# Multiscale Fracture Behaviour of Ice

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SUMMARY. The paper discusses the evidences of fractality in fracture of ice at different scale. Starting from the grain scale at which the physical properties of ice material are described by the fractal geometry, afterwards the paper analyses the formation of crevasses on the glaciers of the Italian Alps. At the geological scale, the application of the box-counting method permits to analyse the distribution of crevasses network within the glacier continuum. This confirms the self-similar fractal properties of ice fracture networks. Finally, the ice behavior at sub-continental scale is detected analyzing the collision between the glacial ice tongue (the Drygalski Ice Tongue) and a massive iceberg (the B-15A). Studying the evolution of the phenomenon, the comparison between the data obtained at different scales - very large scale of Antarctic icebergs; medium scale of Alpine glaciers [1] and laboratory and slab scale [2] – is made and the scaling of the fracture properties of ice is discussed [3, 4].

## 1 INTRODUCTION

The research on ice mechanics is motivated by the wide area of the Earth covered by the Cryosphere and its fundamental role on the climate, the water resources and, consequently, the human health. Usually, ice mechanics is studied in context of global warming. In fact, the research on the mechanical and physical properties on ice is very difficult because the experiments are impossible at the geophysical scales [5] and also due to the extreme conditions *in-situ*. Thus, to understand the mechanics of ice objects at different scales, the researchers investigate on the behavior at laboratory scale of ice. In this paper, the scaling of fracture properties is discussed by considering the fractal evidences on ice at different scales.

The first evidence is the constitution of ice material. The ice is a form of solid water characterized by a density range between 800 and 917 kg/m$^3$. Generated from metamorphosis of snow or direct freezing of water, the ice can be modeled as a porous medium consisting of air and a random assemblage of $H_2O$ grains. Like for the case of snow, the behavior of ice proprieties can be studied at different scales with a lacunar fractal model characterized by scale invariant fractal porosity [6].

Increasing the scale, the fracture properties of ice are evaluated at the regional scale studying the distribution of fracture (crevasses) on one Alpine Glacier of Aosta Valley - Italy. The crevasses have usually been studied as simple cracks measuring their regular spacing, arguing that a power law describes the scaling of their lengths and after, analysing a population of crevasses on Argentière glacier – French Alps [1]. In this paper, the spatial distribution of fractures within the glacier continuum is investigated calculating the fractal dimension of the crevasses network in order to detect the prevailing stress rupture mechanism of ice. The ice behavior at large scale is detected thanks to a natural catastrophe: the impact occurred between the Drygalski Ice Tongue and the iceberg B-15A in the Antarctic Sea.

2 THE EVIDENCE OF FRACTALITY AT GEOLOGICAL SCALE

To investigate the fractality of ice at the geological scale, the spatial distribution of damage on glaciers is studied calculating the fractal dimension of the crevasses network on one of glaciers of the Italian Alps, by the box-counting method.

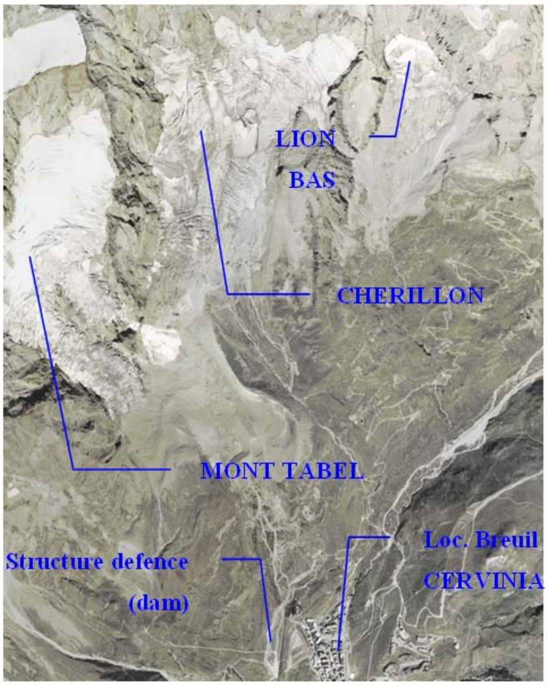



Figure 1. Localization of the Cherillon glacier - Valtournenche (AO) – Italy
(Photo by Cabina di Regia Ghiacciai Valdostani - Fondazione Montagna Sicura).

Continuously monitored to study the effects on global warming [7] and the ice falls and dynamics [8], the Cherillon glacier was chosen. It is localized in Valtournenche, in the South-East sector of the Aosta Valley, Italy. It is placed between Mont Tabel and Lion Bas glacier, near the Mount Cervino (Fig. 1), not far from to the Breuil – Cervinia (AO) city centre. Its principal characteristics are detailed in http://www.nimbus.it/glaciorisk/ .

To show the evidence of the fractality of glacier, the spatial distribution of fractures within the glacier continuum is investigated. Among the various definitions of dimension of a fractal set, the Minkowski-Bouligand is a special case of the Hausdorff-Besicovitch dimension and represents the best definition for numerical implementation. To obtain the fractal dimension of a certain domain, it is necessary to cover it by means of regular Euclidean sets (usually square or rectangular grids) with decreasing linear size. The fractal dimension is obtained by computing the logarithmic density of the measure of these coverings [9]. This is called the box-counting method and, as a result, it gives the box-dimension $\Delta$ depending only on the metric characteristics of the set [10]. The mathematical definition of the box-dimension is given by:

$$\Delta = \lim_{b_i \to 0} \left( \frac{\log N(b_i)}{\log\left(\frac{1}{b_i}\right)} \right) \tag{1}$$

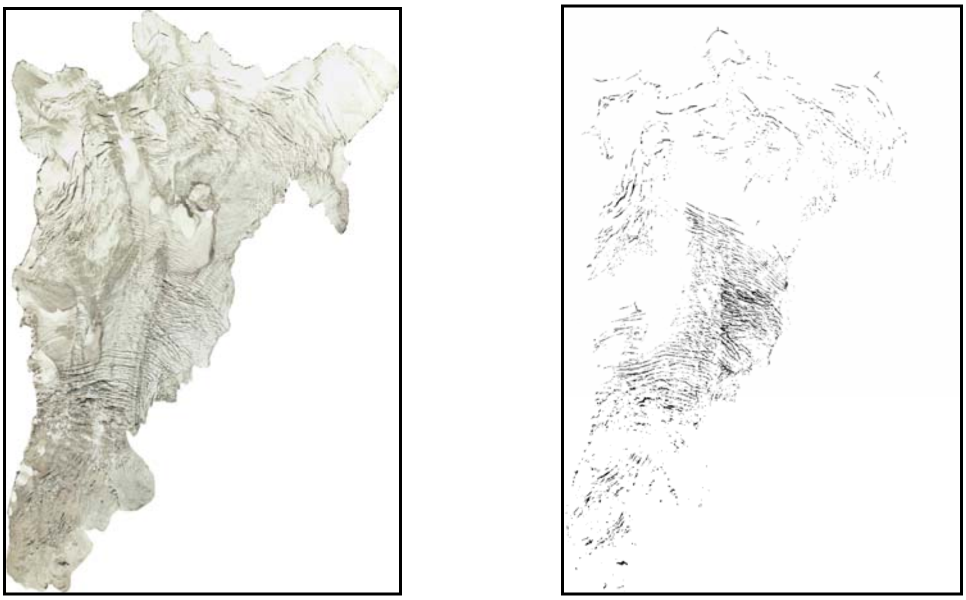
where $b_i$ is the (progressively decreasing) linear size of the covering grid and $N_i$ is the (progressively increasing) number of boxes that cover a part of the fracture network.

The box-dimension can be calculated in a straightforward manner by considering the slope $\alpha$ of a linear regression in the log*N* vs. log*b* plot. In this case, the box-dimension is equal to:

$$\Delta = -\alpha \tag{2}$$

In the presence of self-affine scaling (as is the case of simple profile roughness), this value depends on the shape of the covering grids. Generally, natural fractals do not exhibit an univocal value of the slope (mono-fractality), but show a geometrical multi-fractality [11] with a continuous variation of the $\alpha$ parameter (some authors define this scaling behaviour as self-affinity). This behaviour is caused by the presence of two transition scales of phenomenon [12]. In particular, when the object is a digitalised image (i.e., a discrete set of pixels), it is necessary to define the lower limit of scaling to avoid to consider the external cut-off length of the object [12]. From this the lower limit of the scaling must be clearly defined.

2.1 *The Box-Counting Method applied on the Cherillon glacier*



(a) (b)

Figure 2. The Cherillon glacier:
(a) Aerial image; (b) Crevasses network skeleton on glacier surface.

In order to study the patterns of fracture networks on glacier, a general-purpose version of the method originally developed for the fractal analysis of 2D lattices [13] is used.

From the aero-photogrammetric view and the geo–referenced images of the Cherillon glacier (Figs. 1 and 2.a), the map of fracture networks on the ice surface can be outlined by image analysis, i.e. by skeletonising the areas corresponding to the crevasses (paying attention to eliminating shadows due to ice hills and valley) (Fig. 2.b).

The box-counting method is then applied to this 2D pattern of fractures with dimension [1772 x 1024 pixels], by varying the linear size *b* of the square or rectangular grids (Figs. 3.a, 3.b, 3.c and 3.d). The ice fracture patterns on the Cherillon glacier possess a box-dimension *Δ* closed to 1.6.

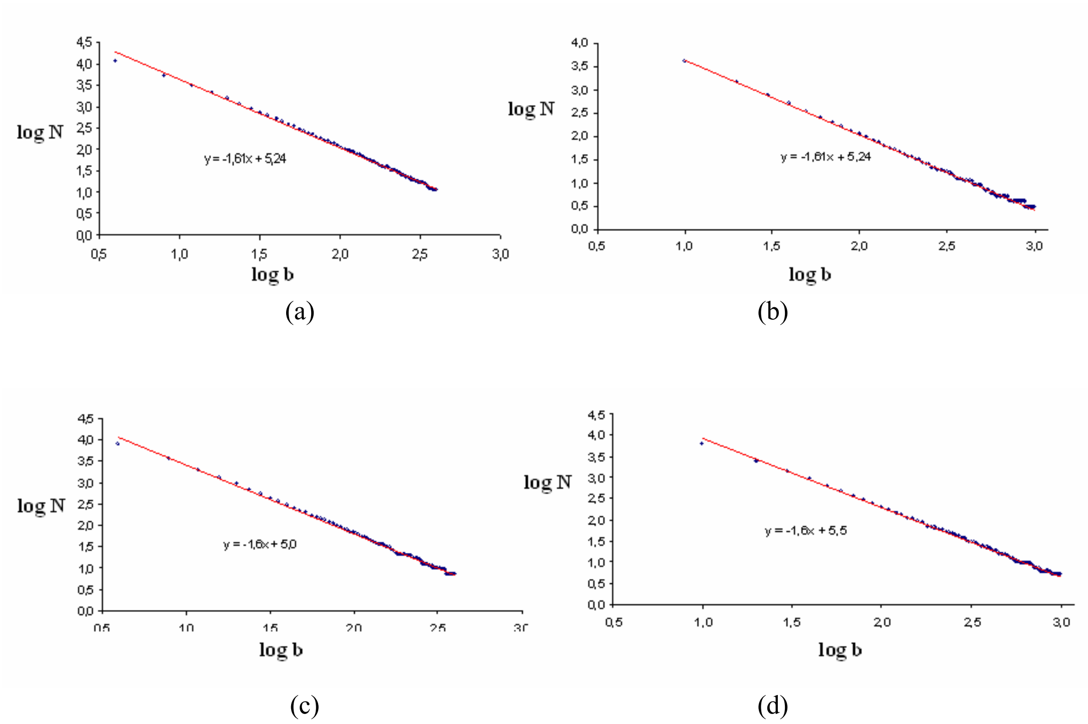



Figure 3. Application of the Box-Counting Method to the patterns of ice fractures: a) and b) square grids; c) and d) rectangular grids with different linear size *b* (the larger size of the rectangle is vertical and horizontal, respectively).

We note that varying the maximum dimension of square elements grids from 400 to 1000 pixels (respectively, Fig 3.a and Fig. 3.b), the box-dimension does not change. However the use of square grids (isotropic scaling implies the self-similarity of the network) could be in contrast with possible self-affine properties of fracture networks on ice.

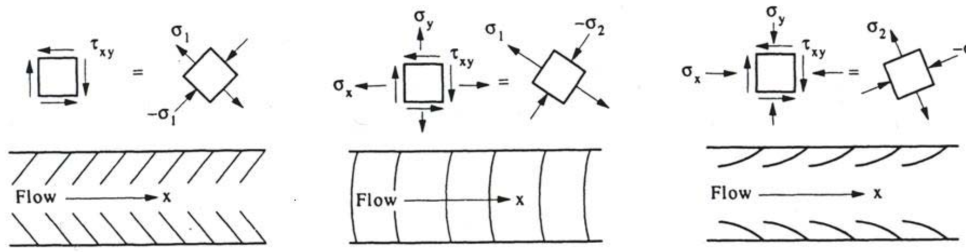
To check this, rectangular grids have also been used to capture the possible direction-dependent scaling, adopting iterative rescaling as a function of the grid direction [13]. Fig. 3.c and 3.d show the box-counting dimension of the Cherillon glacier calculated by rectangular grids (vertical/horizontal ratio equal to 2 and 0,5, respectively). The box-dimension, calculated according to a self-affine grid, results again equal to $\Delta = 1.6$ .

By varying the linear size and the shape of box grids, crevasse networks on the Cherillon glacier yield a value of $\Delta$ close to 1.6, confirming the self-similar fractal properties of ice fracture networks.

### 2.2 *Results*

The dynamic of glaciers is mostly governed by gravity and ice metamorphism which induce different states of stress and strain influenced by the local geo-morphology of the mountain slopes.

When the stress overcomes the ice strength, ice fracturing starts and develops in different forms. A typical appearance of ice fracturing is represented by crevasses, each one generated by high tensile stress, influenced by the presence of adjacent ones and by their depth [14, 15]. As in the case of damage of quasi-brittle materials (e.g., concrete), the direction of fracture propagation obeys to the principal tensile stress, i.e. it is approximately perpendicular to it. For a glacier moving in a valley, different stress states usually induce three kinds of crevasses [15], reported in Fig. 4.




(a) (b) (c)

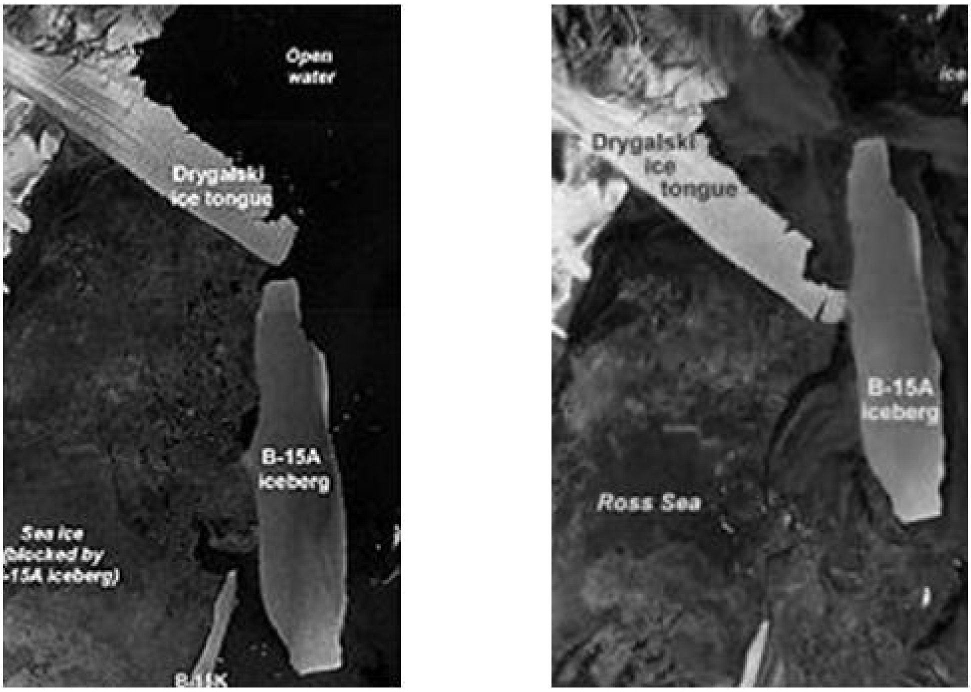
Figure 4. Crevasses patterns in a valley glacier with corresponding stress state [15]: a) shear stress exerted by valley walls only; b) shear stress and tensile flow; c) shear stress and compression flow.

From the mechanical point of view, the self-similar fractal properties of ice fracture networks imply that the three prevailing stress rupture mechanisms are all active within the ice continuum, due to the particular geo-morphological shape of the valley.

## 3 THE EVIDENCE OF FRACTALITY AT LARGE SCALE

To detect the ice behavior at sub-continental scale, the collision occurred in the Terra Nova Bay of the Ross Sea (Antarctic) on the 15$^{th}$ April 2005, between a permanent and floating wharf of pure glacial ice (the Drygalski Ice Tongue) and a massive iceberg (the B-15A) is analyzed (Figure 5.b). The impact caused a crack of 5 km length onto the Drygalski Ice Tongue and provoked the detachment of a new little iceberg (10,5 km long per 5,7 km wide) equal to 3% of Drygalski surface.

The B15-A iceberg was a fragment of the massive iceberg B15 (295 km x 37 km and thickness between 200 m and 350 m) originated from the Ross Platform, in the year 2000. The surface of B15-A was about 3000 $km^2$ with a 120 km of length and an average height of 275 m. The Drygalski Tongue is the floating terminal part of the David Glacier on the Ross Sea with a surface close to 225.000 $km^2$ and height varying between 2000 m and 200 m.




(a) (b)

Figure 5. Two subsequent Envisat ASAR images of the B15-A iceberg and the Drygalski ice tongue: a) on January 2005; b) on 15 April 2005.

3.1 *The fracture energy of the event*

To estimate the fracture properties of the ice at large scale, the evolution of the impact is studied and the fracture energy consumed during crack propagation can be calculated. Thanks to the peculiarity of the natural phenomenon, the moving B15-A iceberg was detected and all its displacements were monitored in time by satellite ENVISAT images from November 2004 to April 2005.

Applying the Principe of Energy Conservation under the hypothesis of no energy loss during the impact between iceberg and ice tongue (i.e. no crushing during the collision , no friction, etc… occurred), the fracture energy consumed during crack propagation on Drygalski Tongue can be estimated as the variation of the total potential energy [16]. The latter can be calculated from the dissipating work done by B15-A at the impact and can be estimated as:

$$L_{B15A} = \int F \, ds = m_{B15A} \int a_{B15A} \; dz \quad (3)$$

where $a_{B15A}$ is the acceleration and $m_{B15A}$ is the constant ice mass of the iceberg. The B15-A mass is estimated considering ice density equal to 917 kg/m$^3$. From the ENVISAT images, the speed and the acceleration of the iceberg have been calculated by the incremental ratio of the space and speed in time, respectively.

The integral part of eq. [3] is evaluated graphically by measuring the subtended area between the trend of acceleration of B15-A in space with and without impact (Fig. 6 in dashed red line).

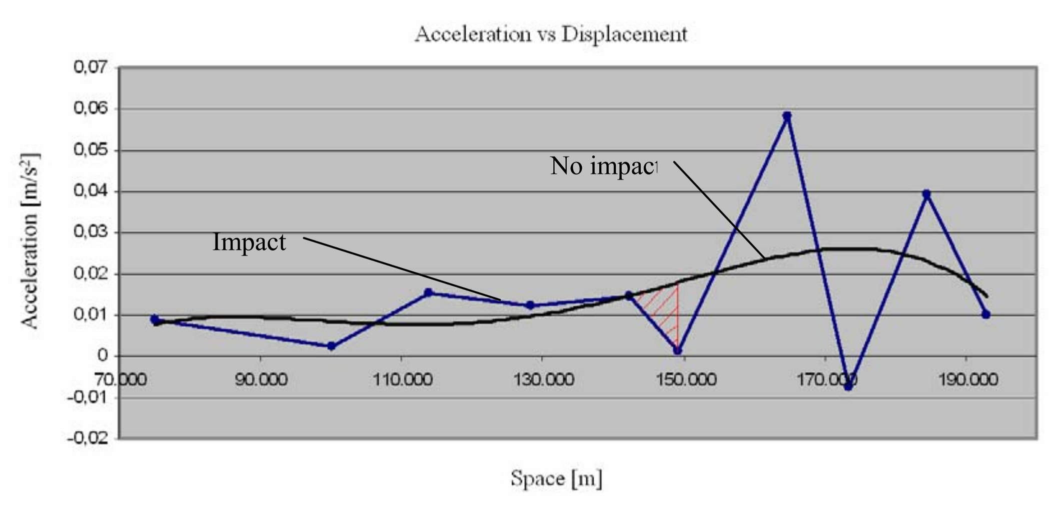



Figure 6: The acceleration trend of B15-A plotted vs. space from November 2004 to April 2005.

The strain energy release rate can be calculated as a ratio between the variation of the total potential energy of B15-A and the area of the induced fracture in Drygalski Tongue. The latter is estimated as the product of the fracture length (5 km) and the thickness of Drygalski Tongue (estimated about 200 m). The strain energy release rate $G_{B15A}$ is estimated around 40804,31 MJ/m$^2$. Considering the elastic modulus of the ice equal to $E_{ice}$ = 9 GPa (around 1/3 of elastic modulus of concrete), the stress intensity factor $K_{ice}$ at the large scale can be calculated as:

$$K_{ice} = K_{B15A} = \sqrt{G_{B15A} \cdot E_{ice}} \quad (4)$$

and results equal to 19.163.475,42 [kPa m$^{1/2}$ ].

### 3.2 *The size effect on ice fracture properties*

To evaluate the scaling of the mechanical properties of ice a comparison between the data obtained at the *in-situ* scale and at small laboratory scale reported by Dempsey et al. [2] on freshwater ice and the phenomenology of fracture evidences at the very large scale of Antarctic sea icebergs is carried out. Table 1 reports the experimental data of stress intensity factors and the related to the size of the specimen at *in-situ* and at laboratory scales ([2] – Table 3 and 4).

Table 1. Experimental data on ice at *in-situ* scale and at laboratory scale [2].

| Authors | b [m] | K [kPa $m^{1/2}$ ] |
|---|---|---|
| Dempsey et al. (1999) | 0,98 | 97 |
| Dempsey et al. (1999) | 0,27 | 76 |
| Dempsey et al. (1999) | 3,19 | 144 |
| Dempsey et al. (1999) | 0,245 | 134 |
| Dempsey et al. (1999) | 0,65 | 190 |
| Dempsey et al. (1999) | 7,24 | 186 |
| Dempsey et al. (1999) | 2,19 | 193 |
| Dempsey et al. (1999) | 2,21 | 144 |
| Dempsey et al. (1999) | 19,66 | 297 |
| Kollé (1981) | 0,05 | 240 ± 79 |
| Danilenko (1985) | 0,015-0,080 | 335 ± 50 |
| Danilenko (1985) | 0,015-0,080 | 439 ± 72 |
| Dempsey, Nigam, Cole (1988) | 0,032 | 465 ± 182 |
| Dempsey, Nigam, Cole (1988) | 0,048 | 280 ± 135 |
| Dempsey, Nigam, Cole (1988) | 0,064 | 250 ± 72 |
| Dempsey, Nigam, Cole (1988) | 0,080 | 282 ± 136 |
| Stehn, DeFranco, Dempsey (1995a) | 0,010 | 126 ± 41 |

Reporting in the Log –Log plot the values of the stress intensity factor of ice at different scales, Fig. 7 shows the increase of $K_{ice}$ , as the scale of the specimen increases.

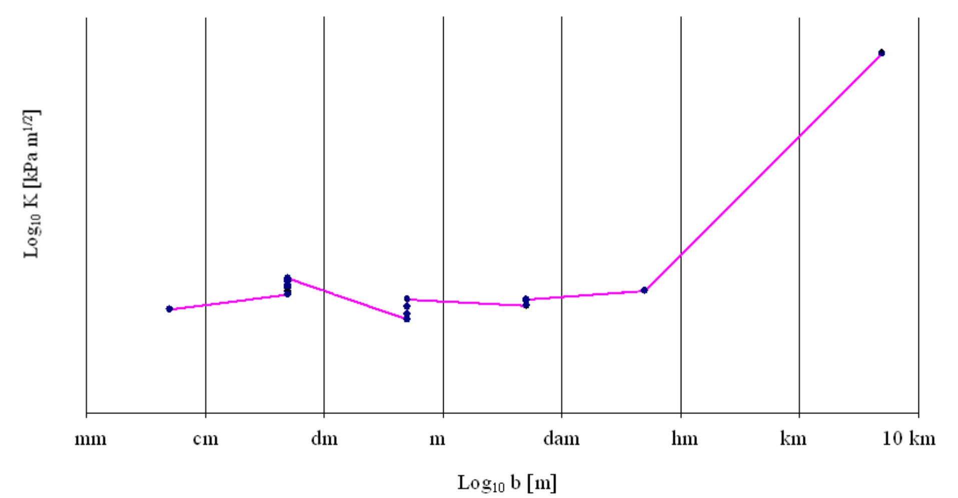



Figure 7: Stress Intensity factor of ice from mmetres scale to kmetres scale.

This confirms that as a consequence of the fracture domain, we can explain the decrease of ultimate tensile strength $\sigma_u$ and the increase of toughness $\mathcal{G}_F$, as the scale of the specimen increases. In the future, it will be interesting to apply the Multi Fractal Scaling Laws - MFSL – [3, 4] which were originally developed for concrete-like materials.

## 4 CONCLUSIONS

It is well known that some typical phenomena on the strength of materials are only the macroscopic results of the structural disorder. Also in ice mechanics, the investigation confirms that fracturing on glaciers surfaces shows a self-similar behaviour and induces the invasive fractality ($\Delta > 1$) of the crack networks. This means that ice behaves as a brittle material at small scales ($\Delta \approx 1$ with regular brittle fractures single paths) whereas becomes more ductile at large scales ($\Delta \cong 1.6$, with a dense fracture network on the ice surface) with a distributed damage and an increase of toughness. A further confirmation of this behaviour is obtained by comparing the data obtained at small scales with those at large scales, like the impact of icebergs.

## 5 ACKNOWLEDGMENTS

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