Analysis of handling and vertical dynamics of a 4 steering wheels electric vehicle powered by 2 in-wheel motors

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SUMMARY. The subject of the present study is a lightweight car designed and prototyped by the Polytechnic University of Marche in collaboration with an Italian manufacturer. The car is provided with four steering wheels and two electric radial flux brushless motors placed on the rear axle [1] and is addressed to a competitive demonstration where the main evaluation criteria promote the use of new technologies in support of sustainable and efficient mobility.
Because of the difficulties linked to the design of a completely new car, especially when topics of innovation are introduced in the problem, the project has been addressed to the redesign of an existing vehicle that was equipped with an electric DC motor and a traditional transmission using differential gear. Indeed some adjustments are needed to make the car fitting with the in-wheel motors and the four wheel steering architecture. These adjustments include the redesign of rear suspension, taking into account that this will be a steering axle and that the rear wheels are driven by in-wheel motors, and the redesign of the whole chassis in order to collocate the new suspensions and the auxiliary components as the battery pack.

1 INTRODUCTION

The research of alternative technologies in the field of the automotive propulsion is an important topic of the modern scientific scenery. Because of this trend, many issues like energy efficiency and environmental impact represent a primary objective in automotive design together with the fundamental aspects concerning the user's safety and consequently the whole vehicle stability. A particular attention is payed towards electric vehicles driven by in-wheel engines. This technology adduces several benefits since several mechanical and non-efficient parts, as gearbox and clutch, are left out of the architecture of transmission.
For this reasons the ATA association has set up a competition between universities, called EHI formula, for vehicles adopting innovative solutions and having purposes of ecological sustainability and energy efficiency. The vehicle which is the main object of this paper has been designed and prototyped in order to take part at the next edition of this challenge.

2 DESCRIPTION OF THE MODEL

The design and the optimization of a complex mechanical system, as a vehicle is, evolves through the definition of different models in order to achieve the different aims that the project reveals during his development. In this work different models have been carried out to study the different aspects of the suspensions and the chassis design.

2.1 *Quarter car model*

The first problem faced was the choice of suitable suspension parameters in terms of stiffness of the spring and damping coefficient. The problem has been solved by creating a mechanical model for each suspension (*quarter car model*) able to simulate the vertical oscillations of chassis and wheels. Figure 1 shows the adopted model, in which m_s is approximately one quarter of the mass of the chassis and is connected to the wheel via a spring of stiffness k_s and a damper of constant c_s. The exact value of m_s should be set as the mass supported by each suspension, evaluated by solving the statics of the vehicle. Furthermore the tire is modeled as a spring of constant k_u and a mass m_u.

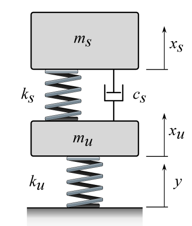


Figure 1: Mathematical model for the quarter car.

Such system is governed by the equations:

$$m_s\,\ddot{x}_s + c_s\,(\dot{x}_s - \dot{x}_u) + k_s\,(x_s - x_u) \quad = \quad 0 \tag{1}$$

$$m_u\,\ddot{x}_u + c_s\,(\dot{x}_u - \dot{x}_s) + (k_u + k_s)\,x_u - k_s\,x_s \quad = \quad k_u\,y \tag{2}$$

The system have been studied and optimized trough analyses in the frequency domain by imposing a harmonic displacement to the wheel ($y = Y\,e^{j\omega t}$). Responses have been calculated for acceleration u and relative displacement of the chassis η:

$$u^2 \quad = \quad \left(\frac{\omega_s^2}{\omega_u^2}\,\frac{\omega}{\omega_s}\right)^2 \left(\frac{4\,\zeta^2\,\frac{\omega^2}{\omega_s^2} + 1}{Z_1^2 + Z_2^2}\right)^2 \tag{3}$$

$$\eta^2 \quad = \quad \frac{\left(\frac{\omega}{\omega_s}\right)^4}{Z_1^2 + Z_2^2} \tag{4}$$

where Z_1 and Z_2 are defined as:

$$Z_1 \quad = \quad \left[\frac{\omega^2}{\omega_s^2}\left(\frac{\omega^2}{\omega_s^2}\,\frac{\omega_s^2}{\omega_u^2} - 1\right) + \left(1 - \left(1 + \frac{m_s}{m_u}\right)\,\frac{\omega^2}{\omega_s^2}\,\alpha^2\right)\right]$$

$$Z_2 \quad = \quad 2\,\zeta\,\frac{\omega}{\omega_s}\left(1 - \left(1 + \frac{m_s}{m_u}\right)\,\frac{\omega^2}{\omega_s^2}\,\frac{\omega_s^2}{\omega_u^2}\right)$$

In previous equations ω_s and ω_u are the natural pulsations of the two masses while ζ is the damping constant. The optimization strategy that was used is based on the calculation of root mean square of acceleration and relative displacement in the range of working frequencies of the suspension ($0 < f < 10\,Hz$). By defining:

$$S_u \quad = \quad RMS(u) = S_u\left(\frac{m_s}{m_u}, \frac{\omega_s}{\omega_u}, \zeta\right) \tag{5}$$

$$S_\eta \quad = \quad RMS(\eta) = S_\eta\left(\frac{m_s}{m_u}, \frac{\omega_s}{\omega_u}, \zeta\right) \tag{6}$$

the optimal design set of parameters is the result of the following optimization strategy:

$$Minimize\ S_u\ with\ respect\ to\ S_\eta \tag{7}$$

which states that the minimum absolute acceleration with respect to the relative displacement, if there is any, makes a suspension optimal [2]. Using equations 5 and 6 an optimization chart has been drawn that illustates how S_u behaves with respect to S_η by varying $\frac{\omega_s}{\omega_u}$ and ζ. It should be noted that the ratio $\frac{m_s}{m_u}$ is determined by the characteristics of the vehicle and it is not possible to modify its value. The values of ζ and $\frac{\omega_s}{\omega_u}$, given their dependence on c_s and k_s, are parameters interested by the optimization procedure and should be chosen according to the criterion adopted. By keeping $\frac{\omega_s}{\omega_u}$ constant it is possible to reach , varying ζ, the optimal point that is the minimum point of the curve.

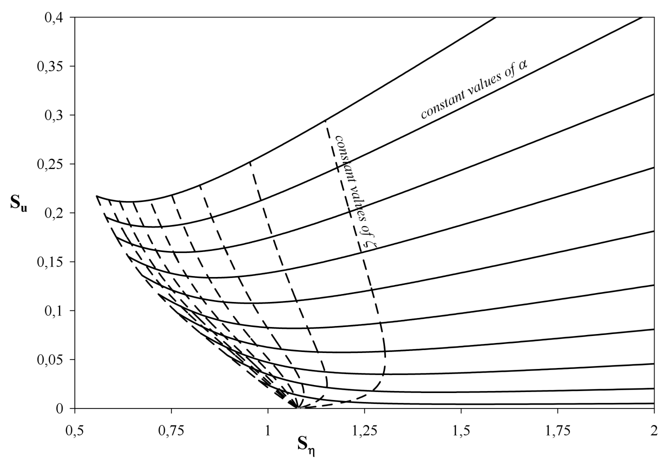


Figure 2: Design chart for optimal suspension parameters of equipments ($\alpha = \frac{\omega_s}{\omega_u}$).

The optimization procedure has been applied for the sizing of both the front and rear suspension. Because of the different unsprung mass due to the presence of in-wheel motors on the rear suspension, the two cases have led to different results in terms of stiffness and damping.

2.2 *Multibody model*

The dynamic analysis needed to design the suspension and chassis were performed using the multibody software LMS VirtualLab. This tool allows to study the dynamics of the vehicle once the whole geometry and some basic parameters such as masses, moments of inertia and suspension properties are known. In order to achieve these parameters a preliminary model of vehicle was built which had the dual purpose of being the starting point for the sequent refinement driven by the results obtained by dynamic analysis and providing the mass properties and the geometry on which analysis could be performed.

The analysis performed made it possible to compare the dynamic behavior of the car with the kinematic one. Indeed it is known that on a curve the hypothesis of kinematic steering is valid only when the feed rate of the car is low enough to consider the lateral acceleration equal to zero. If this occurres the slip angles of the tires may be not considered and Ackerman equation is valid; otherwise it is true that:

$$\delta = \frac{360}{2\,\pi}\,\frac{w\,r}{V} + K\,\dot{v}_y \tag{8}$$

where δ is the steer angle (for 4 wheel steering is a function of the steering angle of the front and the rear), w is the wheelbase of the car, r is the yaw rate, V is the feed rate and $\dot{v}_y$ is the lateral acceleration [3]. K is the **stability factor**: it depends on the position of the center of mass and on the cornering stiffness of the tyres [4]. The value it assumes is indicative of vehicle behavior (over-steering, under-steering or neutral) [5].
Once a value of speed is fixed (usually $V = 20\ m/s$), the curve described by the equation 8 can be obtained by carrying out several simulations with a constant feed rate and different steer angles. For each simulation the value of lateral acceleration have been registered and plotted with respect to the respective steer angle δ.
To achieve this aim, it has been necessary to control two fundamental magnitudes in the model: the steering angle and the feed rate [6]. Actually, only the speed has been controlled: in fact the degree of freedom of the steer has been constrained so that it could take only one fixed configuration. To maintain the speed constant, instead it was necessary to implement a simple feedback control system which provided the wheels with the torque required to maintain a constant speed even with a not straight trajectory (*cruise control*).
In this way the control system simulates the task that in real vehicles is performed by mechanical differential (Figure 3).

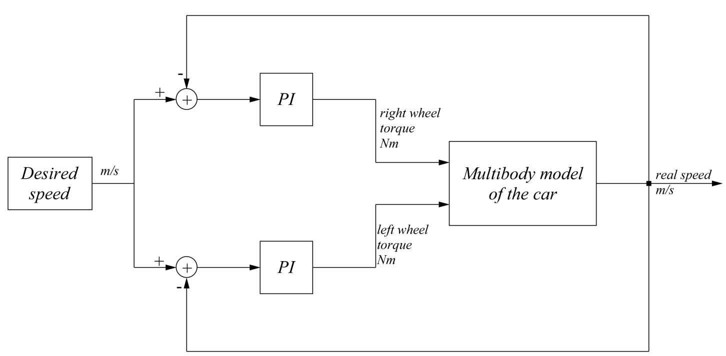


Figure 3: Control scheme adopted in dynamic analysis.

2.3 *Steering management on the rear axle*

If the rear axle is equipped with a steering system it needs to be actuated according with a control strategy that usually depends on vehicle advance velocity and steer angle of the front axle [7]. In general, two design solutions could be possible:

- *positive four wheel steering vehicle*: in this situation the front and rear wheels steer in the same direction and the center of rotation is situated behind the rear axle;

- *negative four wheel steering vehicle*: the front and rear wheels steer opposite to each other, the center of rotation is situated between the two axles.

Both solutions include pros and cons: the solution with negative steering is not recommended at high speed because of the high yaw rate involved, negative steering is not recommended at low speed because it reduces the ability to maneuver. For these reasons it was chosen to study a management system that reduced the negative aspects of both cases by changing the ratio between front and rear steering angle depending on feed rate. Such a system is called *Active Steering System* and the benefits that it entails are:

- increased stability when cornering at high speeds because of higher radius of curve obtained and the resulting lower centrifugal acceleration;
- increased maneuverability at low speeds, when the most important characteristic of the car is the ability to make curves with a radius as small as possible.

To implement this system it has been necessary to trace a curve describing how ratio between front and rear steering angles behaves when speed varies. Usually the rear axle is not built to enable steering angles as large as the front axles. It is rather common that the rear wheels do not exceed angles of about $5°$, while the limit for front axles can also reach angles of $30°$. Because of these practical considerations it was agreed that the ratio between front and rear angle should not go beyond 16.7% either in the case of positive or negative steering.
This ratio represents the maximum allowed and it is reached only when the car is stopped or moving at full speed that is expected about $20\,m/s$. The points at intermediate speed between the two limit values are interpolated and then, for a large tract, the rear steering angle becomes almost zero or less than $5°$. However it has been verified that the effect of the 4 wheel steering solution is not negligible if compared to the traditional case of 2 wheel steering. Figure 4 shows the relation adopted between V and $\frac{\delta_f}{\delta_r}$ (where δ_f and δ_p are the front and the rear steering angles, respectively). This pattern of behavior was introduced in the multibody model to perform the dynamic analysis in line with the physical configuration of the vehicle.

3 DESIGN OF SUSPENSION AND CHASSIS

The models so far described have been useful for chassis and suspension design since they have permitted to take into account the influence of the variation of some quantities on the dynamic performances of the vehicle. Some of these quantities concerned the disposition of the suspension within the chassis: in particular it has been investigated how the toe angle of the wheels, the camber angle and the caster angle affect the steering stability. In this way it has been possible to find the proper configuration for the kingpin axis, which is the axis around which the wheel rotates during steering maneuvers.

3.1 *Suspension optimization*

As was explained above, the optimization of the parameters of stiffness and damping of the suspension can be reached through the study of the vertical dynamics of the vehicle and in particular through the analysis of the frequency response of acceleration and relative displacement of the chassis.
Firstly the parameters needed to calculate equations 3 and 4 have been assessed for several values of ζ and $\frac{\omega_s}{\omega_u}$. Sequently equations 5 and 6 have been applied to evaluate the root mean square of each

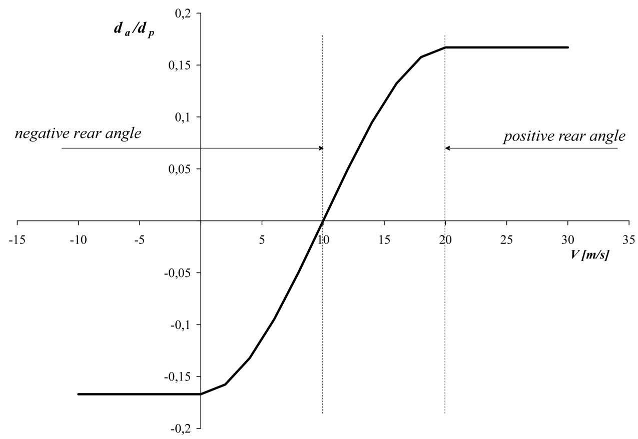


Figure 4: Management policy of the rear steering.

frequency response. These values have been plotted on a graph S_η-S_u where the curves show the mutual evolution of the two parameters by varying ζ and $\frac{\omega_s}{\omega_u}$. The same procedure was performed for both the front and the rear suspension in order to find, in the two different cases, the optimization chart that is necessary for the choice of stiffness k_s and damping c_s (Figures 5 and 6). The optimal point for a given stiffness k_s, and then for a given value of $\frac{\omega_s}{\omega_u}$, is the minimum of the corresponding curve. Interpolating the minimum points the optimal design curves have been obtained through which suspensions were decided. The design results are presented in Table 1.

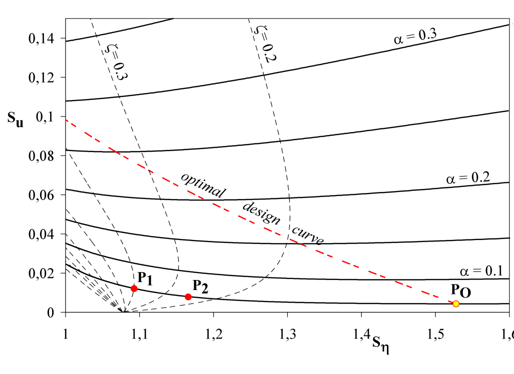


Figure 5: Optimal curve for the front suspension: difference between the starting point P_1, the optimal point P_o and the chosen point P_2.

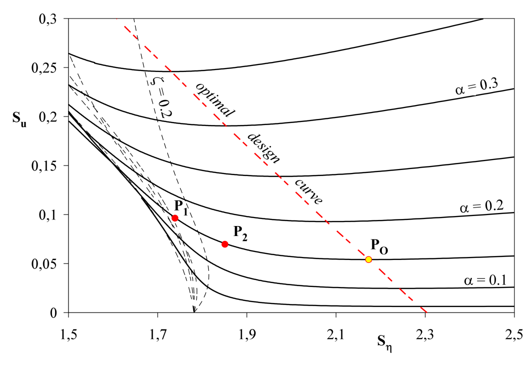


Figure 6: Optimal curve for the rear suspension: difference between the starting point P_1, the optimal point P_o and the chosen point P_2.

The optimization procedure led to results for the damping parameter that were not acceptable for an automotive application. However, since the curves are nearly horizontal in the area interested by the project, values of c_s almost equivalent in terms of S_u have been chosen: these values, even if not

Front Suspension		
problem data	m_s	$97.4\,kg$
	m_u	$10\,kg$
	k_u	$200\,kN/m$
design data	k_s	$6000\,N/m$
	c_s	$400\,kg/s$
	$\frac{\omega_s}{\omega_u}$	0.05
	ζ	0.26

Rear Suspension		
problem data	m_s	$72.6\,kg$
	m_u	$40\,kg$
	k_u	$200\,kN/m$
design data	k_s	$8000\,N/m$
	c_s	$500\,kg/s$
	$\frac{\omega_s}{\omega_u}$	0.15
	ζ	0.22

Table 1: Design results

optimal, seemed to be more indicated for the application.
In order to verify that a right choice has been actually made, frequency response were calculated for the relative displacement of the chassis and even for other variables of interest as pitch and roll angles.

3.2 *Analysis of the steering dynamics*

The study of the vertical dynamic is not sufficient to determine if the road behavior is also acceptable for the security, the stability and the maneuverability of the car. For this reason, the structure of the chassis and the suspension architecture has been refined through the study of the steering dynamics.

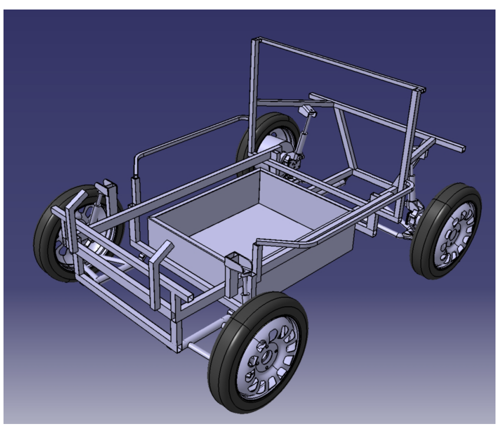


Figure 7: CAD model used for determination of dimensions and mass properties.

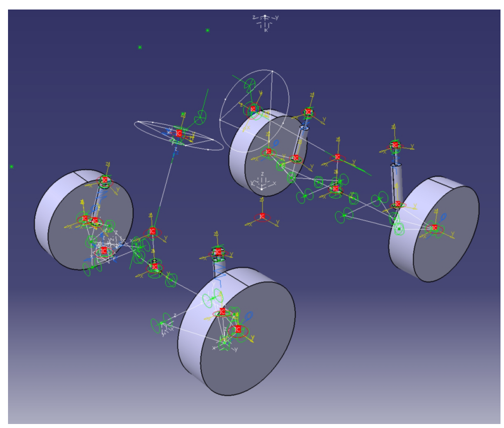
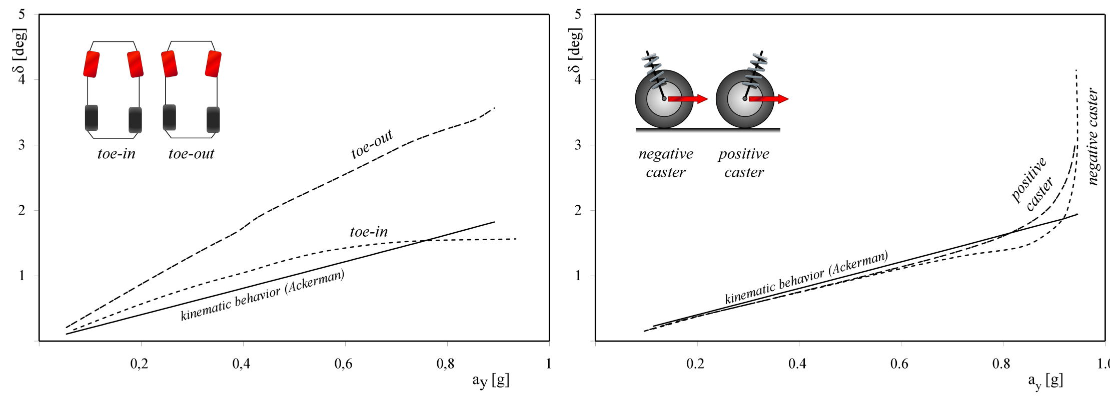
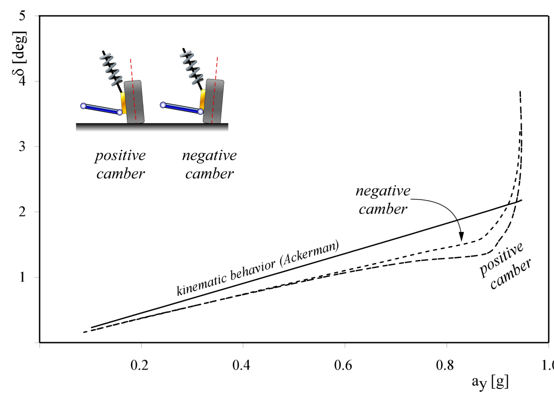


Figure 8: Multibody model used in dynamic analysis.

To make appropriate choices with regard to arrangement of the suspension, the influence of the variation of each angle has been studied separately. The information collected during the dynamic analysis allowed to select the appropriate value of the parameters that still were under design (*toe*, *caster* and *camber* angles). As examples, some $\dot{v}_y$-δ graphs are shown which were obtained with the variation of the three angles; the set of parameters chosen for the front and rear suspension were as follows (Figure 9).





	Front	***Rear***
Toe-in	0.6°	0°
Caster	5°	0°
Camber	1°	0°

Figure 9: Analysis of influence of the arrangement of the suspension on cornering stability. Results are shown about the designed angles for front and rear suspensions.

The influence of wheel motors have been studied similarly to the variation of the angles of the suspension and with this aim the curve $\dot{v}_y$-δ has been plotted for a model which involves a traditional type of actuation. The multibody model has been changed accordingly: masses and moments of inertia of the rear wheels have been modified and a weight equivalent to that of the engines has been added to the chassis. The behavior in absence of wheel motors was improved if compared to the case where these were not used. In fact the car without wheel motors took a behavior comparable to that kinematic for a wide stretch of the field of lateral acceleration. The cause of this fact lies in the lower unsprung mass on the rear axle. In fact the decrease in the weight of the wheel and hub makes the frequency of its vertical vibration grow and the amplitude of oscillation decrease. However it has to be stressed that at large accelerations the car equipped with hub-motors behaves more stably, as indicated by the lower tendency to over-steer, as shown in Figure 10. In Figure 11 it has been shown the frequency response of relative displacement η. It should be noted that the difference between the two curves is in the value of the natural frequency of the wheel which is shifted to lower frequencies in the case of traditional power train.

The presence of the steering system on the rear axle has changed significantly the maneuverability mainly because of use of the active steering system. This can be noted by observing the curvature

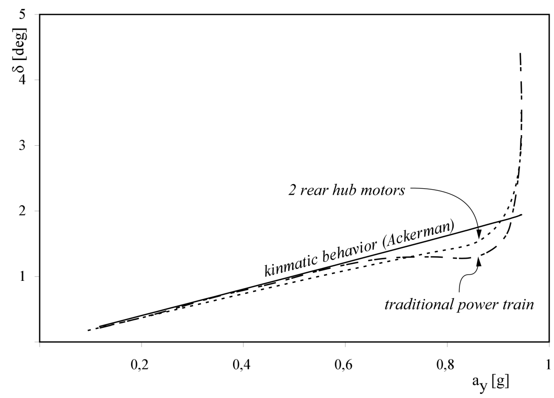


Figure 10: Influence of usage of in wheel motors on cornering stability.

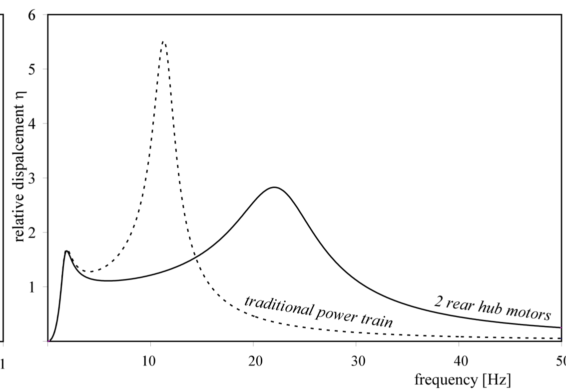


Figure 11: Frequency response of the relative displacement η.

response of the car for various advance speed (Figure 12). The curvature response is defined as $S_\kappa = \frac{\kappa}{\delta}$ where κ is the curvature of the trajectory (inverse of the radius of curve) caused by the steering angle δ. To obtain this parameter, simulations have been performed in conditions of constant speed and steer angle (exactly as it has been done for $\dot{v}_y$-δ curves) either using rear steering axle or not. The resulting curves showed the improved performances of car. In fact we noted that for low velocities S_κ was greater in case of 4 wheels steering with respect to the traditional 2 wheels steering (index of higher maneuverability). On the other hand it was lower for high velocities (higher stability). In the intermediate range of speeds the two curves overlapped and therefore the influence of the rear steering was negligible.

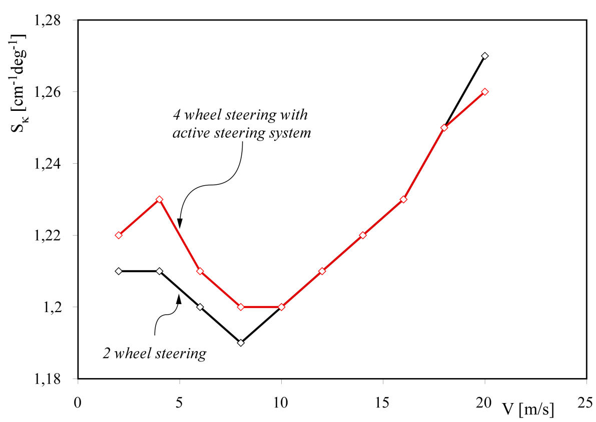


Figure 12: Comparison of the curvature response S_κ between 4 wheel steering and 2 wheel steering vehicles.

4 CONCLUSIONS

Remark 1. The suspension were optimized as required by the common practice. Vertical dynamics was investigated to produce results in terms of stiffness and damping. Once this parameters were estimated, it has been possible to study the dynamics of whoole vehicle in order to refine the architecture of chassis and suspensions.

Remark 2. The design of the arrangement of the suspension angles was carried out thanks to the study of the steering dynamics of the vehicle. The simulations run in multibody environment demonstrated that there was a real advantage in using a 4 wheel steering architecture. Actually, this advantage was appreciable only if an active steering system was used. Such system was designed in order to reach both the aims of stability at high speeds and high maneuverability at low speeds.

Remark 3. It is known that the presence of a huge unsprung mass could have a negative effect on the vehicle behavior. This phenomenon was studied either in term of vertical dynamics or in terms of cornering stability. Results obtained showed that the usage of this kind of motors increases the natural frequency of wheel vibration even though their amplitude decreases. However the stability of the vehicle didn't seem to have very negative effects as shown by comparison with a model devoid of this type of actuation.

Remark 4. At the end of the work some finite element analysis [9] have been performed to verify resistance of those parts that resulted critical because of the stress they support during work.

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