymbol{\sigma}^d\right) = 0 \tag{10}$$

where λ is the inelastic multiplier. A model which accounts for the coupling of mode I and mode II of fracture is considered for the evolution of the damage parameter D. The two quantities η_N and η_{NT}, which depend on the first cracking strains $\varepsilon_{N,0}$ and $\gamma_{NT,0}$, on the peak value of the stresses $\sigma_{N,0}$ and $\tau_{NT,0}$ and on the fracture energies G_{cI} and G_{cII}, respectively, are introduced in the form:

$$\eta_N = \frac{\varepsilon_{N,0}\,\sigma_{N,0}}{2G_{cI}}, \qquad \eta_{NT} = \frac{\gamma_{NT,0}\,\tau_{NT,0}}{2G_{cII}} \tag{11}$$

The equivalent strain measures Y_N and Y_{NT} are defined as $Y_N = \langle\varepsilon_N\rangle^2$, $Y_{NT} = \left(\gamma_{NT}\right)^2$, where the bracket operator $\langle\bullet\rangle$ gives the positive part of the quantity $\bullet$. Then, the strain ratios are determined as:

$$\eta = 1 - \frac{1}{\alpha^2}\left(Y_N\,\eta_N + Y_{NT}\,\eta_{NT}\right), \qquad \beta = \sqrt{\frac{Y_N}{\varepsilon_{N,0}^2} + \frac{Y_{NT}}{\gamma_{NT,0}^2}} - 1 \tag{12}$$

with $\alpha = (Y_N + Y_{NT})^{1/2}$. Finally, the damage is evaluated according to the following law:

$$D = \max_{history}\left\{\min\left\{1, \frac{1}{\eta}\left(\frac{\beta}{1+\beta}\right)\right\}\right\} \tag{13}$$

3 NONLINEAR HOMOGENIZATION TECHNIQUE

In order to derive the macroscopic behavior of periodic masonry, a compatible nonlinear homogenization procedure, based on a proper formulated kinematic map linking the macro- and micro-levels, is presented [6]. The heterogeneous masonry UC is subjected to the macroscopic Cosserat strain $\mathbf{E}$ applied to the whole UC and to the inelastic strain $\boldsymbol{\pi}^i$, with $i = 1,\ldots,m$, applied to each of the m mortar joints.

The displacement field for the Cauchy micromechanical medium at the point $\mathbf{x} = (x_1, x_2)^T$ of the UC is expressed in the following representation form:

$$\mathbf{u} = \bar{\mathbf{u}}(\mathbf{x}) + \tilde{\mathbf{u}}(\mathbf{x}) \tag{14}$$

where $\bar{\mathbf{u}}(\mathbf{x})$ is the assigned displacement field, depending on the macroscopic deformation $\mathbf{E}$ and $\tilde{\mathbf{u}}(\mathbf{x})$ is the periodic field, satisfying periodicity conditions at the UC boundary ([2], [3]). Consequently, the compatible strain field for the Cauchy medium results in the form:

$$\boldsymbol{\varepsilon} = \bar{\boldsymbol{\varepsilon}}(\mathbf{x}) + \tilde{\boldsymbol{\varepsilon}}(\mathbf{x}) \tag{15}$$

where $\bar{\boldsymbol{\varepsilon}}$ and $\tilde{\boldsymbol{\varepsilon}}$ are the strain fields compatible with $\bar{\mathbf{u}}$ and $\tilde{\mathbf{u}}$, respectively. Herein, by considering a rectangular cell, the following form of the assigned displacement $\bar{\mathbf{u}}$ results in compact notation:

$$\bar{\mathbf{u}} = \mathbf{A}(\mathbf{x})\mathbf{E} \qquad \mathbf{A} = \begin{bmatrix} x_1 & 0 & \frac{1}{2}x_2 & -\alpha(x_2^3 - 3\rho^2 x_1^2 x_2) & -x_1 x_2 & -\frac{1}{2}x_2^2 \\ 0 & x_2 & \frac{1}{2}x_1 & -\rho^2\alpha(\rho^2 x_1^3 - 3x_1 x_2^2) & \frac{1}{2}x_1^2 & x_1 x_2 \end{bmatrix} \tag{16}$$

where $\alpha = 5\left(a_1^2 + a_2^2\right)/\left(4a_1^4\right)$ and $\rho = a_2 / a_1$.

It has to be underlined that the Cosserat strain vector $\mathbf{E}$ in Eq. (16) differs from the one defined in Eq. (2); in fact, the fourth component is redefined as:

$$\hat{\Theta} = \Theta + \frac{1}{2}\frac{\rho^2 - 1}{\rho^2 + 1}\Gamma_{12} \tag{17}$$

In such a way, the Cauchy deformation modes can be activated independently from the Cosserat modes. The stress component associated to $\hat{\Theta}$ is denoted with $\hat{Z}$.

The strain vector at the microlevel can be expressed as:

$$\boldsymbol{\varepsilon} = \mathbf{B}(\mathbf{x})\mathbf{E} + \tilde{\boldsymbol{\varepsilon}}(\mathbf{x}) \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 6\alpha\rho^2 x_1 x_2 & -x_2 & 0 \\ 0 & 1 & 0 & 6\alpha\rho^2 x_1 x_2 & 0 & x_1 \\ 0 & 0 & 1 & 3\alpha\left(\rho^2 - 1\right)\left(x_2^2 - \rho^2 x_1^2\right) & 0 & 0 \end{bmatrix} \tag{18}$$

where $\tilde{\boldsymbol{\varepsilon}}(\mathbf{x})$ is the periodic strain, satisfying null average condition in Ω.

The total assigned Cosserat macroscopic strain $\mathbf{E}$ is additively decomposed into an elastic part $\mathbf{E}_e$ and an inelastic part $\mathbf{P}$, resulting from the presence of the inelastic strains $\boldsymbol{\pi}^i$ in the m mortar joints.

Firstly, the BVP on the UC subjected to $\mathbf{E}_e$ is formulated. The micromechanical strain field, resulting after numerically solving the BVP, can be written in the following representation form:

$$\mathbf{e} = \mathbf{R}_e(\mathbf{x})\mathbf{E}_e \tag{19}$$

where the localization matrix $\mathbf{R}_e(\mathbf{x})$ allows to evaluate the Cauchy local strain, **e**, at any point of the microscopic medium, corresponding to the application of the Cosserat strain $\mathbf{E}_e$. The average strain in each mortar joint M^i, with Ω^{M^i} denoting its volume, results as:

$$\overline{\mathbf{e}}^{M^i} = \frac{1}{\Omega^{M^i}} \int_{M^i} \mathbf{e}(\mathbf{x})\, d\Omega = \frac{1}{\Omega^{M^i}} \int_{M^i} \mathbf{R}_e(\mathbf{x})\, d\Omega\, \mathbf{E}_e = \overline{\mathbf{R}}_e^{M^i}\mathbf{E}_e, \quad i = 1, 2, \ldots, m \tag{20}$$

The corresponding homogenized Cosserat stress, $\boldsymbol{\Sigma}_e$, in the whole UC volume Ω is obtained by applying the generalized Hill-Mandel principle, so that:

$$\boldsymbol{\Sigma}_e = \mathbf{C}\mathbf{E}_e \tag{21}$$

where $\mathbf{C}$ represents the overall elastic constitutive matrix. Similarly, the average stress in the mortar joint M^i may be evaluated as $\overline{\boldsymbol{\sigma}}_e^{M^i} = \mathbf{C}^M \overline{\mathbf{e}}^{M^i} = \mathbf{C}^M \overline{\mathbf{R}}_e^{M^i}\mathbf{E}_e$.

Solving the micromechanical problem of the UC subjected to an inelastic strain $\boldsymbol{\pi}^i$, prescribed in the mortar joint M^i, the resulting local strain field, $\mathbf{p}^i$, is expressed in the form:

$$\mathbf{p}^i = \mathbf{R}_{\pi^i}(\mathbf{x})\boldsymbol{\pi}^i \tag{22}$$

being $\mathbf{R}_{\pi^i}(\mathbf{x})$ the associated localization matrix, which relates the Cauchy inelastic strain $\boldsymbol{\pi}^i$ to the Cauchy local strain $\mathbf{p}^i$. The elastic strain in the typical mortar joint M^j is obtained as the difference between its total deformation $\mathbf{p}^{i,M^j}$ and the inelastic strain $\boldsymbol{\pi}^i$ as:

$$\boldsymbol{\eta}^{i,M^j} = \mathbf{p}^{i,M^j} - \delta_{ij}\boldsymbol{\pi}^i = \left(\mathbf{R}_{\pi^i}^{M^j} - \delta_{ij}\mathbf{I}\right)\boldsymbol{\pi}^i \tag{23}$$

where $\mathbf{p}^{i,M^j}$ and $\mathbf{R}_{\pi^i}^{M^j}$ are the restriction to the mortar M^j of the fields $\mathbf{p}^i$ and $\mathbf{R}_{\pi^i}$, respectively.

By enforcing the average stress corresponding to the presence of the inelastic strain $\boldsymbol{\pi}^i$ equal to zero, since it is a self-equilibrated stress state, the macroscopic strain associated with the inelastic strain $\boldsymbol{\pi}^i$ in the UC is determined as $\mathbf{P}^i = \boldsymbol{\Pi}^i\boldsymbol{\pi}^i$, where $\boldsymbol{\Pi}^i$ is a linear operator. The average stress in the mortar joint M^j is $\overline{\boldsymbol{\sigma}}_{\pi^i}^{M^j} = \mathbf{C}^M\,\overline{\boldsymbol{\eta}}^{i,M^j}$, with $\overline{\boldsymbol{\eta}}^{i,M^j} = \left(\overline{\mathbf{R}}_{\pi^i}^{M^j} - \delta_{ij}\mathbf{I}\right)\boldsymbol{\pi}^i$.

When the UC is subjected to the overall elastic strain $\mathbf{E}_e$ and the inelastic strains $\boldsymbol{\pi}^i$, $i = 1, 2, \ldots, m$, the superposition of the effects can be performed. In such a way, it is possible to compute:

- the total overall strain $$\mathbf{E} = \mathbf{E}_e + \boldsymbol{\Pi}^1\boldsymbol{\pi}^1 + \cdots + \boldsymbol{\Pi}^m\boldsymbol{\pi}^m \tag{24}$$

- the overall stress $\mathbf{\Sigma} = \mathbf{\Sigma}_e$ (25)
- the total average strain in the m mortar joints and in the brick

$$\overline{\boldsymbol{\varepsilon}}^{M^j} = \overline{\mathbf{R}}_e^{M^j}\mathbf{E} + \sum_{i=1}^{m} \overline{\mathbf{L}}_{\pi^i}^{M^j} \boldsymbol{\pi}^i \qquad \overline{\mathbf{L}}_{\pi^i}^{M^j} = \overline{\mathbf{R}}_{\pi^i}^{M^j} - \overline{\mathbf{R}}_e^{M^j} \overline{\mathbf{R}}_{\pi^i}^{M^j} \tag{26}$$

- the average stress in the m mortar joints

$$\overline{\boldsymbol{\sigma}}^{M^j} = \mathbf{C}^M \left(\overline{\mathbf{e}}^{M^j} + \overline{\boldsymbol{\eta}}^{1,M^j} + \cdots + \overline{\boldsymbol{\eta}}^{m,M^j} \right) = \mathbf{C}^M \left(\overline{\boldsymbol{\varepsilon}}^{M^j} - \boldsymbol{\pi}^j \right) \tag{27}$$

Herein, it is assumed that the inelastic strain is constant in each mortar joint and the nonlinear behavior of the UC depends on the average stresses and strains evaluated in each of the m mortar joints.

4 NUMERICAL RESULTS

Hereafter, some numerical applications are reported. The number m of mortar joints considered in the analysis is set equal to 8. The geometrical parameters of bricks and mortar are assumed as follows: size of the brick $b = 240$ mm, $h = 120$ mm; thickness of the mortar joints $s = 10$ mm. Furthermore, the material mechanical parameters for the bricks are: $E = 18000$ MPa, $\nu = 0.15$; for the mortar: $E = 1000$ MPa, $\nu = 0.15$, $\varepsilon_{N,0} = 0.0005$, $\gamma_{NT,0} = 0.001$, $G_{cI} = 0.00125$ MPa, $G_{cII} = 0.00217$ MPa, $\mu = 0.5$.

The validation of the nonlinear numerical homogenization is performed comparing the results obtained by the proposed procedure with the ones determined by micromechanical Finite Element Analyses (FEA). In particular, four tests are performed, applying to the UC tensile, symmetric shear, unsymmetric shear strain and micro-curvature macroscopic loading histories. In particular, the last two applications aims to show the performance of the UC when it is subjected to Cosserat typical macroscopic strain components. As for the micromechanical analyses, a 2D displacement-based 4-node finite element is formulated based on the presented damage-friction law for the mortar. In order to avoid strain and damage localization in the mortar joints, a nonlocal integral model is adopted, based on the integral criterion with the characteristic length $\rho = 15$ mm.

Initially, the tensile test is performed on the UC subjected to the following loading history:

t	$E_1\left(10^{-4}\right)$	$E_2\left(10^{-4}\right)$	Γ_{12}	$\hat{\Theta}$	K_1	K_2
0	0	0	0	0	0	0
1	0	–4.0	0	0	0	0
2	30.0	–4.0	0	0	0	0
3	–5.0	–4.0	0	0	0	0

(28)

Due to the double symmetry of the geometrical scheme and to the loading conditions, micromechanical FEA is performed by considering only a quarter of the UC under suitable boundary conditions (Figure 1b). A regular mesh of 169 elements and 256 nodes is used for the computations. In the figures reported in the following the solid curve is referred to the proposed procedure results, while the diamond symbols concerns the FEA solution. In Figure 2 the overall response curve is plotted in terms of the macroscopic stress component Σ_1 versus the macroscopic strain E_1. It can be noted that a very satisfactory agreement is obtained. The mechanical behavior of the UC is characterized by an initial linear elastic response, followed by the damaging of the head joints (Figure 2 point A) and the subsequent damaging and frictional slip of the bed joints(Figure 2 point B). During the unloading and reverse loading paths, an elastic and perfect plastic response is observed, followed by the unilateral contact effect of the head joints. It can be noted that only in the path DE the TFA analysis is not in perfect agreement with the FEA solution.

Then, the symmetric shear test is performed considering the following loading history:

$$\begin{array}{ccccccc} t & E_1\left(10^{-4}\right) & E_2\left(10^{-4}\right) & \Gamma_{12} & \hat{\Theta} & K_1 & K_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -3.0 & 20.0 & 0 & 0 & 0 \\ 2 & 0 & -3.0 & -20.0 & 0 & 0 & 0 \\ 3 & 0 & -3.0 & 0 & 0 & 0 & 0 \end{array} \tag{29}$$

Due to the symmetry of the geometrical scheme, micromechanical FEA is performed by considering only half of the UC (Figure 1c). A regular mesh of 336 elements and 488 nodes is used for the FEA. In Figure 3 the macroscopic shear stress component Σ_{12} versus the macroscopic shear strain Γ_{12} is shown. Also in this case, a very good agreement is obtained.

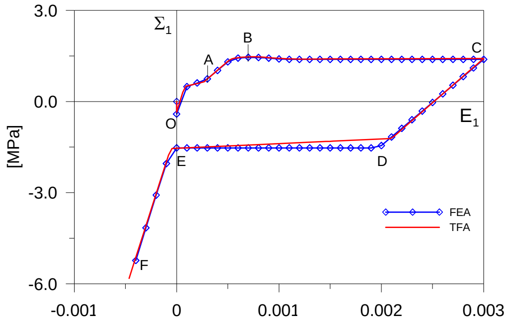

Figure 2: Tensile test: comparison of the results obtained by micromechanical FEA and by the proposed procedure.

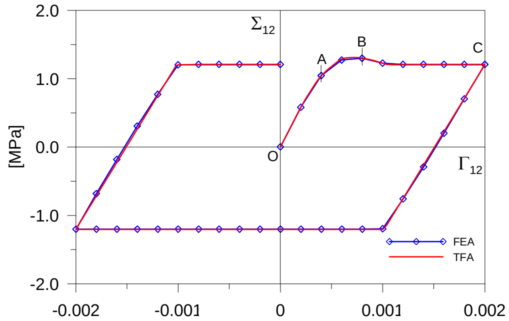

Figure 3: Shear test: comparison of the results obtained by micromechanical FEA and by the proposed procedure.

After an initial linear elastic response, the nonlinear behavior appears due to the activation of both the damaging and plasticity mechanisms in both head and bed joints (Figure 3 point A). After that, damage tends to localize into bed joints up to the complete deterioration, while plasticity spreads trough the bed joints, where a friction mechanism is activated (Figure 3 points B and C). During the subsequent unloading and reloading paths the UC response is characterized by the progression of the friction plasticity.

Afterthat, the response of the UC subjected to the unsymmetric shear component is analyzed by applying the loading history shown in Eq. (30a).

t	E_1	E_2	Γ_{12}	$\hat{\Theta}\left(10^{-4}\right)$	K_1	K_2
0	0	0	0	0	0	0
1	0	0	0	1.0	0	0
2	0	0	0	0	0	0

(a)

t	E_1	E_2	Γ_{12}	$\hat{\Theta}$	K_1	$K_2\left(10^{-6}\right)$
0	0	0	0	0	0	0
1	0	0	0	0	0	4.0
2	0	0	0	0	0	2.0
3	0	0	0	0	0	10.0
4	0	0	0	0	0	0

(b)

(30)

In this case micromechanical FEA is performed by discretizing all the UC by 650 elements and 930 nodes. In Figure 4 the macroscopic unsymmetric shear stress component $\hat{Z}$ versus the macroscopic unsymmetric shear strain $\hat{\Theta}$ is shown. Note that, in this case, the two curves departs a little. Such behavior is clarified by looking at the damage initiation and progression in the mortar joints, which appears a little different in the micromechanical analysis with respect to the TFA. Such differences in the damage evolution could be expected and mainly related to the simplified assumptions on which the TFA procedure is based, i.e. the assumption of uniform distribution of the nonlinearities in the mortar joints and the adopted regularization. After the initial linear elastic response, damage and plasticity are firstly activated in the upper right and bottom left bed joints (Figure 4 point A), then they involve the middle head joints (Figure 4 point B), where damage is very severe, finally spreading over all the bed joints (Figure 4 point C). The response of the UC during the unloading is linear elastic characterized by the damaged stiffness.

Finally, the nonlinear behavior of the UC experienced under the application of the micro-curvature Cosserat component K_2 is investigated by applying the loading history in Eq. (30b).

The same discretization as in the above example is adopted to perform the micromechanical FEA. In Figure 5 the Cosserat macroscopic micro-couple M_2 versus the micro-curvature K_2 is reported. It is worthwhile noting that, also in this case, the two curves obtained by the two procedures are in good but not in perfect agreement, due to the different damage evolution in the mortar joints. The initial elastic response is followed by the initiation of the damage in the two right bed joints (Figure 5 point A), which are located in the region of the UC experiencing a tensile deformation state. Then, damage spreads over the right middle head joint and grows maintaining such mechanism (Figure 5 point B). Indeed, plastic deformations are located mainly in the right middle head joint during all the loading process. The unloading branch is linear elastic.

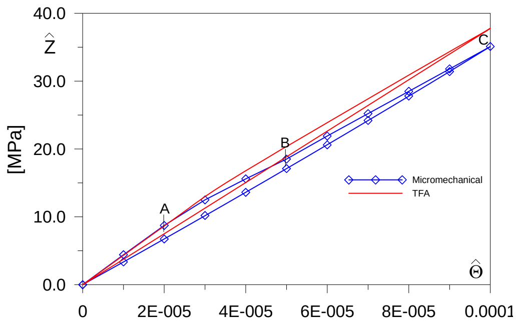

Figure 4: Unsymmetric shear test: comparison of the results obtained by micromechanical FEA and by the proposed procedure.

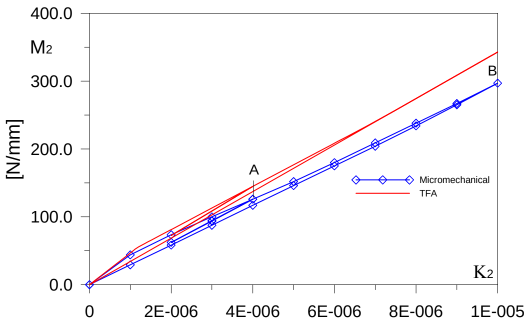

Figure 5: Micro-curvature test: comparison of the results obtained by micromechanical FEA and by the proposed procedure.

5 CONCLUSIONS

A nonlinear homogenization procedure able to describe the constitutive response of regular masonry material has been introduced. In particular, the 2D Cosserat continuum model has been adopted for the equivalent medium at the macro-level, while the standard 2D Cauchy continuum has been employed at the micro-level.

The higher order polynomial expansions used for the macro-micro kinematic map has allowed to analyze micromechanics deformation modes richer than in the classical first order homogenization framework, as also flexural and non symmetric shear modes have been included. Developing a TFA procedure, the overall elastic constitutive matrix and the localization tensors have been evaluated by linear FEAs of the UC, on the basis of which the nonlinear damage and plasticity evolutive problems at the typical point of the macroscopic equivalent medium have been solved by a step-by-step analysis. Furthermore, the analysis of the nonlinear micromechanical response of the UC has been carried out by means of a nonlinear FEA on the basis of the coupled damage-plastic model adopted for mortar joints. The comparison between the numerical results obtained by the proposed procedure and the ones evaluated by the nonlinear micromechanical FEA has shown a very satisfactory agreement, then validating the assumption of uniformly distributed inelastic strains, on which TFA procedure is founded.

Aim of further developments is to implement the proposed nonlinear homogenization technique in a finite element code for uncoupled multiscale analysis.

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