Mode-I parameter identification for an interface model describing damage in adhesive joints

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SUMMARY. The identification of mode-I parameters of a cohesive-zone model for the analysis of adhesive joints is presented. The data set for inverse analysis is provided either by kinematic full-field data, such as those obtained via Digital Image Correlation, or a suitable combination of it with by static global data. The results of numerical identification exercises are presented for a Double Cantilever Beam specimen subject to pure mode-I bending under displacement control.

1 INTRODUCTION

The development of new and advanced materials puts new challenges on processing technology. This is particularly true when different materials have to be joined to make composites in which one aims to retain the individual beneficial properties of the component products. In this perspective, the choice of the joining technique is often as important as the material itself. Adhesive bonding has several advantages compared to mechanical fastening, i.e. high corrosion and fatigue resistance and superior strength properties that often allow structures that are mechanically equivalent to, or even stronger than, conventional assemblies to be built at lower weight and cost. Moreover, adhesives can transmit stresses between structural members with a more uniform distribution with respect to bolts and rivets that, on the contrary, are often responsible of high stress concentrations that can lead to structures having a lower static and fatigue strength than an adhesively bonded system.

As for most structural components consisting of the assembly of individual elements, failure of adhesive joints due to damage growth at bonded interfaces is one of the most important failure modes and for its simulation cohesive interfaces can be usefully resorted to.

The cohesive-zone concept, originally introduced by Barenblatt and Dugdale to describe the near-tip fracture process, has gained major popularity in recent years for simulating delamination, debonding, fracture and fragmentation via finite element methods. This approach is mainly motivated by the consideration that prior to the development of macroscopic fractures there exists a zone in a state of progressive damage located in front of the crack, the so-called *cohesive process zone*, where an interaction across the crack sides is specified via a constitutive relationship relating surface tractions to displacement discontinuities.

In this work the mode-I parameters governing the damage mechanics-based formulation developed in [1] are estimated as the solution of a nonlinear programming problem. A least-squares norm is used as objective function that quantifies the distance between experimental data and the analogous quantities computed via finite elements as a function of the unknown parameters.

The data set of the identification process is provided either by the conventional reaction force data provided by a load cell, or by kinematic full-field data. These last ones can be obtained by a Digital Image Correlation (DIC) procedure, and concern the deformation process of the tested specimens

or of a suitable region of interest extracted from it [2]. In this preliminary validation exercise the data set is only pseudo-experimental, i.e. it has been generated by using a priori known material parameters corrupted via a random noise with zero mean value and varying standard deviation.

The computation of gradients of the response functions, typically displacements or reactions forces, with respect to the parameters to identify is referred to in the literature as sensitivity analysis and represents a key ingredient for identification purposes. In this study the sensitivities are computed using the Direct Differentation Method (DDM); this requires, at each step of the forward finite element analysis, the solution of an auxiliary linear problem whose right-hand side is a pseudo-load vector and the coefficient matrix is the mechanical tangent. Numerical identification exercises are presented concerning a Double Cantilever Beam (DCB) geometry subject to pure mode-I bending under displacement control. The numerical simulations show the effectiveness of the proposed approach and the stability of the identification procedure when increasing the noise-to-signal ratio.

2 COHESIVE MODEL

In the general case the formulation of a cohesive-zone-like model relies upon two main ingredients: the definition of a traction-separation law describing the relationship between the interface tractions $\mathbf{t}$ and the displacement discontinuities $[\![\boldsymbol{u}]\!] = \boldsymbol{u}^+ - \boldsymbol{u}^-$, and the introduction of a damage criterion to be met for the cohesive process zone to grow and the crack to advance. In particular, in this paper we shall make reference to the interface model proposed by Valoroso and Champaney [1] and consider the only one-dimensional (mode I) case, that is governed by the following equations:

$$
\begin{aligned}
t = \quad & \frac{\partial \psi}{\partial [\![u]\!]} = (1-D) k \langle [\![u]\!] \rangle_+ + k^- \langle [\![u]\!] \rangle_- \\
Y = & -\frac{\partial \psi}{\partial D} = \frac{1}{2} k \langle [\![u]\!] \rangle_+^2 \\
\boldsymbol{\phi} = & \, Y - Y^* \leq 0 \\
\dot{Y}^* = & \, \dot{D} \, \frac{\partial F}{\partial D} \\
\boldsymbol{\phi} \leq 0; \qquad & \dot{D} \geq 0; \qquad \dot{D} \, \boldsymbol{\phi} = 0
\end{aligned}
\tag{1}
$$

In the above equations t and Y respectively denote the interface traction and the damage-driving force, $D \in [0,1]$ is the scalar damage variable, $[\![u]\!]$ is the displacement jump in the direction normal to the interface while k and k^- are the undamaged interface stiffnesses in tension and compression, respectively. The impenetrability constraint is introduced in penalty form via the stiffness coefficient k^- and by explicitly distinguishing between the positive $\langle \cdot \rangle_+$ and negative part $\langle \cdot \rangle_-$ of the displacement jump.

The critical damage-driving force Y^* represents a non-decreasing energy threshold whose value is determined by a monotonically increasing positive function F given as:

$$
Y^* = \begin{cases}
G_o & \text{if } D = 0 \\
\displaystyle\int_0^t \dot{Y^*} \, \mathrm{d}t = F(D) & \text{if } D \in]0,1[\\
\displaystyle\max_{\tau \in [0,T]} Y(\tau) & \text{if } D = 1
\end{cases}
\tag{2}
$$

Typical forms of F are that of power laws or exponential functions, and their explicit expressions

are constructed in a way to ensure that the energy dissipated in the formation of a new unit traction-free surface equals the critical strain energy release rate G_c, namely:

$$\int_0^{+\infty} Y\dot{D}(t)\,\mathrm{d}t = G_c \tag{3}$$

In particular, the exponential traction-separation relationship originally contributed in [1], see also Figure 1, can be obtained using the following expression:

$$F(D) = G_o + \frac{1}{\Gamma(N+1)}(G_c - G_o)[-\log(1-D)]^N \tag{4}$$

where $N > 0$ (non integer) is a brittleness parameter and Γ is the complete Gamma function.

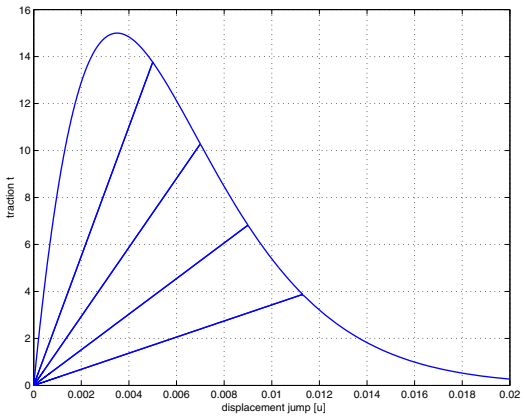

Figure 1: Traction-separation relationship for the exponential model (see also [1]). $k = 1.e4$ N/mm^3, $G_c = 0.125$ N/mm, $G_o = 0.$ N/mm; $N = 1.7$

We emphasize that the formulation at hand requires no time-discretization for the computation of the damage state, that can be evaluated in a completely explicit way. In particular, for damage loading ($\dot{D} > 0$) at each time τ the damage variable is computed as:

$$D(\tau) = \min\left(1,\ \max_{(\sigma \leq \tau)}\{F^{-1}(Y^*(\sigma))\}\right) \tag{5}$$

2.1 *Governing parameters*

The one-dimensional cohesive model possesses four material parameters, namely k, Gc, N, G_o. However, as already put forward in [1], certain choices of the material parameters may lead to traction-separation relationships possessing a substantially different shape with respect to the one reported in Figure 1. This occurs in particular when $N > 1$ and $G_o > 0$ simultaneously, since in this case the material tangent stiffness turns out to be infinite at the onset of damage. In order to rule

out this possibility, we here consider a reduced set of parameters for identification purposes:

$$\boldsymbol{x} = \begin{bmatrix} k \\ G_c \end{bmatrix} \tag{6}$$

while the exponent and the initial energy threshold will be fixed a priori ($N = 1$ and $G_o = 0$).

3 THE DCB TEST

The DCB is the standard test for obtaining the mode-I fracture toughness G_c of adhesives. Different procedures exist for performing the experiment and for data reduction; in particular, among those recommended by ASTM and British Standards are corrected beam theories and compliance calibration methods, which can be modified in various ways in order to compensate material and geometric uncertainties that are present in the schematization of the test [3, 4]. Both data reduction schemes share a common drawback, i.e. the fact that they make some strong assumptions on the test to compute the material parameters from experimental results, and the reliability of these assumptions has a direct impact on the determination of such parameters. For this reason, in the authors' opinion the use of a Finite Element model to compute the material parameters is quite desirable in order to limit the possible incoherencies with the numerical model to be used for computations.

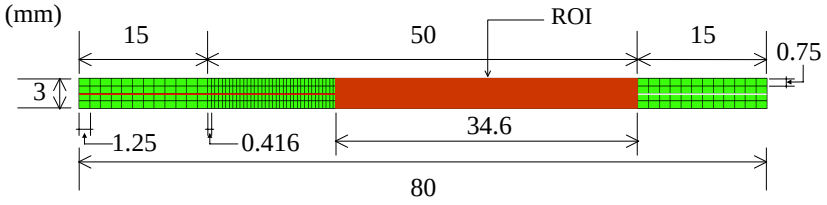

Figure 2: DCB test. Geometry and FE mesh.

The geometry of the test considered is shown in Figure 2, where the FE mesh used in numerical computations is also given. The specimen is made of two $1.5\ mm$ thick, $80.0\ mm$ wide and $1.0\ mm$ deep aluminum arms, bonded with a layer of resin adhesive and separed by an initial crack of length $a = 15\ mm$ that is used as the starting defect.

The response of the DCB during the debonding tests is simulated using the interface model discussed in Section 2. This is taken as the constitutive law for interface elements, which have been implemented as a part of a customized version of the FE code FEAP [5]. In the numerical simulations plane strain conditions are considered; the left-end of the structure (intact part) is free whilst on the right side two supports and increasing vertical displacements are prescribed at the end of each arm. The load-deflection curve corresponding to the target material parameters ($\overline{k} = 800$ N/mm^3, $\overline{G}_c = 0.1$ N/mm) is shown in Figure 3 in terms of reaction force P versus the relative displacement δ. For comparison purposes here is also shown the analytical solution obtained via Timoshenko beam theory with plane strain correction.

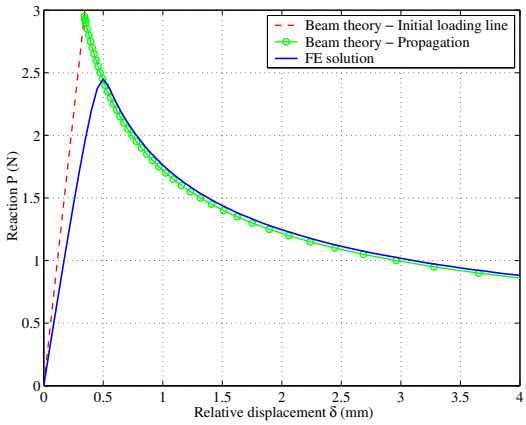

Figure 3: DCB test. Load-deflection curves. $k = 800\,\text{N/mm}^3$, $G_c = 0.1\,\text{N/mm}$.

3.1 *Sensitivity analysis*

The role of the sensitivity information can be crucial for the development of a robust identification strategy in presence of noisy residuals, which may generate spurious local minima and/or solution multiplicity. Moreover, sensitivities could allow one to select the most informative measurable quantities over space and time and to use for identification purposes only those sufficiently sensitive, i.e. exceeding a suitable threshold.

Finite element aspects of sensitivity analysis for nonlinear problems have been given an exhaustive presentation by [6], to which the interested reader may refer for a more detailed discussion; here we only summarize the very essential points of the approach with particular reference to the problem at hand. In a displacement-like formulation the residual form of FE equations reads:

$$\mathbf{R}\left(\mathbf{u}^{(k)}(\boldsymbol{x}), \boldsymbol{x}\right) = \mathbf{0} \tag{7}$$

where the dependence of the residual force vector $\mathbf{R}$ and of the nodal displacements $\mathbf{u}$ from the vector of material parameters $\boldsymbol{x}$ has been made explicit. Equation (7) holds at the converged equilibrium state corresponding to the $k-$th load step. Sensitivities are obtained by differentiating (7) with respect to the parameter vector, i.e.

$$\frac{\partial \mathbf{R}}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \boldsymbol{x}} + \frac{\partial \mathbf{R}}{\partial \boldsymbol{x}} = \mathbf{0} \tag{8}$$

The above equation results in a linear problem in the unknown response sensitivities whose coefficient matrix is the tangent stiffness, that is, the same governing matrix as in the last Newton-Raphson iteration of the equilibrium problem for the $k-$th load step. Accordingly, the solution of the problem (8) only requires the formation of a pseudo-load vector, i.e. the computation of some additional derivatives at the local level to be assembled via the standard FE assembly operator.

It is worth emphasizing that in the present context the evaluation of sensitivities for the vector of material parameters (6) results in a negligible increase of the computing time since use is made of the existing factorized tangent matrix to solve for the pseudo-loads.

4 PARAMETER IDENTIFICATION

Ultimate goal of the present study is the identification of the material parameters of the interface model presented in section 2 via inverse analysis using as data set kinematic full-field data possibly enriched by conventional static data such as reaction forces. In a deterministic framework, the optimal material parameters $\hat{\boldsymbol{x}}$ can be obtained as the solution of a nonlinear programming problem where a suitable cost function is minimized [7]. Such a function can be conveniently defined so to quantify, at all considered instants, the distance between *measured* quantities, i.e. displacements inside the monitored sub-domain and reaction forces, and those *computed* via the mathematical model as a function of the unknown parameters $\boldsymbol{x}$.

In the following a pseudo-experimental data set is considered that has been generated via the FE model by using a priori known material parameters and then corrupting the response by an additive noise with zero mean value and varying standard deviation. Accordingly, in the present work both *measured* and *computed* quantities have been derived from the Finite Element model of the entire DCB sample depicted in Figure 2.

The kinematic data set of the identification process (the *measured* kinematic quantities) is constituted by a time-space sampling of suitable displacements. In particular, it consists of a collection of n_u-dimensional vectors $\boldsymbol{u}_i^{meas}$ each referring to a particular instant t_i, $i = 1\,,\,...\,,\,n_t$, of the loading process. The values of the same displacements that are instead *computed* via the FE model will be denoted as $\boldsymbol{u}_i^{comp}$. In the same way, we shall denote by $\boldsymbol{P}_i^{meas}$ the n_P-dimensional vectors ($n_P = 1$ for the DCB problem) collecting the *measured* values of the reaction forces at instants t_i, $i = 1\,,\,...\,,\,n_t$, whilst the static quantities *computed* via the FE model will be denoted as $\boldsymbol{P}_i^{comp}$.

With this notation in hand the identification problem can be stated as follows:

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}\in\mathcal{S}} \omega(\boldsymbol{x}) \tag{9}$$

$\mathcal{S}$ being the constraint set and $\omega(\boldsymbol{x})$ the cost function:

$$\omega(\boldsymbol{x}) = \sum_{i=1}^{n_t} \left\{ \alpha \, ||\boldsymbol{u}_i^{meas} - \boldsymbol{u}_i^{comp}(\boldsymbol{x})||^2_{\mathbf{W}_u^{-1}} + \beta \, ||\boldsymbol{P}_i^{meas} - \boldsymbol{P}_i^{comp}(\boldsymbol{x})||^2_{\mathbf{W}_P^{-1}} \right\} \tag{10}$$

In the previous equation $\mathbf{W}_u$ and $\mathbf{W}_P$ denote weighting matrices that scale and make non-dimensional the static and kinematic residuals, respectively, while α and β are factors that allow the definition (10) of the cost function to encompass the different possibilities for the data set of the identification problem, namely: (i) $\alpha = 1, \beta = 0$, kinematic data only; (ii) $\alpha = 0, \beta = 1$, static data only; (iii) $\alpha \neq 0, \beta \neq 0$, linear combination of static and kinematic data. As for the situation (iii), we note in passing that the minimization of the objective function is governed by the ratio $\frac{\beta}{\alpha}$ that, consequently, has to be chosen in a way to ensure the simultaneous minimization of both the addends within the same tolerance.

In the numerical simulations documented in the following the pseudo-experimental data $\boldsymbol{u}^{meas}$ and $\boldsymbol{P}^{meas}$ have been numerically generated assuming the following target parameters:

$$\overline{\boldsymbol{x}} = \begin{bmatrix} \overline{k} \\ \overline{G}_c \end{bmatrix} = \begin{bmatrix} 800\,(N/mm^3) \\ 0.1\,(N/mm) \end{bmatrix} \tag{11}$$

The minimization of the objective function (10) is performed through a gradient-based Trust Region algorithm, available in the Matlab package [8], that communicates with the FE code via an *ad-hoc* developed interface. In order to assess the robustness of the inverse procedure, the pseudo-experimental vectors $\boldsymbol{u}_i^{meas}$ and $\boldsymbol{P}_i^{meas}$ have been subsequently generated by corrupting the response

with uncorrelated (over time and space) additive Gaussian noises independently for displacements and reaction forces. The added noise has zero mean values and time-independent standard deviations. The latter are denoted by the symbols σ_u and σ_P, as for the displacements and the reaction force, respectively. In presence of background noise it is important to assess the experimental information required to achieve a sufficient accuracy on the parameter estimates. For this reason in the following we examine the two possible sources of information and three basic possibilities of combining them for constructing the cost function to be minimized. In other words, we shall consider cost functions depending upon: (a) only displacements $\boldsymbol{u}_i$, ($\alpha = 1, \beta = 0$); (b) only the reaction force P ($\alpha = 0, \beta = 1$); (c) a combination of kinematic and static data ($\alpha = \beta \neq 0$).

4.1 *Identification via kinematic data*

In this section the results of identification exercises based on kinematic data only are analyzed. In view of a thorough validation of the inverse procedure, in the following the effects of quality and quantity of input data on the parameter estimates have been investigated by considering a three-inputs two-outputs parametric study. The three independent input variables are time sampling, space sampling and and the standard deviation of the additive Gaussian white noise σ_u,while the output variables are the relative errors of the two material parameters. The numerical results of the inverse analysis exercises are shown in the following.

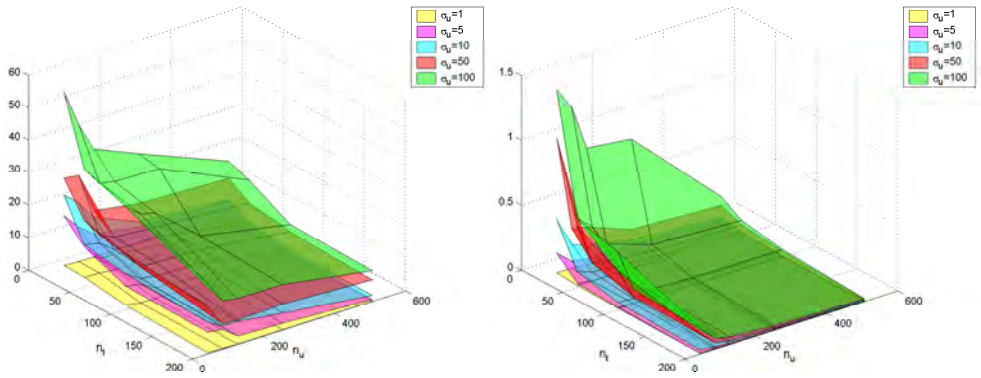

Figure 4: Identification via kinematic data. Relative errors on parameters k (left) and G_c (right), as function of space and time sampling. Surfaces refer to equal noise levels.

Figure 4 shows the relative errors on the parameter estimates, as function of the input nodal displacements n_u and measurement instants n_t. It is noted that the estimation of the fracture energy G_c is always robust and the relevant error is very low, almost independently from the quantity and quality of processed data. On the contrary, accuracy in the estimation of the initial stiffness k decreases as the noise standard deviation σ_u exceeds 10 μm. Moreover, the relative error on the initial stiffness grows rapidly if the time-space sampling is too poor ($n_u < 100$ and $n_t < 100$, respectively).

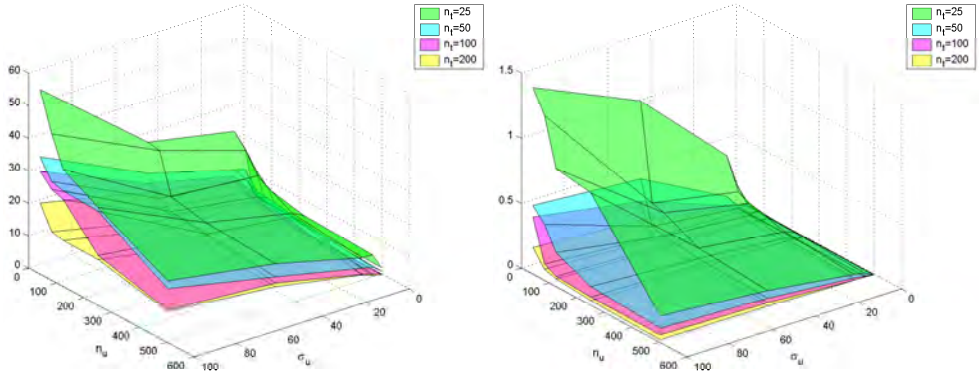

Figure 5: Identification via kinematic data. Relative errors on parameters k (left) and G_c (right), as function of space sampling and noise level. Surfaces refer to equal time sampling.

Figure 5 shows the relative errors as functions of the number of input displacements n_u and noise standard deviation σ_u. Each surface in the Figure corresponds to a given number of measurement instants n_t from 25 to 200. In this case it is noted that, even in the presence of a very poor time-space sampling, the relative error on the initial stiffness remains under $15\,\%$ if the noise standard deviation σ_u is lower than 10μm. In other terms, the high quality of data (i.e. their information content) can balance their shortage.

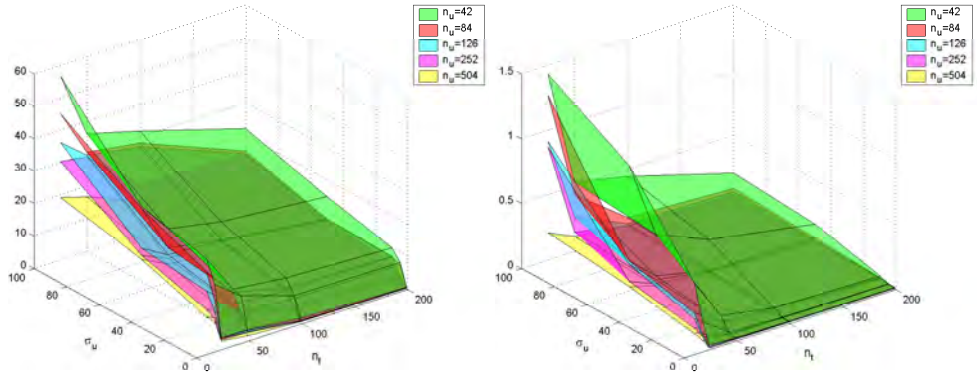

Figure 6: Identification via kinematic data. Relative errors on parameters k (left) and G_c (right), as function of time sampling and noise level. Surfaces refer to equal space sampling.

Figure 6 shows the relative errors E_k and E_{G_c} as functions of the measurement instants n_t and noise level σ_u. Each surface corresponds to a fixed number of input displacements n_u. In this case it can be noted that time sampling can be coarsened without altering the estimation accuracy if the spatial sampling is sufficiently rich ($n_u > 100$) and the noise level is sufficiently low ($\sigma_u \leq 10\,\mu$m).

4.2 *Identification using mixed kinematic and static data*

Identification exercises have also been performed including both kinematic and static data in the cost function to minimize, as indicated in (10). In this case the choice $\alpha = \beta = 1/2$ in equation (10) leads to kinematic and static residual norms of the same order of magnitude. The results of the identification are shown synoptically in Figure 7. Here are shown the relative errors on parameter estimates as a function of the noise standard deviations σ_u and σ_P. As in the previous cases, the critical fracture energy is always correctly estimated with a maximum relative error not exceeding $1\,\%$. In this case a significant improvement is observed in the estimated values of the interface stiffness k, which are much more accurate compared to those obtained based on use of kinematic data only (see Figure 4). In particular, even in presence of high noise ($\sigma_{reac} = 0.25\,N$ and $\sigma_u = 100\,\mu$m), the relative error remains well under $10\,\%$.

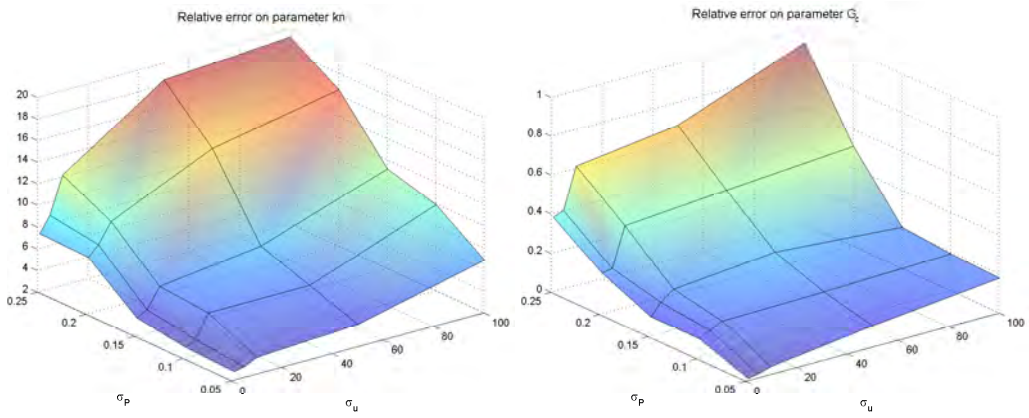

Figure 7: Identification based on the simultaneous use of static and kinematic data. Relative errors on parameters k (left) and G_c (right), as a function of noise standard deviations (σ_u) and (σ_P).

The results of the identification exercises can be summarized as follows.

As for the evaluation of the material parameters, the critical energy release rate is, in practice, correctly estimated regardless of the noise level and the time-space sampling of kinematic data. Unfortunately, in the examined cases the estimation of the interface stiffness provided by the inverse procedure is not so satisfactory as for G_c. In particular, a good evaluation of the parameter k based on purely kinematic data has been achieved only for very low noise levels, $\sigma_u \sim 10\mu\mathrm{m}$. However, this apparent lack of robustness in the identification of the stiffness parameter has not to be understood as a weakness of the inverse procedure, since the information contained in k in the examined case is much less characterizing the structural response than the fracture energy.

Improvements in the estimation of both material parameters can however be achieved by properly using the results of the sensitivity analysis. The sensitivity information can indeed be very helpful in the development of inverse analysis procedures since it can be taken as the basis of a selection criterion for the choice of the measurable quantities (displacements and reaction forces) with the highest information content to be included in the cost function.

5 SUMMARY AND CONCLUSIONS

A theoretical and numerical investigation has been presented aiming at the characterization of an interface damage model for the de-cohesion analysis of adhesive joints.

An inverse procedure has been used for the identification of the cohesive relationship under pure mode-I conditions starting from pseudo-experimental data generated via a Finite Element model and corrupted with additive white noise. Sensitivity analysis has also been implemented in view of the application of the developed methodology to the identification and calibration of the model parameters starting from true experimental data.

The proposed inverse analysis rests on kinematic full-field data at different instants of the load history such as those provided by DIC procedures. For this reason the noise effects that have been introduced to corrupt the pseudo-experimental kinematic data have the characteristics of an additive white noise, which is rather close to the experimental noise arising in DIC measurements.

The numerical results presented in the paper have demonstrated the effectiveness of the inverse procedure as well as its stability properties with respect to noise and time-space sampling.

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