Damage and healing effects in inflated rubber balloons

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Keywords: Damage, Rubber-like Materials, Membranes.

SUMMARY. A recently developed constitutive model which describes damage and healing effects in rubber materials is applied to the inflation problem of a thin, initially spherical balloon. The model efficiently describes the overall, cyclic response of the balloon. The occurrence of asphericity during inflation is then taken in consideration and some preliminary results in this direction are discussed in the last section.

1 INTRODUCTION

Theoretical and experimental analysis of the inflation of thin balloons constitute a traditional subject of mechanics of materials. In particular, inflation experiments represent a widely used tool for the constitutive characterization of soft materials. Moreover, this field of research is of interest for several technological applications, ranging from aerostatic balloons [12] and inflatable structures [14], to pneumatic micro-actuators and sensors [15].

Inflation experiments on thin spherical rubber-like balloons show a complex, history-dependent behavior, with a possible occurrence of asphericity. These macroscopic effects may be ascribed to the complex phenomena of damage and recrosslinking phenomena at the micro-scale level. In particular, an accurate analysis of the experimental pressure-strain and stress-strain responses highlights that, for successive cyclic inflation experiments, the occurrence of healing for previously damaged material may play a crucial role.

In the paper [3], the authors apply a recently proposed microstructure-based model for damage and healing in rubbers to the inflation of a thin spherical balloons. More specifically, the constitutive model (see [1],[2]) is based on the assumption that the material is constituted by a fraction of elastic material and a fraction of damageable material. The presence of links with variable activation and breaking lengths is accounted by the introduction of a suitable probability density function. The model, while keeping a computational efficiency, is in significant agreement with the experimental behavior of rubber materials.

A preliminary analysis of the occurrence of non-homogeneous configurations in thin inflated balloons is presented in the last section. In order to investigate the occurrence of non-homogeneity, we introduce an additive decomposition of the left stretch tensor $\mathbf{V}$ into its spherical and deviatoric parts

$$u := \left(\frac{\mathrm{tr}\mathbf{V}}{2}\right)^2, \qquad \delta := \frac{||\mathbf{V}_D||^2}{2},$$

where $\mathbf{V}_D$ is the deviatoric part of $\mathbf{V}$. A purely kinematical analysis shows that this decomposition is useful for the study of non-homogeneous configurations of the thin balloon; more in detail, we show that the insurgence of distortions on a finite portion of the balloon is a necessary condition for the occurrence of non-homogeneities. Well known results of global differential geometry and the kinematical characterization in terms of the fields (u, δ) address attention on the fact that the occurrence of non-homogeneities might be interpreted as a consequence of a damage induced softening with respect to u and/or δ.

The topic of shape transitions in cyclic inflations of thin balloons is documented in the experimental as well in the theoretical literature (see, e.g., [8], [10], [13]), but a theory relating such phenomena to damage is still lacking. Within this point of view, the main goal of our approach will be, in future development of this work, to relate the shape transition phenomena to damage and healing effects in rubbers.

2 THE MODEL

In this section we briefly recall our constitutive model for rubbers undergoing damage and healing; further details can be found in [1], [2] and [3].

Let $\boldsymbol{f} : \boldsymbol{X} \in \mathscr{B}_0 \mapsto \boldsymbol{x} \in \mathscr{B}$ be the deformation of a body $\mathscr{B}_0$ with $\mathbf{F} := \nabla \boldsymbol{f}$ the corresponding deformation gradient and $\mathbf{B} := \mathbf{F}\mathbf{F}^T$ the left Cauchy-Green tensor. The key assumption of the model is that at each point $\boldsymbol{X} \in \mathscr{B}_0$ a fraction $\alpha \in (0, 1)$ of the amorphous material is described by an hyperelastic, isotropic and incompressible constitutive law (*elastic matrix*), whereas the remaining fraction $(1 - \alpha)$ takes care of the activation, breaking and recross-linking effects at the micro scale level (*damageable material*).

The constitutive response of the elastic matrix is assigned in terms of an elastic energy density $\varphi_e = \varphi_e(I, II)$, where $I := \mathrm{tr}\mathbf{B}$ and $II := \mathrm{tr}\mathbf{B}^{-1}$ are the first and second invariants of $\mathbf{B}$. The expression of the Cauchy stress in the elastic matrix is given by

$$\mathbf{T}_e = -\pi\mathbf{I} + 2\varphi_{e,1}\mathbf{B} - 2\varphi_{e,2}\mathbf{B}^{-1}, \tag{2.1}$$

where $\varphi_{e,1}$ and $\varphi_{e,2}$ denote the derivatives with respect to the first and second invariant, respectively, and π is the reactive stress maintaining the incompressibility constraint.

The constitutive response of the damageable material is based on an activation and breaking criterion, which is expressed in terms of an isotropic scalar function of the two invariants of $\mathbf{B}$, so that the damageable material is *activated* when $s(I, II) = s_a$ and it is *broken* when $s(I, II) = s_b$. We assume that the response of the damageable material elastically depends on the deformation gradient measured from the activation state $\mathbf{F}_a$ up to the reaching of a breaking state $\mathbf{F}_b$, in correspondence of which the stress falls to zero.

More precisely, let $\hat{\mathbf{F}} := \mathbf{F}\mathbf{F}_a^{-1}$ be the deformation gradient measured from the activation state and let $\hat{I}, \hat{II}$ be the invariants of $\hat{\mathbf{B}} := \hat{\mathbf{F}}\hat{\mathbf{F}}^T$ and $\varphi_d = \varphi_d(\hat{I}, \hat{II})$ be the strain energy density which describes the behavior of the damageable material; with these positions the stress in such fraction attains the values

$$\mathbf{T}_d := \begin{cases} \mathbf{0} & \text{when} \quad s(I, II) < s_a \\ 2\varphi_{d,1}\hat{\mathbf{B}} - 2\varphi_{d,2}\hat{\mathbf{B}}^{-1} & \text{when} \quad s_a \le s(I, II) \le s_b \\ \mathbf{0} & \text{when} \quad s(I, II) > s_b \end{cases},$$

where $\varphi_{d,1}, \varphi_{d,2}$ represent derivatives with respect to the first and second invariant, respectively.

In order to take care of the microstructure disorder, the values of s_a and s_b are considered to be locally regulated by a general probability density $f = \hat{f}(s_a, s_b)$, which can be determined by simple cyclic uniaxial loading experiments (see [1] for details). A possible simplified assumption consists in assuming that $s_b = \hat{s}_b(s_a)$, with $\hat{s}_b$ invertible. This allows us to reduce to a one parameter distribution function $f(s_a) := \hat{f}(s_a, \hat{s}_b(s_a))$.

The overall stress in a given point is the sum of the stresses in the elastic matrix and in the damageable material, and it clearly depends on the past deformation history. If breaking is considered as an irreversible event (*irreversible damage*) then the stress is zero for all strain histories after $s > s_b$.

The possibility of a partial recovery of the broken material under cyclic deformations is also admitted (*healing effect*); in this case (*reversible damage*) we admit that during an unloading path, a fixed fraction $k < 1$ of previously broken material can be healed when the local value of s equals the activation value s_a. Such fraction of material can be re-broken upon reaching of the breaking threshold s_b.

Without entering details on the derivation of the stress in the most general reversible case (a detailed description of which can be found in [3]), the overall stress during a strain history $\mathbf{F} = \mathbf{F}(t)$ is given by the compact expression

$$\begin{aligned} \mathbf{T}(t) = \alpha \mathbf{T}_e(t) \quad &+ \quad (1-\alpha)\mathcal{H}(s(t) - s_f(t) \int_{s_f(t)}^{s(t)} f(s_a) \mathbf{T}_d(\hat{\mathbf{B}}(t))\, ds_a + \\ &+ \quad k(1-\alpha)\mathcal{H}(s(t) - s_h(t)) \int_{s_h(t)}^{s_u(t)} f(s_a) \mathbf{T}_d(\hat{\mathbf{B}}(t))\, ds_a. \end{aligned} \tag{2.2}$$

where $s_i(t)$ are functions determined during the deformation path and where $\mathcal{H}$ is the Heaviside function.

3 EQUILIBRIUM IN THE HOMOGENEOUS CASE

Several interesting phenomena deserve attention when observing the behavior of inflated rubber balloons. As a first step, we here restrict attention to equibiaxial deformations: it should be remarked that by this assumption the possible occurrence of non-homogeneities in initially spherical balloons is neglected. We thus show the feasibility of the model in describing the main experimental effects observed during the inflation of rubber balloons.

In Fig.1 we represent the results of numerical simulations for the pressure/engeneering stress versus strain curves. The model shows a good qualitative agreement with the experimental results carried on, for example, in [6] and it gives a micromechanical interpretation of the macroscopic mechanical response of the balloon under repeated inflations of increasing amplitude. It should be remarked that our model neglects residual deformations and this is the main reason for the difference at the origin ($\lambda = 1$) of the experimental pressure/stress - strain curves produced by [6] and our numerical results here reported. As described in [1], the model here proposed is predictive, in the sense that simple experimental analysis allow to deduce the probability density properties of the material, which will hopefully lead to a quantitative description of the experimental behaviors.

4 PERSPECTIVES: OCCURRENCE OF NON-HOMOGENEOUS CONFIGURATIONS

In this section we present some recent unpublished results [4] which, in our point of view, give insights on the stability for the inflation problem of a thin, initially spherical balloon.

The numerical analysis described in the previous section refers to a one-variable problem, since the kinematics of the spherical balloon was restricted to homogeneous conformal deformations. With an eye toward the more realistic non-homogeneous cases, it is well documented in the experimental literature (e.g. [7], [9], [13]) that inflated spherical balloons often exhibit transitions to spherical configurations, characterized by a non-homogeneous thickness, and transitions to aspherical modes. Interestingly enough, after appearing in the first loading cycle (*primary loading path*) such phenomena are eventually mitigated in successive loading paths; in some cases, during a monotonic increase of pressure, after exhibiting transition to non-homogeneous configurations the balloons reattains its homogeneous, spherical configuration (*closed loop behavior*).

On the other side, plane stress experiments carried on square, rubber plane membranes subjected to hydrostatic tractions at the boundary show a sudden transition to non-conformal shapes (which

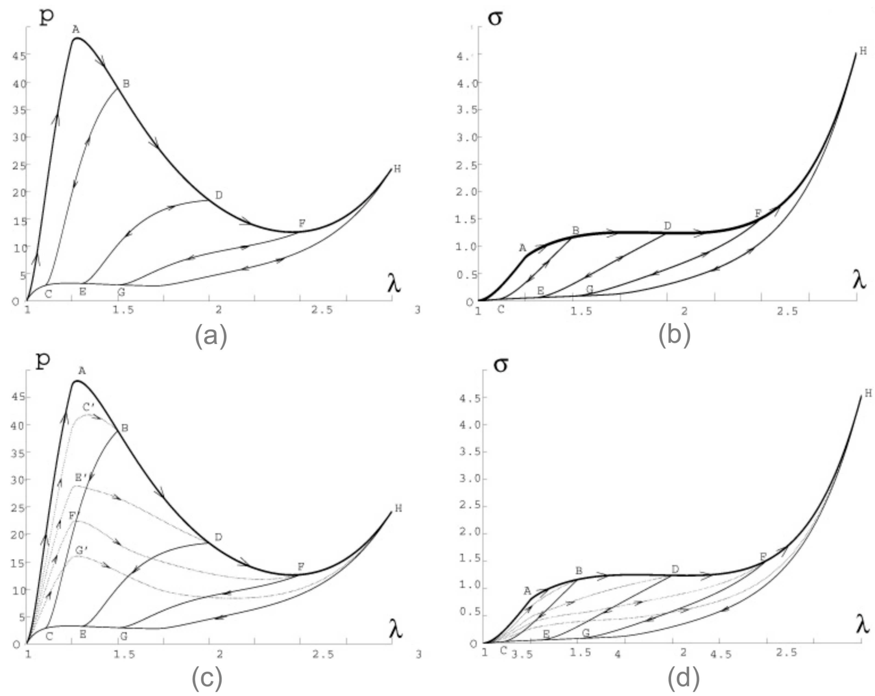

Figure 1: Pressure-strain and engineering stress-strain curves have been obtained by means of our constitutive model under numerical simulation of cyclic inflation experiments. We here considered an Ogden type constitutive law for the elastic matrix and a Neo-Hookean damageable material. The distribution of damageable material is assigned through a probability density of the *beta-type*. We considered a percentage $\alpha = 0.2$ of elastic matrix and an elastic range $\Delta = s_b - s_a$ linear in s_a. In a) and b) we neglect healing (i.e. we assume $k = 0$) while in c), d) we take healing effect into consideration by assuming that a fraction $k = 0.35$ of material broken during previous cycles may reforms upon unloading.

is referred to as the classical *Treloar effect*). The question if Treloar effect might be related to the spherical-aspherical transition in thin balloons was addressed in [5] and [8], but within the purely elastic contexts where damage is ignored.

The idea behind our approach is that damage cannot be neglected in a description of the possible instabilities observed in cyclic inflation experiments of balloons. This idea is supported by the experimental evidence that the aforementioned transitions can be appreciated during primary loading paths, but these eventually disappear along successive loading paths: purely elastic model are clearly unable to describe such phenomena. These evidences point attention on the fact that the activation of links at the microscale-level might induce non-convexities in the energy density, such that during a first inflation path the transition to distorted states might be energetically preferred; the breakage of the activated links during increasing inflations might lead to the cancelling of such non-convexities, so that during successive cycles the transition to distorted states becomes energetically unfavored, and the conformal deformation is preserved.

It is easy to show that the occurrence of distortions at the local level on the balloon represents a necessary condition for the transition to aspherical configurations. This is shed in evidence by considering the balloon membrane as a two-dimensional body, and projecting the equilibrium equation of the current surface $\mathrm{div}\mathbf{T} + \boldsymbol{b} = \boldsymbol{0}$ on the normal direction $\boldsymbol{n}$; by decomposing the stress $\mathbf{T}$ in its spherical part $\mathbf{T}_0 = \sigma(\mathbf{I} - \boldsymbol{n} \otimes \boldsymbol{n})$ and deviatoric part $\mathbf{T}_D$, the *generalized* Laplace's equation of equilibrium holds

$$2\sigma H + \mathbf{T}_D \cdot \mathbf{L}_D + p = 0,$$

where $\mathbf{L}_D$ is the deviatoric part of the curvature tensor $\mathbf{L} = -\mathrm{grad}\,\boldsymbol{n}$ of the current balloon surface, σ the hydrostatic membranal stress and $H = \mathrm{tr}\mathbf{L}/2$ the current mean curvature. If $\mathbf{T}_D = \mathbf{0}$ on the whole membrane, the equilibrium equation of the balloon reduces to the classical Laplace's equation $2\sigma H + p = 0$ with $\sigma = const$, whose unique solution for a regular, compact surface is represented by a sphere. Under physically reasonable constitutive assumptions (such as, for example, in elastic isotropic membranes) the only possibility for $\mathbf{T}_D$ to vanish is that the deviatoric part of $\mathbf{B}$ vanishes; this implies that the occurrence of distortions on a finite part of the membrane represents a necessary condition for the occurrence of asphericities.

With these conclusions in mind, it is interesting to observe that the occurrence of transitions to non-homogenous states can be profitably afforded by the introduction of the strain measures

$$u := \left(\frac{\mathrm{tr}\mathbf{V}}{2}\right)^2, \qquad \delta := \frac{||\mathbf{V}_D||^2}{2},$$

where $\mathbf{V} = \sqrt{\mathbf{B}}$ and $\mathbf{V}_D$ is the deviatoric part of $\mathbf{V}$, where it is easy to check that the areal stretch J is equal to $u - \delta$. Both u and δ are greater or equal to zero and since $J > 0$, it results that $u > \delta$. Let in particular the reference surface of the balloon be the spherical surface S of area A and let v be the volume of the current configuration of the balloon. The area a of the current surface of the balloon is necessarily greater or equal of a_v, namely the area of the sphere of volume v, so that

$$a = \int_S J\, dS = \int_S (u - \delta)\, dS \geq a_v = \sqrt[3]{36\pi v^2}. \tag{4.3}$$

Let now u_v to be the homogeneous, spherical dilatation corresponding to the sphere with volume v, so that $a_v = u_v\, A$. Denoting by the *tilde* the average over the reference surface S, so that $\tilde{\phi} := (\int_S \phi) A^{-1}$, the condition (4.3) can be recast in terms of the averaged values $\tilde{u}$ and $\tilde{\delta}$, as

$$\tilde{u} - \tilde{\delta} \geq u_v. \tag{4.4}$$

With reference to Fig.2, we will say that a point $\tilde{P} = (\tilde{u}, \tilde{\delta})$ is *representative* of the system, since its knowledge allows to determine some important qualitative properties of a configuration corresponding to a given volume v. Such properties are collected in the following proposition (which is here given without proof) and these do not deal with compatibility or equilibrium, but just with purely geometrical considerations.

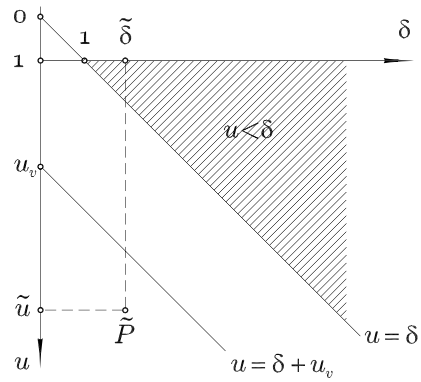

Figure 2: The (u, δ) plane.

• **Proposition 1** *Letting v the volume of the current configuration, it results that:*

(i). *$\tilde{P}$ cannot belong to the half-space $u - \delta < u_v$;*

(ii). *if $\tilde{P}$ belongs to the line $u - \delta = u_v$, then the current configuration is spherical;*

(iii). *homogeneous configurations are only those surfaces with $u = u_v$ and $\delta = 0$;*

(iv). *configurations with $\tilde{P} = (u_v, 0)$ are spherical; these can be nonhomogeneous, but it has to result $\delta = 0$ everywhere;*

(v). *all configurations with $\tilde{u} - \tilde{\delta} > u_v$ are necessarily aspherical.*

In light of the above proposition, we believe that the choice of variables (u, δ), together with considerations coming from the global theory of surfaces and with the requirements of kinematical compatibility and equilibrium, will be useful in describing the occurrence of non-homogeneities on inflated spherical balloons, within the context of damage and healing effects in rubber materials.

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