# Effects of polypropylene fibres in concrete under fire conditions

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SUMMARY. Polypropylene (PP) fibres contribute to the reduction of pore pressures in concrete during heating, and hence the reduction of the probability of explosive spalling. Pressure reduction depends upon (a) the molecular structure of PP fibres during heating in relation to the microstructure of the concrete; (b) the mechanical properties of the fibres as functions of temperature; and (c) the type, dosage and dimensional characteristics of the fibres. This work forms part of an international research programme called NewCon [1] carried out under the Eureka umbrella involving six organisations in four European countries. The data produced is input into a thermo-hydro-mechanical numerical model for the prediction of concrete spalling with and without the presence of PP fibres.

### 1. INTRODUCTION

While Polypropylene (PP) fibres have been employed in concrete to combat explosive spalling in fire for over a decade, the mechanisms underlying this benefit are not well understood. This is because the studies to date have surprisingly not investigated the behaviour of the PP fibre itself and the influence of heating on its molecular structure and its properties in relation to those of heated concrete. Such an investigation should, in fact, form a starting point of any comprehensive investigation. Large-scale spalling tests have been essentially of an industrial nature involving relatively few specimens, and even some of the results have been kept commercially confidential and are thus not in the public domain. In addition, there is a lack of a predictive tool to enable engineers to predict the dosage and type of fibres to use in order to reduce or eliminate explosive spalling that has caused major problems in recent tunnel fires with significant loss of section.

## 2. POLYPROPYLENE CHARACTERISTICS

One of the first features to note is that water and polypropylene are grossly dissimilar in polarity. This polarity mismatch also applies between polypropylene and concrete during mixing, hardening and in the hardened state. The PP fibres are coated with a dispersant agent to allow good dispersion during casting. After casting, the dispersant agent and fibres would have been dispersed within the concrete leaving uncoated hydrophobic fibres in contact with the hardening concrete with poor wetting at the interface between the fibre and concrete that could contain water in free, physically bound and interlayer forms as well as chemically bound water - all of which contribute to the build-up of vapour pressures during fire. The poor interfacial adhesion with concrete can allow for capillary transport of water between the fibre and the concrete because the weakly

bonded region is always vulnerable to disruption by water/steam) pressure. On this basis, water vapour does not need to pass through the polymer to escape this composite but could easily travel around the fibre thus providing a mechanism for vapour transport under pressure even before the fibre has melted. Hitherto, it has normally been assumed that water vapour is transported through the "vacated" channels after melting of the fibres.



Figure 1. Representation of a water molecule and a polypropylene repeating unit (C3H6) in the space filling format [2]. Units in Angstroms = 0.1 nm

The second feature to note is the molecular structure. Polypropylene is a high molecular weight hydrocarbon of up to 2,400 monomer units and with a repeating unit of C3H6 (repeat distance 6.5 Angstroms - Figure 1) and comprising, at ambient temperatures, about 55% crystalline and 45% amorphous phases with an overall density of 0.91. The likely end-to-end distance for a single i-PP molecule of 100 MW is 520 nm if the molecule is 100% crystalline (polarity relevant) and 14 nm, as r.m.s. length, if the molecule is 100% amorphous (polarity relevant). Given the high viscosity of polypropylene in the melt phase, its relatively large molecular size (Figure 2), entanglement with sister molecules, the lack of affinity with concrete and the small size of the concrete pores even at temperatures of 200-500°C (peak at about 10-100nm) a considerable effort would be required to force the molecules through the concretes pores even for pores of the size 1-10 µm. In the vapour phase, the effective diameter of the volatile products is about 0.68 to 1.08 nm (polarity irrelevant) which is small and volatile enough to penetrate the concrete pore structure. Therefore, contrary to common belief, the melted fibre would not easily transport within the concrete structure even under pressure. It is only when the melted fibres vaporise to low molecular size hydrocarbons of irrelevant polarities that flow of the hydrocarbon becomes easy through the concrete pore structure [2].

Also, contrary to common belief there are no single transitional temperatures during heating. Polypropylene undergoes non-stepwise transitional phase changes as it is heated starting with the glass transition softening phase (-10°C to  $+10^{\circ}$ C), followed by the endothermic melting phase that starts at 150°C, peaks at 165°C and completes at 176°C. The process of melting polypropylene involves the destruction of the crystallinity to create a wholly amorphous polymer. This is followed by an endothermic pyrolysis involving the break-up of the carbon backbone into smaller molecules. Detectable pyrolysis starts at about 325°C with the release of hydrocarbon vapour species. Pyrolysis peaks at about 460°C and completes at 475°C. Volatile products would combust at 550°C in the presence of oxygen but when exposed to a direct flame they would combust at about 350-400°C. During fire, combustion is more likely to take place for hydrocarbon vapours released from the surface of the concrete [2]. A third feature to note is the dramatic influence of heating upon the mechanical properties. At temperatures below the glass transition temperature the

modulus of elasticity of polypropylene lies in the GPa region (Figure 3).



Figure 2. Representation of (a) Single turn of i-PP helix and (b) packing of i-PP helices showing van der Waal's radius.



Figure 3. Dynamic modulus versus temperature (°C) for "solid" polypropylene [2] developed from test by Sauer and Pae [4].

At ambient temperatures it is about 1-4 GPa. It falls sharply with increase in temperature such that at 80°C it reaches the upper service temperature and then collapses at about 150°C. The modulus of concrete at room temperature is about 40-50 GPa for concrete compressive strength in the region 60-100 MPa. Concrete heated under compressive load (as in practice) suffers little decline in the modulus for temperatures up to 165°C when that of polypropylene has collapsed. Therefore, the modulus of concrete is greater than that of polypropylene at ambient temperature by about 5-30 times, and this gap increases significantly with increase in temperature. By 100°C concrete would have a modulus of elasticity some 100+ times that of polypropylene and this increases further to about a 1,000+ times at 150°C. Above 165°C, the modulus of polypropylene collapses while that of concrete may be not much altered from its initial unheated value [2]. The situation with tensile strength is the reverse. Given that individual intermolecular forces in polypropylene crystalline phase are based on weak van der Waal's forces it is noteworthy that tensile strength is in excess of 30 MPa and is even 560 MPa for some commercially produced fibres. In contrast, tensile strengths in direct tension are as low as 3-4 MPa for concrete of compressive strength 60-80 MPa. The tensile strength of polypropylene fibres at ambient temperatures is, therefore, some 10-100 times greater than the tensile strength of concrete [2]. The coefficient of linear thermal expansion of polypropylene is anisotropic and ranges from 100  $\mu\epsilon/^{\circ}C$ at 20-60°C, to 210  $\mu\epsilon/^{\circ}$ C at 100-140°C. The coefficient of thermal expansion of concrete is in the range up to about 13  $\mu\epsilon/^{\circ}C$  depending on the type and content of aggregate used. The coefficient of thermal expansion of polypropylene at ambient temperatures is therefore some 10 times greater than that of concrete [2]. Description of the different mechanisms that contribute to the reduction of pore pressures are beyond the scope of this paper and are presented in detail in reference [3] including the new concept of the "pressure induced tangential space" (PITS) mechanism. In brief, PITS is tangential space that is created between the poorly wetted fibre and the surrounding concrete induced by steam pressures. This allows steam to travel along this space even before the fibre is melted. Toxicity analysis indicates negligible addition to the toxic products of the combustion of carbon monoxide, optically obscure smoke and mixed organic irritants which are well below 1% of the total toxicity from the fire itself. This is fully described in reference [5] including toxicity analysis for a tunnel example. The foregoing brief outline illustrates that significant differences exist between polypropylene and concrete materials when subjected to heating. Such an insight is necessary as the starting point for the better understanding of the interaction between them and the role that polypropylene fibres play in reducing pore pressures, and hence explosive spalling, during fire.

#### 3. PRESSURE RELIEF MECHANISMS

Polypropylene fibres play a role in reducing pore pressures via a number of possible mechanisms [3] which include reservoirs (e.g. air bubbles and micro-cracks) to accommodate expanding steam as well as continuous channels for moisture-vapour migration (e.g. via pressure induced tangential space PITS or through spaces vacated by vaporisation of the polypropylene fibres). In the melt stage polypropylene would not easily transport through the concrete pore structure (owing to its high viscosity and large molecular size [2]) but in the vapour stage it would be. The effectiveness of the pressure release mechanisms depends, not only on the characteristics of polypropylene material, but also on the characteristics of the fibre itself in terms of its dimensions. Aspect ratios by themselves are meaningless and the important parameters are the actual fibre diameter and length. Key parameters that could influence pore pressure reduction are the number of fibres, the cumulative length and cumulative surface area of the fibres as well as fibre interconnectivity - all of which increase with reduction in fibre diameter for a given individual fibre length. An optimum individual fibre length would exist that allows both interconnectivity and good dispersion. Criteria for thermo-hydro-numerical modelling would depend upon the physical, chemical, thermal and mechanical properties and their interrelation with the surrounding matrix.

Smeared properties can be assumed, although meso-level modelling at the micro-metre scale simulating individual fibres and other concrete constituents would be useful especially in modelling PITS. The assumptions made for modelling would depend upon the transformations taking place as temperature increases during a fire. These are divided into five temperature ranges of 20-100°C, 100-165°C, 165-475°C, 475-550°C and above 550°C [3].

#### 4. NUMERICAL MODELLING

#### 4.1 Modelling PP Fibres

The data from this comprehensive micro-structural and materials investigation provided by the

last author are input into the NEWCON3D code. The purpose is to provide a predictive tool. The following couple of equations have been used [6]

$$k = k_0 \cdot 10^{f(T)} \cdot \left(\frac{p_g}{p_0}\right)^{A_p} \cdot 10^{A_p D} \cdot \beta_{pp}(m_{pp}, T)$$
(1)

$$n = n_0 + A_n (T - T_0) + n_{pp} (m_{pp}, T)$$
<sup>(2)</sup>

where two new parameters have been included to the standard NEWCON3D (i.e without PP fibres) model for permeability and porosity. In the following, the modelling aspects related to the porosity contribution due to PP fibres only are presented. The permeability changes due to the PP fibres follows similar developments, but they are not included because of space limits.

#### 4.1.1 Porosity Change Due to PP fibres

As far as the porosity change due to the PP fibres is concerned, it has been evaluated that contribution given by the PP can be considered an additive parameter to the usual porosity relationship, mainly dependent on the PP mass and by the temperature. Hence the phenomenon has been modelled examining what happens between key temperatures. One of these temperatures is the PP vaporisation temperature: indeed the porosity increment at this point can be easily calculated, since the free volume given by the PP can be expressed by means of the relationship

$$n_{pp} = \frac{m}{\rho V} \quad for \quad T \ge 475^{\circ}C \tag{3}$$

where  $n_{PP}$  represents the additive contribution given by the PP in the global porosity relationship, m is the PP mass in the concrete,  $\rho$  its density and V the considered volume of concrete. This relationship is valid for all temperatures higher than 475°C, temperature beyond which the PP is considered fully vaporised. But the vaporisation process does not happen at a constant and fixed temperature, it appears within a temperature range starting from 325°C, temperature at which the PP is melted and volatile hydrocarbon molecules are present, that means the vaporisation process has begun. Another important temperature can be considered the PP melting temperature, that is a phase change starting at 165°C and ending with the whole fibre melting, that appears at about 200°C. In this phase, the volume occupied by the PP is the same, because the PP maintains a noticeable viscosity even if in the fluid state, hence without an apparent increasing of the porosity. However it is possible to consider a contribution to the porosity increment given by the elastic recovery of the material: indeed during the preparation of the concrete and PP mix and in the successive hardening phase, the PP can be subjected to tensile stresses. Such stresses in the fusion stage of the fibre slightly disappears, leading the fibre to recover the initial length. For this reason a contribution to the porosity directly proportional to the length of the single fibres can be considered valid, i.e.

$$n_{pp}^{I} = \beta l \quad for \quad 165^{\circ}C \le T \le 200^{\circ}C \tag{4}$$

where  $n_{PP}^{I}$  is the contribution to the porosity given by the fibre melting and I is the length of the single PP fibre, while  $\beta$  is a proportionality coefficient to be characterised experimentally. In the intermediate stages where a phase change does not occur, the porosity increment can be considered

practically zero: the only contribution to the porosity can be justified by the fact that the PP is an hydrophobic material, hence it is not able to be strictly connected to the concrete, maintaining a very small distance from it able to be crossed through by the vapour in pressure. Thanks to this phenomenon it is possible to consider a further increment to the porosity, even if small, given by the relationship

$$n_{pp}^{II} = \alpha \frac{m}{\phi l} \quad for \quad 100^\circ \le T \le 165^\circ \quad and \quad 200^\circ \le T \le 325^\circ \tag{5}$$

where  $n_{PP}^{II}$  is the contribution to the porosity given by the PITS (Pressure Induced Tangential Space) phenomenon, m is the PP mass, here considered directly proportional to the porosity increase, while  $\phi$  and 1 are the chosen diameter and PP length, respectively, since it has been considered that such phenomenon is prevented as much as the fibre surface is large. Hence it has been determined the relationship between  $n_{PP}$  parameter and the temperature, that is composed as follows: two linear strictly increasing functions, with similar slopes since they are determined by the same phenomenon, for temperatures comprised between 100°C and 165°C end between 200 and 325°C; a constant function for temperatures higher than 475°C while for intermediate temperatures, where the increasing porosity peaks are given by the state changes; in absence of further information it was decided to describe them by means of simple 3rd degree polynomial expansions (Hermite's cubic functions) making use of the knowledge of the functions at the interval extremes and of their derivatives at the same points. By interpolation the following conclusions can be found:

If temperatures are comprised between 100 and 165°C the relationship found is

$$n_{pp} = \alpha \frac{m}{\phi l} \left( T - 100 \right) \quad for \quad 100^\circ \le T \le 165^\circ \tag{6}$$

This leads the parameter  $n_{PP}$  to assume at the temperature 165°C a value equal to

$$n_{pp} = 65\alpha \frac{m}{\phi l} \quad for \quad T = 165^{\circ} \tag{7}$$

that will be used in the next interpolation. Indeed, posing together with the above condition, the next one, that is at 200°C the parameter assumes the following value

$$n_{pp} = 65\alpha \frac{m}{\phi l} + \beta l \quad for \quad T = 200^{\circ}$$
(8)

and imposing that at both the extremes (165°C and 200°C) the tangent slope is equal to

$$\frac{\partial n_{pp}}{\partial T} = \alpha \frac{m}{\phi l} \quad for \quad T = 165^{\circ} \quad and \quad T = 200^{\circ} \tag{9}$$

It is possible to establish a system of 4 equations with four unknown variables, leading to

$$n_{pp} = \left(1,633 \times 10^{-3} \alpha \frac{m}{\phi l} - 4,665 \times 10^{-5} \beta l\right) T^{3} - \left(8,939 \times 10^{-1} \alpha \frac{m}{\phi l} - 2,554 \times 10^{-2} \beta l\right) T^{2} + \left(1,626 \times 10^{2} \alpha \frac{m}{\phi l} - 4,618 \beta l\right) T - \left(9,768 \times 10^{3} \alpha \frac{m}{\phi l} - 2,762 \times 10^{2} \beta l\right) for \quad 165^{\circ} \le T \le 200^{\circ}$$

$$(10)$$

The second straight piece, for temperatures comprised between 200 and 325°C, having the same slope of the previous one, can be easily found using the following relationship

$$n_{pp} = \alpha \frac{m}{\phi l} \left( T - 200 \right) + 65\alpha \frac{m}{\phi l} + \beta l \quad for \quad 200^\circ \le T \le 325^\circ$$
(11)

As in the case of the first interconnecting curve, also in this case, taking advantage of the fact that the previous relation for a temperature equal to 325° assumes the value

$$n_{pp} = 190\alpha \frac{m}{\phi l} + \beta l \quad for \quad T = 325^{\circ}$$
(12)

with slope of the tangent line equal to

$$\frac{\partial n_{pp}}{\partial T} = \alpha \frac{m}{\phi l} \quad for \quad T = 325^{\circ}$$
(13)

while at the other extreme (475°C) it assumed the known value of

$$n_{pp} = \frac{m}{\rho V} \quad for \quad T \ge 475^{\circ} \tag{14}$$

with horizontal slope. It is possible to set up a system of four equations with four unknowns, from which the following final relation can be found

$$n_{\rho\rho} = \left(1,570 \times 10^{-4} \alpha \frac{m}{\phi l} + 5,926 \times 10^{-7} \beta l - 5,926 \times 10^{-7} \frac{m}{\rho V}\right) T^{3} - \left(1,918 \times 10^{-1} \alpha \frac{m}{\phi l} + 7,111 \times 10^{-4} \beta l - 7,111 \times 10^{-4} \frac{m}{\rho V}\right) T^{2} + \left(7,589 \times 10^{1} \alpha \frac{m}{\phi l} + 2,744 \times 10^{-1} \beta l + 2,744 \times 10^{-1} \beta l + 2,744 \times 10^{-1} \frac{m}{\rho V}\right) T - \left(9,610 \times 10^{3} \alpha \frac{m}{\phi l} + 3,343 \times 10^{1} \beta l - 3,443 \times 10^{1} \frac{m}{\rho V}\right) for \quad 325^{\circ} \le T \le 475^{\circ}$$

$$(15)$$

Finally we know that for temperatures higher than 475°C the porosity remain unchanged.

In the following, a diagram is shown demonstrating a possible picture of the porosity behaviour as a function of the temperature, where  $\alpha$  and  $\beta$  parameters have been assigned equal to  $10^{-8}$  mm<sup>2</sup>/kg and  $10^{-5}$  mm<sup>-1</sup>. Such values have been given paying attention to the reference system used, because they are not simple non-dimensional coefficients, and moreover not all the points of the

functions are assigned in parametric form. For example, it can be observed a well defined maximum value, i.e. the  $n_{PP}$  value for temperature higher than 475°C; hence the above parameters are given in a way that the porosity increment is equal for about 90% by the fibre vaporisation, being this one the main involved mechanism. As can be noticed from Figure 4, a variation of the mass gives rise to a quite similar behaviour up to the temperature of 325°C, at which the vaporisation process starts and the  $n_{PP}$  parameters are different because the final porosity increasing given by the vaporisation can be considered proportional to the PP mass.

It is interesting to note that proportionally higher porosity peaks are attained due to a mass variation, and an increasing of the slope can be observed where the PP fibre vaporises. This indicates that the porosity increase in the concrete mix is more rapid as far as the hydrocarbon quantity at high molecular weight is incremented. From this point of view, the addiction of the higher quantity of fibres appears reasonable, with the aim of reducing the explosive spalling risk inside the concrete. The dependence of  $n_{PP}$  parameter with the temperature, at different values of the fibre length, is shown in Figure 5: it can be clearly seen that these values have some impact only in the domain comprised between 165°C and 475°C. In this interval, the two graphs are slightly different and outside they are practically coincident. Hence, in the above interval there is a small porosity increment that is not further cumulated with another increment after the fibre vaporisation. The porosity increment of the fibre in the above interval, going from the melting point of the PP fibre (165°C) to the complete vaporisation of the same (475°C), is probably due to the fact that an higher length of the single fibre is important for that mechanism, called in the previous sections "vacated channels" (and at a lower extend also for the other mechanisms giving rise to a dissipation of the overpressures), since it plays a fundamental role for the interconnectivity of the same fibres. There is also another key parameter in the concrete porosity dependence from temperature, if PP fibres are added, that is the fibre diameter. From this point of view it is interesting to comment the relationship shown in Figure 6, as reported below.



Figure 4. Behaviour of the  $n_{PP}$  parameter for different values of m

From the three curves shown in Figure 6, it is clear how at the increase of the diameter the porosity decreases in the temperature interval from the transition of the PP to the liquid state (165°C) to the beginning of its vaporisation (325°C), while outside that domain, at higher temperatures, practically uniform levels are shown for each curve near the complete vaporisation temperature of the PP. This observation about the fibre diameter, should be considered in addition

to the above remarks about the ratio between the diameter and number of employed fibres, as well as the ratio between the diameter and the cumulative length of the fibres or cumulative area of the fibres. In particular, in that context it was observed that an increment of the diameter leaded to a decrease of the three above parameters, obtaining a worsening in the PP efficiency, in terms of escaping of the vapour at the high overpressures developing inside the concrete subjected to high temperatures, especially for high strength concrete, responsible for the "pore pressure spalling" phenomenon. It can be concluded that the fibre to be used should have a diameter as small as possible, according to the limits imposed by the molecular arrangement of the PP. The mathematical / numerical approach offers a tool able to quantify this effect.



Figure 5. Relationship between n<sub>PP</sub> parameter and temperature for different values of the fibre length "L".



Figure 6. Relationship between  $n_{PP}$  parameter and temperature, for different values of  $\phi$  diameter.

All the effects described in Section 4, plus contribution to the porosity by the elastic recovery and the PITs effect, along with the similar ones for permeability changes (not reported here for sake of brevity), are introduced in NEWCON code for simulation purposes [7-12]. The found relationships are intended to be appropriately scaled against experimental results, to generate correct pressure relief do to real PP fibres content per m<sup>3</sup> of concrete.

#### 5. CONCLUSIONS

- Molecular characteristic of polypropylene need to be understood in relation to water and concrete in addition to their thermal, chemical and physical properties.
- Water and polypropylene are grossly dissimilar in polarity causing poor interfacial adhesion between the pp fibre and which allow for capillary transport of water between the fibre and the concrete.
- The high molecular weight and dimensions combined with the high viscosity of the melted fibre do not allow for easy transport of the melted fibre through the concrete pores.
- Cracks could form because of the differential thermal expansion of the two materials at the lower temperature range and also during fibre melting.
- Pressure relief is related to the dimensional characteristics of the fibres (not aspect ratio) which determine the total number of fibres, interconnectivity of the fibres and the total cumulative surface area.
- Connection between experimental data and established mechanical theories can give reasonable answers for predicting the behaviour of concrete under heavy loading and thermal conditions. Appropriate theory and modelling of PP fibres effects in high temperature concrete are developed, which are especially important if high performance or high strength concrete is adopted.

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