Optimal Four-Bar Linkage for the Stability and the Motion of the Human Knee Prostheses

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SUMMARY. The optimal design of human lower limb prostheses, in particular of knee devices, is fundamental in order to restore the lost functionality and aesthetic aspect of the amputee's locomotion. Among all knee devices, the four-bar linkage is still the most widespread mechanism, since, despite its simplicity, it allows the prosthesis to be sufficiently stable and, at the same time, to replicate the natural motion of the joint with a sufficient accuracy. This paper presents an optimization procedure for the synthesis of a four-bar linkage for knee prosthesis. Starting from an experimental reference motion and given some patient-specific requirements related to his capacity to control and to stabilize the prosthesis, the procedure identifies the four-bar linkage that best-fits the experimental motion, at the same time respecting the given specifications. A case study is also presented which shows the potentialities of the proposed procedure.

1 INTRODUCTION

Human lower limb external prostheses must restore the amputee's ability of walking and supporting the body weight on the amputated limb during the walking stance phase. Prostheses must also guarantee the amputee's self-sufficiency by restoring the lost functionality of the lower limb. In particular, instability conditions, which could compromise the amputee's safety, must be avoided. The knee prosthesis represents the fundamental component of a lower limb prosthesis since it both allows flexion between the femur and shank and considerably influences the fulfilment of the technical specifications requested by the amputee. In particular, the most requested specifications to allow a fast and optimal rehabilitation are the functionality of the prosthetic knee (i.e. the level of stability and of voluntary control of the prosthesis) on one side, and the aesthetic aspect of the generated motion (i.e. the capability of the prosthesis to simulate the relative motion between the femur and shank) on the other side. Lower limb prostheses and, in particular, knee prostheses must be carefully designed and analysed in order to satisfy all these requirements.

Most knee prostheses available on the market and reported in the literature can be divided into two large categories: prostheses with a single axis of rotation (these devices are basically two links connected by a revolute joint) and polycentric prostheses (articulated mechanisms with one degree of freedom). Single axis prostheses have a fixed instantaneous centre of rotation (ICR) of the relative motion between the femur and shank, are cheap and replicate knee motion with a good accuracy. However, such prostheses have a low functionality and do not provide high stability and voluntary control. On the contrary, in polycentric prostheses the position of the ICR changes at each flexion angle. The latter are more expensive than the former and guarantee a higher functionality, but in general to the detriment of a lower ability to simulate the natural knee motion.

In other words, the position of the ICR has a conflicting influence on the functionality of the prostheses versus the aesthetic aspect of the generated motion.

The best compromise solution, considering cost, functionality and the aesthetic aspect, is the polycentric prosthesis based on the four-bar mechanism (4BM). The 4BM is a very simple mechanism and, in the literature, was widely described and analyzed for prosthetic applications [1, 2, 3]. With reference to Figure 1, member 1, with length x_1, of the 4BM is stiffly linked to the shank; members 2 and 4, with lengths x_2 and x_4, are the posterior and anterior links between the femur and shank respectively; finally, member 3, with length x_3, is stiffly linked to the femur. The synthesis of the 4BM could be performed by considering the positions and orientations of the coupler link 3 at several flexion angles (a well-known methodology in the literature) or by considering the positions of the ICR of the relative motion between the members 1 and 3. In particular, two different methods were presented in the literature for the synthesis of a 4BM for prosthetic applications:

- synthesis by means of precision points [4]. This method makes it possible to obtain a 4BM with a centrode passing through seven precision points arbitrarily selected as a reference. However, it is not possible to define the knee flexion angle corresponding to each precision point of the centrode. Moreover it is not possible to specify the relative motion of the femur and shank in order to satisfy the aesthetic aspect of the prosthesis motion;
- synthesis using optimization methods [5]. These methods determine the optimal 4BM starting from a certain number of given centrode points and coupler points corresponding to arbitrarily chosen reference knee flexion angles. However, a centrode must be chosen and used as a reference.

In this paper, starting from the optimization method introduced in [5], a modified synthesis procedure addressed to the knee prosthesis is presented. Likewise in [5], the new procedure considers both the amputee requirements (functionality and aesthetic aspect). Moreover, unlike the procedure proposed in [5], it is not necessary to provide a reference centrode. Starting from experimental data which represent the natural motion of the human knee, the proposed optimization procedure finds a 4BM whose relative motion between members 1 and 3 is similar to the experimental motion and, at the same time, whose centrode satisfies some functional constraints which provide the knee prosthesis with the level of stability and of voluntary control required by the amputee. As a result, the new procedure makes it possible to identify a 4BM that satisfies the specification of functionality, at the same time replicating the experimental motion with a higher accuracy with respect to a 4BM obtained by previously presented procedures. Finally, a case study is reported and discussed.

2 STABILITY OF EXTERNAL KNEE PROSTHESES

Lower limb prostheses should be designed in order to restore the stability during walking (especially when the amputee weight is borne by the prosthesis) and the voluntary control, in terms of allowing the amputee to completely control the position and the stability of the flexed prosthetic knee in each stance phase stage. In particular, a knee prosthesis should be designed in such a way that it can be extended by the amputee during the stance phase (thus sustaining the amputee weight) and it can be flexed at the end of the stance phase. As presented in previous studies [1, 2, 3] the knee prosthesis' stability and voluntary control are highly influenced (i) by the residual hip musculature of the amputated limb and thus by the maximum hip moment the amputee can apply to the lower limb, (ii) by the position of the ICR between the femur and shank during the stance phase, and (iii) by the possible flexional moment generated by a device inserted into the knee prosthesis. In particular, the knee prosthesis can be extended when the ICR between the femur and shank is located posteriorly with respect to the line of the resultant ground reaction force (GRF) applied at the centre of pressure (COP) of the prosthetic foot; on the contrary, the prosthesis can be flexed when the ICR is located anteriorly with respect to the GRF line. Considering the flexional moment of the prosthesis negligible (perfectly passive prosthesis), the prosthesis is statically balanced if the line of the GRF coincides with the line connecting the COP and the ICR.

In general, from a static viewpoint, the GRF on the lower limb is balanced by an opposite force applied at the hip joint centre and by a hip moment given by the product between the GRF intensity and the distance between its line and the hip joint centre. The angle between the GRF direction and the line passing through the hip joint centre and the COP is thus superiorly limited by a value α_{max} which depends on the maximum hip moment the amputee can apply to the lower limb and on the GRF intensity, namely:

$$M_{\max} = F \cdot l \cdot \mathrm{sen}(\alpha_{\max}) \tag{1}$$

where M_{max} is the maximum hip moment, F is the intensity of the GRF and l is the distance between the COP and the hip joint centre. In different words, in order for the amputee to balance the GRF, the GRF line must lie within a cone with vertex in the COP, axis on the line passing through the hip joint centre and the COP, and vertex angle equal to $2\alpha_{max}$. Since the GRF intensity, COP and hip joint centre positions change during the stance phase, this cone changes at each stance phase stage.

It should be noted that the amputee is able to change the GRF direction acting on the voluntary hip moment and thus he is able to voluntarily control the flexion and extension of the prosthesis as long as he is able to move the GRF direction more posteriorly or more anteriorly with respect to the ICR, as previously clarified. As a consequence, the knee prosthesis is stable and can be voluntarily controlled if at each stance phase stage the ICR is inside the cone defined above, which can be seen as the voluntary control zone.

Given these considerations and starting from the optimization method proposed in [5], a new bounded optimization procedure can be devised for the synthesis of a 4BM for a knee prosthesis. In particular, the new procedure finds the 4BM which best-fits the natural experimental motion of the knee, among all the 4BMs whose ICRs are inside the voluntary control zone at all the considered stance phase stages. The 4BM parameters are also bounded in order to obtain a physically acceptable solution.

3 THE OPTIMIZATION PROCEDURE

Figure 1 shows the reference 4BM considered in this paper. It is known that a 4BM can be identified by 10 geometrical parameters. In this paper, all the 10 parameters are free and will be defined as a result of the optimization procedure. In the following they are identified by x_j (j=1,2,…,10). In particular:

- the four link lengths are $x_1 = ||O_3\text{-}O_1||$, $x_2 = ||A\text{-}O_1||$, $x_3 = ||B\text{-}A||$, $x_4 = ||O_3\text{-}B||$;
- the four revolute joint axes are centred at points O_1, A, B and O_3 which are identified by two Cartesian coordinates: $O_1 = (x_5, x_6)$, $A = (x_A, y_A)$, $B = (x_B, y_B)$, $O_3 = (x_{O3}, y_{O3})$;
- the angles considered in the 4BM are referred to the direction of the vector (O_3-O_1) (positive if counterclockwise) and are defined as follows: θ_2 is the angle between vectors (A-O_1) and (O_3-O_1); θ_3 is between vectors (B-A) and (O_3-O_1), and θ_4 is between vectors (O_3-B) and (O_3-O_1);
- the angle x_7 represents the orientation of the link 1 with respect to the x-axis of the global coordinate system fixed to the shank (positive if counterclockwise);

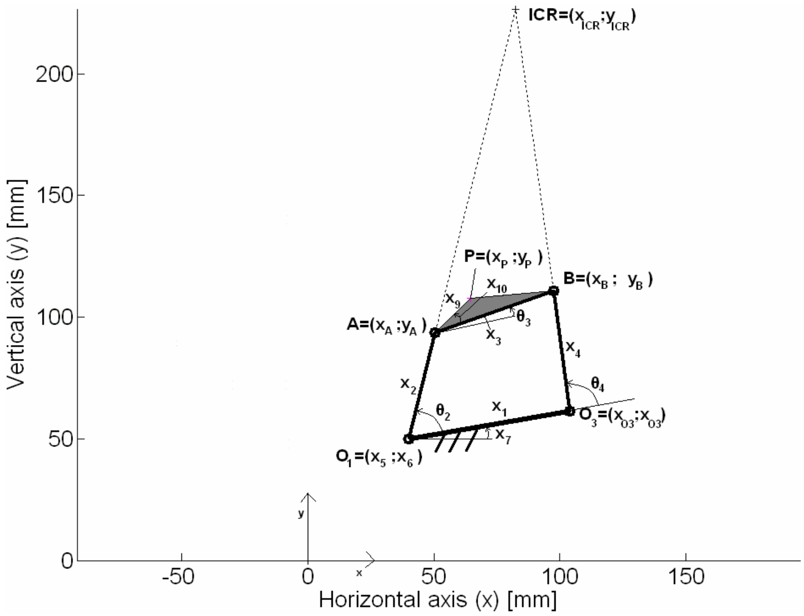

Figure 1: Four-bar linkage considered in the optimization procedure.

- the flexion angle θ_F of the knee is connected with the angle θ_3 by the relation:

$$\theta_3 = x_8 + \theta_F \tag{2}$$

  where x_8 is the starting inclination of the coupler 3 when the knee flexion angle θ_F is equal to zero;
- the coupler is defined as a rigid triangular body with vertices in A, B and P. The coupler point P = (x_P, y_P) in the 4BM corresponds to the experimental reference point used to describe the relative motion of a point of the femur with respect to the shank. The parameter x_9 represents the length of the vector (P-A) whereas the parameter x_{10} represents the angle between the vectors (P-A) and (B-A) (positive if counterclockwise);
- the instantaneous centre of rotation at the i-th considered stance phase stage, ICR_i, is represented both in Cartesian coordinates, $ICR_i = (x_{ICRi}; y_{ICRi})$, and in Polar Coordinates, $ICR_i = (\rho_{ICRi}; \beta_{ICRi})$. The origin of the Polar coordinates is variable and located at the COP_i relative to each considered stage of the stance phase; the radius is $\rho_{ICRi} = \|ICR_i - COP_i\|$; the angle β_{ICRi} is the angle between the vector ($ICR_i - COP_i$) and the horizontal axis of the global coordinate system fixed to the shank.

In order to define the objective function of the proposed optimization procedure, three sub-problems have to be solved at each considered stance phase stage: the 4BM configuration has to be obtained by solving the mechanism's closure equations; from the obtained 4BM configuration, the ICR and the P point positions have to be computed. These problems are analysed in the following subsections.

3.1 *Four-bar Linkage Closure Equations*

The 4BM showed in Figure 1 is studied considering the well-known closure equations [6]:

$$x_2 \cdot \cos(\theta_2) + x_3 \cdot \cos(\theta_3) = x_1 + x_4 \cdot \cos(\theta_4) \tag{3}$$

$$x_2 \cdot \sin(\theta_2) + x_3 \cdot \sin(\theta_3) = x_4 \cdot \sin(\theta_4) \tag{4}$$

The input of the 4BM is θ_3 (function of θ_F) and from the equations (3) and (4) a relation between the angles θ_2 and $\theta_3$is obtained as follows:

$$F_1 \cdot \sin(\theta_2) + F_2 \cdot \cos(\theta_2) = F_3 \tag{5}$$

where:

$$F_1 = \sin(\theta_3) \tag{6}$$

$$F_2 = \cos(\theta_3) - \frac{x_1}{x_3} \tag{7}$$

$$F_3 = -\frac{x_1^2 + x_2^2 + x_3^2 - x_4^2}{2 \cdot x_2 \cdot x_3} + \frac{x_1}{x_2} \cdot \cos(\theta_3) \tag{8}$$

From equation (5) it is possible to obtain the angle θ_2 as a function of θ_3:

$$\theta_2 = 2 \cdot \arctan\left(\frac{F_1 \pm \sqrt{F_1^2 + F_2^2 - F_3^2}}{F_2 + F_3} \right) \tag{9}$$

It is worth noting that the equation (9) provides two different solutions for the angle θ_2 that are the two possible configurations of the 4BM. Once the value of the angle θ_2 is chosen, it is possible to define the angle θ_4 as follows:

$$\theta_4 = \arcsin\left(\frac{x_2 \cdot \sin(\theta_2) + x_3 \cdot \sin(\theta_3)}{x_4} \right) \tag{10}$$

and

$$\theta_4 = \arccos\left(\frac{x_2 \cdot \cos(\theta_2) + x_3 \cdot \cos(\theta_3) - x_1}{x_4} \right) \tag{11}$$

Once the angle θ_2 is computed the angle θ_4 is univocally determined by (10) and (11).

3.2 *ICR Position Computation*

The ICR between the femur and shank can be obtained as the intersection of two straight lines: the first line passes through O_1 and A whereas the second one through O_3 and B. The equations of the two straight lines are represented in the global coordinate system as follows:

$$y_{line1} = \tan(\theta_2 + x_7) \cdot (x_{line1} - x_5) + x_6 \tag{12}$$

and

$$y_{line2} = \tan(\theta_4 + x_7) \cdot (x_{line2} - x_5 - x_1 \cdot \cos(x_7)) + x_6 + x_1 \cdot \sin(x_7) \tag{13}$$

It should be noted that the variables θ_2 and θ_4 are functions of θ_3 as expressed in the equations (9), (10), (11). Thus, the intersection between the two straight lines (12) and (13) represents the ICR between the femur and shank at each given angle θ_3 or, equivalently, at each given knee flexion angle θ_F. From equations (12) and (13), the ICR coordinates can be obtained as follows:

$$x_{ICR} = x_5 + \frac{\tan(\theta_4 + x_7) \cdot \cos(x_7) - \sin(x_7)}{\tan(\theta_4 + x_7) - \tan(\theta_2 + x_7)} \cdot x_1 \tag{14}$$

and

$$y_{ICR} = x_6 + \frac{\tan(\theta_2 + x_7)\cdot[\tan(\theta_4 + x_7)\cdot\cos(x_7) - \sin(x_7)]}{\tan(\theta_4 + x_7) - \tan(\theta_2 + x_7)}\cdot x_1 \tag{15}$$

3.3 *Coupler Point Position Computation*

The following equations describe the position of the coupler point P as a function of the angle θ_3 or, equivalently, the flexion angle θ_F, according to the symbols shown in Figure 1:

$$x_P = x_5 + x_2\cdot\cos(\theta_2 + x_7) + x_9\cdot\cos(\theta_3 + x_{10} + x_7) \tag{16}$$

and

$$y_P = x_6 + x_2\cdot\sin(\theta_2 + x_7) + x_9\cdot\sin(\theta_3 + x_{10} + x_7) \tag{17}$$

3.4 *Objective Function*

The objective of the proposed optimization procedure is to find a 4BM which replicates the experimental knee motion as better as possible and, at the same time, whose ICR satisfies some constraints which make it possible to satisfy the specification of functionality (section 2). As a consequence, the objective function value OF (i.e. the function to minimize) can be obtained as the sum of the squared distances between the i-th experimental position of the reference point (x_{RPi}, y_{RPi}) at the i-th experimental flexion angle and the respective position of the coupler point P at the same flexion angle (x_{Pi}, y_{Pi},), computed according to the equations (16) and (17). In order to force all ICR_i to remain inside the voluntary control zone, a penalty value is added to the objective function each time a ICR_i falls outside this zone. Thus, a possible objective function could be expressed by:

$$OF = \sum_{i=1}^{n}(f_i) + \sum_{i=1}^{n}\left((x_{P_i} - x_{RP_i})^2 + (y_{P_i} - y_{RP_i})^2\right) \tag{18}$$

where n is the total number of knee flexion angles or, equivalently, the number of considered stance phase stages; f_i is a function equal to zero if the ICR_i computed from the equations (14) and (15) belong to the voluntary control zone, otherwise it is equal to a high value. The value OF is minimized using a genetic algorithm since the objective function domain is highly discontinuous due to the discontinuities of the f_i function.

The input data are:

- the positions of the experimental reference point (x_{RPi}, y_{RPi}) at each experimental knee flexion angles θ_{Fi}, i=1,2,…,n;
- the constraints on each parameter x_j, j=1,2,…,10, which is bounded by a minimum value, x_{minj}, and a maximum value, x_{maxj}, by the following formulation:

$$x_{\min_j} < x_j < x_{\max_j} \tag{19}$$

- the constraints on the voluntary control zone.

It is worth noting that the voluntary control zone for each stance phase stage is described by polar coordinates with the origin located at the COP (which changes at each stance phase stage); as a consequence, also the Cartesian coordinates of each ICR_i are converted into Polar coordinates (ρ_{ICRi}; β_{ICRi}), as described at the beginning of this section. Moreover, a minimum centrode distance ρ_{Hi} from COP should be also considered for each stance phase stage to improve the stability and voluntary control (in particular at the beginning of the stance phase [1, 2]). The constraints for the voluntary control zone could thus be expressed by the following relations:

$$\rho_{ICR_i} \geq \rho_{H_i} \tag{20}$$

and

$$\left(\alpha_{Hip_i} - \alpha_{\max_i}\right) \leq \beta_{ICR_i} \leq \left(\alpha_{Hip_i} + \alpha_{\max_i}\right) \tag{21}$$

where α_{maxi} is the angle obtained from equation (1) at each considered stance phase stage and α_{Hipi} is the corresponding angle between the line hip joint centre-COP and the horizontal axis of the global coordinate system. The voluntary control zone is therefore represented by an annular sector centered at COP and delimited by the straight lines with inclination angles (α_{Hip}- α_{max}) and (α_{Hip}+ α_{max}) respectively, and by the circumference with radius ρ_H.

4 CASE STUDY

The presented synthesis procedure has been applied on a specimen, in order to verify the potentiality and the reliability of the proposed method. The knee passive motion of the specimen, i.e. the motion of the knee in virtually unloaded conditions, has been used as a reference: actually the passive motion is not exactly the same than the relative motion between the femur and shank during walking, but the differences between these two motions are not significant in this context and can be ignored. Relative spatial positions and orientations of the femur and shank during passive motion were collected on a human amputated lower limb. The path in the sagittal plane of the femur reference point (i.e. the mid-point between the lateral and medial epicondyles) relative to a shank anatomical frame (centred at the deepest point in the sulcus between the medial and lateral tibial intercondylar tubercles) is shown in Figure 2. The experimental data were obtained on a flexion range starting from about 5 degrees up to about 120 degrees. In order to apply these data to the stance phase, only the sagittal plane positions of the femur reference point corresponding to a knee flexion between 5 degrees and 50 degrees have been considered, according to the knee flexion angles reported in Figure 3 [7].

The voluntary control zone could be defined considering the average maximum hip moment which can be exerted by a sound person. A reference average hip moment could be from about 60 Nm up to 120 Nm [8], to which is associated an angle α_{max} from about 5 degrees up to 7 degrees (considering an average length of the lower limb [9, 10] and amputee's weight). It should be noted that for the evaluation of the mentioned angle α_{max}, the maximum GRF exerted during normal walking (equal to about 120% the weight of the amputee [11]) has been considered constant during the whole walking cycle for safety's sake: α_{max} also remains constant at all the considered stance phase stages in this case. Figure 4 shows the voluntary control zone for a knee flexion angle of about 7 degrees, the femur and shank (the shank is considered fixed), and the reference line hip joint centre-COP (the COP changes its position during the stance phase [12] so the reference line hip joint centre-COP moves during the whole walking cycle).

Starting from these data, a 4BM has been synthesized by means of the proposed optimization procedure. Figure 5 shows the synthesized 4BM with the ICR and the coupler point P relative to each considered stance phase stage. It is worth noting that the constraint given by (20) relative to the height of the ICR has been applied only for the first part of the walking cycle, by imposing ρ_H equal to the shank length plus half the femur length: in fact it is important for the amputee to have a high-stable knee prosthesis at heel contact [1, 2]. This latter constraint is shown in Figure 4 as a dashed line in the voluntary control zone relative to the heel contact stage of the stance phase.

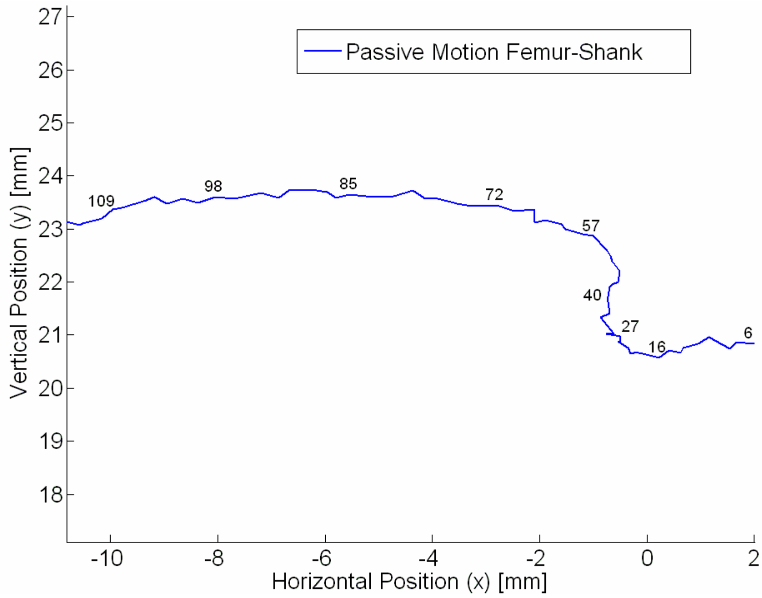

Figure 2: Path of the femur reference point during passive motion on tibial sagittal plane (the numbers near the curve represent the corresponding knee flexion angle [degrees]).

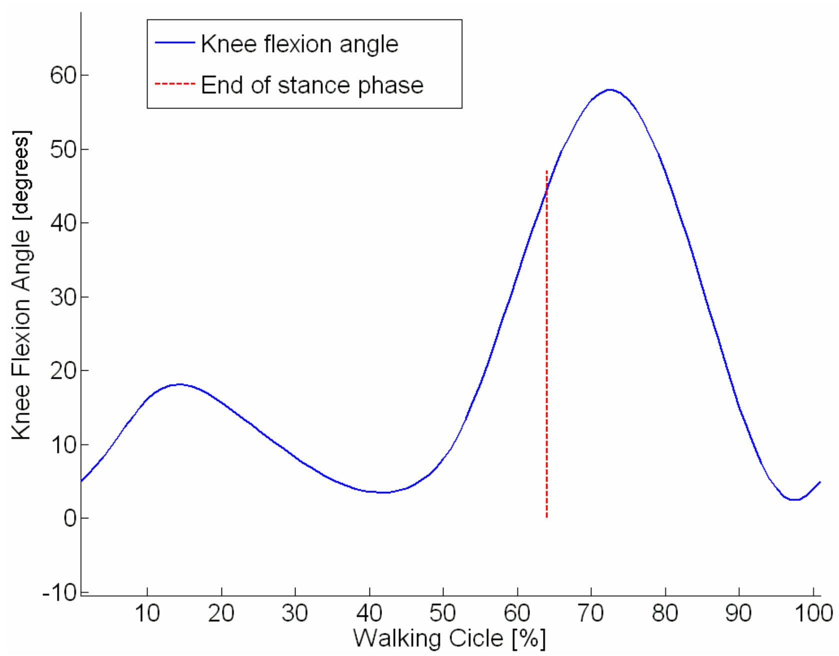

Figure 3: Path of the knee flexion angle during walking cycle.

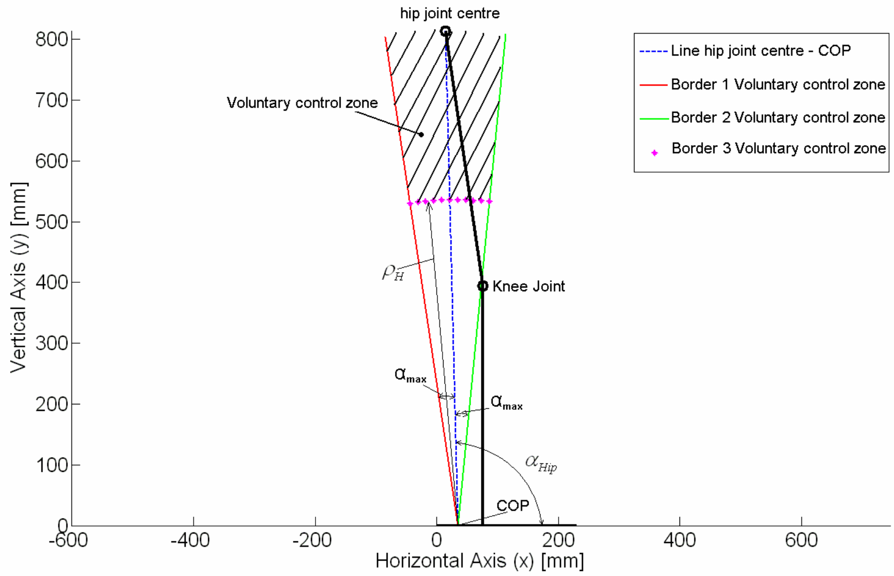

Figure 4: Voluntary control zone relative to the initial stance phase (knee flexion angle of about 7 degrees).

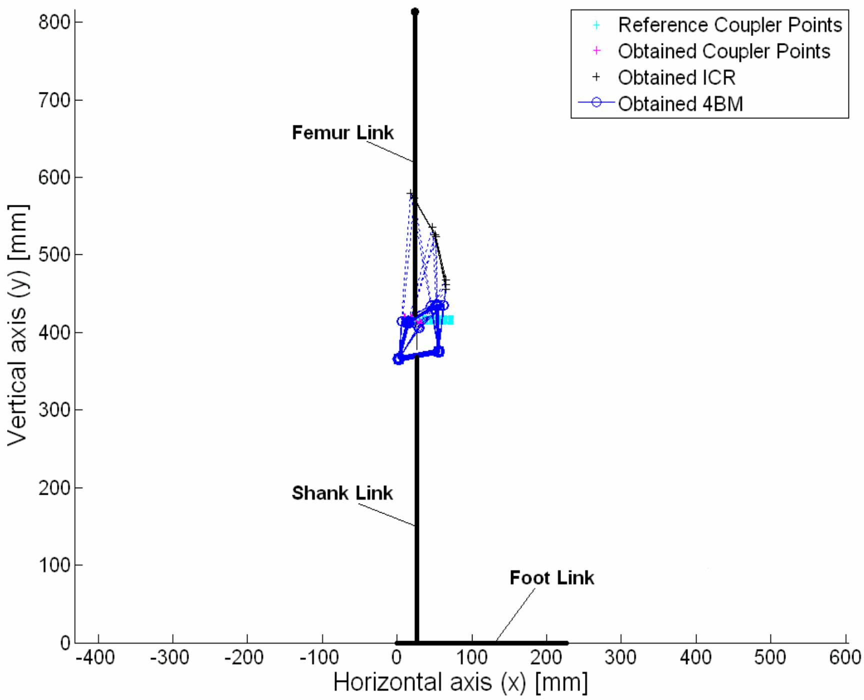

Figure 5: 4BM obtained from the optimization procedure

5 CONCLUSIONS

An optimization procedure has been presented for the synthesis of a four-bar mechanism addressed to lower limb prostheses. The procedure makes it possible to synthesize a 4BM which replicates the natural motion of the knee and, at the same time, provides the lower limb prosthesis with a requested level of stability and voluntary control. Unlike previously presented methods, the proposed procedure does not require a reference centrode: this aspect makes it possible to identify a 4BM that replicates the natural motion of the knee with a sufficiently good accuracy.

A case study has been reported considering as a reference the passive motion between the femur and shank measured on a specimen. The results confirm the accuracy and the potentiality of the proposed technique.

The next step is the application of the procedure for the design of a new knee prosthesis considering a reference patient profile.

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