A Proposal of Unified Elastic Moduli-Matrix Volume Fraction Law for Porous RVEs

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SUMMARY. On the base of some exact solutions available in literature with reference to overall mechanical properties of porous materials with prescribed microstructures, say dilute distributions of voids and interconnected lamellae, an analytical model is proposed, in order to predict stiffness of porous media with intermediate volume fractions. The model is finally validated by means of Finite Element-based numerical tests, performed over a wide range of void shapes and distributions inside the RVE.

1 INTRODUCTION

Heterogeneous media, such as advanced man-made materials (composites, multilayers or graded materials) or systems with cracks and voids, have a great importance in many engineering applications. The overall mechanical behaviour of such materials is hard to predict because it depends on properties, volume fraction, shape and spatial distribution of each constituent. The methodology commonly used for determining average response of heterogeneous objects is usually based on the *Theory of Homogenization*. In this framework, it is assumed that the effective material properties of the media - at the macro-scale level - are homogeneous and they can be obtained by statistical averaging of properties of the representative volume element (RVE) at the micro-scale level.

To the authors knowledge, with specific reference to heterogeneous media characterized by an elastic solid matrix weakened by voids, no many investigations have been performed on the sensitivity of the RVE's overall elastic moduli with porosity, by exploring the whole range of the volume fraction and making variable sizes and shapes of voids.

Despite the problem is crucial in several fields – also limiting the interest to porous materials whose behaviour is expected to be isotropic – Literature seems to offer only constitutive laws for very particular cases (Rho et al., [1]). With the aim of overcoming the above described limitations, an exploration of the elastic response of porous RVEs characterized by a wide range of void geometries is presented, under the hypothesis of cubic symmetry of the void arrangements. The corresponding matrix percentage ranges from high porosity up to high matrix volume fraction, spanning the entire admissible interval of void sizes.

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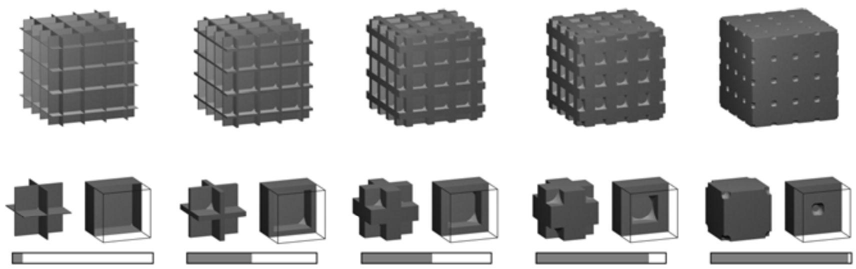
2 THE MODEL

In the present work FE method is firstly adopted with the aim of establishing a general relationship between averaged elastic moduli and matrix volume fraction for a copious amount of porous RVE materials. The geometrical evolution of the RVE porosity is obtained by means of three-dimensional periodic arrangement of voids, embedded in a linearly elastic isotropic solid matrix and spatially distributed so that cubic symmetry is reached, so exploring a significant number of cavity shapes (see Figure 1).



Then, by collecting all the numerical results, sensitivity analyses are performed in order to detect possible influences of cavity shapes on the overall elastic moduli (for both the cases of displacement and stress prescribed), as well as graphics *stiffness* vs *volume fraction* are created for all the study cases.

3 NUMERICAL RESULTS

A third polynomial regression $\chi_E(\gamma)$ is hence used to fit data for Young, shear and bulk moduli in the form:

$$\chi_E(\gamma) = A_E + B_E \cdot \gamma + C_E \cdot \gamma^2 + D_E \cdot \gamma^3 \qquad \gamma \in [0,1] \qquad (1)$$

where A_E , B_E , C_E , D_E represent algebraic coefficient to be determined and γ is the volume fraction. The settings of these coefficients is finally obtained by recalling literature closed-form solutions in terms of overall elastic moduli for the limit cases of high porosity (Flugge, [2]), and small porosity (Christensen, [3]), exploiting both the conditions on the elastic values and their derivative respect to the volume fraction. By following this way, the four coefficients can be obtained as explicit functions of the matrix Poisson’ratio and a very good agreement of the analytical law with the numerical tests is highlighted by all the FE data.

4 CONCLUSIONS

In the paper it has been investigated the influence of cavity shapes and volume fraction on the overall stiffness of RVEs depleted by voids. The strategy on which the final model is based exploits some analytical results related to the evaluation of elastic moduli in porous materials with high and low volume fractions. Therefore, by starting from these results, constraints on the values of third-order polynomial functions and their derivatives have been imposed, these functions representing the normalized elastic moduli of the porous RVE. In order to show the effectiveness of the formulas, a significant number of FE simulations have been conducted, highlighting the very good agreement of the numerical results with the analytical predictions.

References

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