Synthesis of single-loop kinematotropic mechanisms

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SUMMARY. In this work we present 3 single loop kinematotropic mechanisms, i.e., mechanisms that can be assembled in different closures in which they present different numbers of degrees of freedom. The mechanisms can be located in their different closures by assembling their pairs in different ways consistent with the kinematic constraints. The three chains have been synthesized using the theory of displacement groups and their mobility characteristics are analyzed. All the chains are 4-bar non planar linkages, i.,e., RRRR, RPRP, RFRF.

1 INTRODUCTION

Kinematotropy is a topic in mechanism kinematics that has been investigated for several years . Its interest is mainly theoretical, because it represents an exception to mobility rules and constitutes a challenge to any research for comprehensive mobility equations (or algorithms) [1]. In recent years, an interest is growing toward metamorphic mechanisms, specially for robotic applications, of which kinematotropic mechanisms are a subset [2, 3, 4]. In fact, the possibility of obtaining mechanisms that, like an anthropomorphic hand, change their number of degrees of freedom when performing different tasks - e. g., for grasping - is of notable interest.

Kinematotropic mechanisms can be divided in two types according to the following definitions:

A: chains in which variations in the position variables can result in changes in the permanent mobility of the chains. This definition, given by Wohlhart [5]) is the most known and useful for robotic applications. A kinematotropic mechanisms of this type can be driven in a continuous branch of positions with a certain number of drivers, applied in certain location of its kinematic chain; then, passing through a singular position, a different number of drivers, often applied in different locations are necessary to drive the mechanism.

B: chains in which different closures present different numbers of degrees of freedom. According to this definition the mechanism has a certain number of degrees of freedom, but it can be disassembled and reassembled in such a way that its number of degrees of freedom is different.

A more general definition of kinematotropy will contains two other types of chains:

C and D: chains of type A or B in which the permanent mobility does not change, but the displacement group of at least one link or its invariant properties are changed [6] in different branches.

Single-loop and multiloop kinematotropic mechanisms according to definition A have been studied extensively, basic set of kinematic chains with this property have been found [7] and rules to form complex chains have been developed [8]. Minor attention has been devoted to kinematotropic mechanisms according to definitions B and C [9]. This paper deals with mechanisms of type B that will be called B-kinematotropic chain.

It is to be noted that it is very easy to obtain B-kinematotropic chains if one allows one or more links of the chain to be stretched. Obviously this is not permissible in a consistent rigid body model, then, this possibility is banned in this work. The only permissible assembly operations are three (Fig. 1):

- i) the links of a prismatic pair can be assembled by reversing their axes or giving them any rotation consistent with the pair shape; for our purposes only rotations multiple of 90° are considered;

- ii) the links of a revolute pair can be assembled giving their axes an orientation equal to zero or to 180°;

- iii) the links of a planar pair can be assembled giving their axes an orientation equal to zero or to 180°.

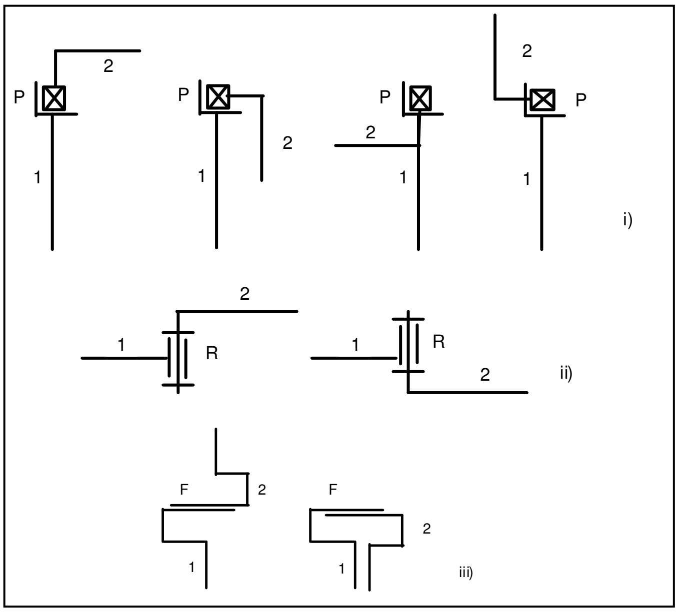

Figure 1: Permissible assembly of prismatic (i), revolute (ii) and planar pairs (iii).

In this paper we present 3 single-loop kinematotropic mechanisms of type B that are synthesized by means of the displacement group theory, whose foundations are discussed in [10]. In Section 2 we outline a small subset of the group properties that are used for the synthesis, in Section 3 we discuss how to form the 3 B-kinematotropic chains and how to assemble them in two branches of positions in such a way that they exhibit different degrees of freedom. In Section 4 we draw our conclusions.

2 DISPLACEMENT GROUPS

Since 1978 [10] the theory of displacement groups has had interesting applications in various field of mechanism kinematics and dynamics: mobility analysis, setting up and solution of closure equations, manipulations of equations of motion, and so on. For the purpose of this work we need only a small amount of the results obtained in [11] regarding the mobility that arises when two groups are intersected. In particular only few conditions are utilized, concerning the groups: revolute, R, prismatic, P, cylindrical, C, and planar, F. They are reported in Table 1.

Table 1: Intersections of groups

		G1	G2	Geometric condition	Intersection	Connectivity
1	G*	R	R	coincident axes	R	1
2	G**	R	R	non coincident axes	-	0
3	G*	P	P	parallel axes	P	1
4	G**	P	P	non parallel axes	-	0
5	G*	F	F	parallel planes	F	3
6	G**	F	F	non parallel planes	P	2
7	G*	C	C	coincident axes	C	2
8	G**	C	C	parallel non-coincident axes	P	1

In rows 1 and 2 of Table 1 we see that the intersection of two revolutes establishes a revolute with connectivity 1 if the axes of the revolutes are coincident (the intersection group is G*). Otherwise, the intersection is empty and the connectivity is 0 (case G**). Rows 3 and 4 show that a P intersection exists if two prismatic pairs are parallel (case G*), otherwise the intersection is empty. In rows 5 and 6 the case of two planar pairs F is reported: if the planes are parallel the intersection is the plane F and mobility is 3, otherwise the intersection is a prismatic group with connectivity 1, whose direction lies on both planes. Rows 7 and 8 show the intersections of two cylindrical groups: when the axes coincide the intersection is the same group with connectivity 2, if the axes are parallel but non-coincident the intersection is a prismatic group with connectivity 1.

3 SYNTHESIS OF THE CHAINS

The single-loop chains can be synthesized using the results reported in Section 2, with the following procedure.

S1) from Table 1, take into consideration two rows where the intersections of two groups G1 and G2 can be G* or G** with different connectivities;

S2) form an open kinematic chain c1 made up with a body 1, a pair A generating the group G1, a body 2;

S3) form an open kinematic chain c2 made up with a body 3, a pair D generating the group G2, a body 4;

S4) verify if it is possible to connect 2 and 3 with a pair C, and 4 and 1 with a pair B in such a way that by different permissible assemblies of pairs C and B the groups G1 and G2 have the intersection G* or G**.

S5) If the result of S4 is true, the closed chain 1-A-2-C-3-D-4-B-1 is kinematotropic according to definition B.

Following this procedure 3 B-kinematotropic chains are easily synthesized.

3.1 *RRRR chain*

Figure 2 reports a chain synthesized by considering the intersection cases stated by the first two rows of Table 1. All links have equal lengths. The chain assembled in the position in Fig. 2-a has mobility equal to zero (non-empty intersections do not exist between any couple of pairs, according to row 2 of Table 2, and no other intersections are possible). The pairs C and B are disassembled (Fig. 2-b) and the two open chains are displaced until the axes of pairs A and D coincide. The subchain 3-D-4 is rotated (180°) according to Fig. 2-c, in such a way that the axis of pair D is reversed (Fig. 1-d). In this position pair C can be assembled. Link 3 is rotated (90°), Fig. 2-d, and a new position is achieved (Fig. 2-e) in which also pair B can be reassembled. In such position the chain is closed. but now pairs A and D have coincident axes. According to row 1 of Table 1, the chain has mobility equal to 1 and can rotate around the axis common to A and D.

The connectivities of the chain in the mobile branch are reported in Table 2.

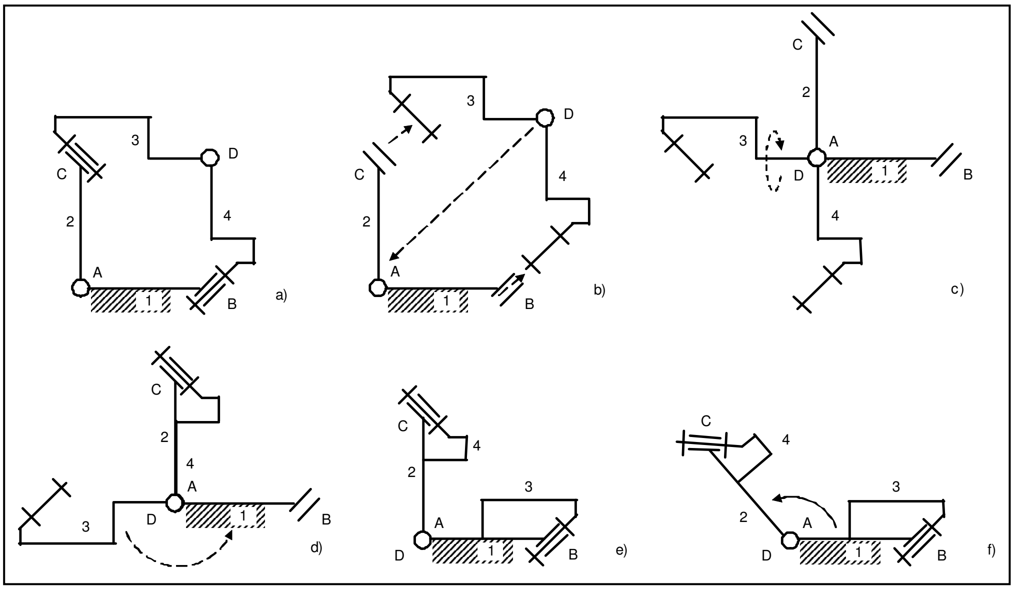

Figure 2: RRRR chain.

Table 2: Connectivities of RRRR chain in the mobile branch.

	1	2	3	4
1	-	R	0	R
2	R	-	R	0
3	0	R	-	R
4	R	0	R	-

3.2 *RPRP chain*

This chain is obtained by considering again the change of mobility between revolute groups according to rows 1 and 2 of Table 1, as in the previous case. Now, prismatic pairs C and B are used instead of revolutes (Fig. 3). The chain is assembled in the position shown in Fig. 3-a. The revolute A and prismatic pair C generate a cylindrical group, revolute D and prismatic pair B generate a second cylindrical group; the two groups have parallel non-coincident axes, and according to row 8 of Table 1, their intersection is a prismatic group. The mobility of the chain is 1. The types of connectivities among links are reported in Table 3.

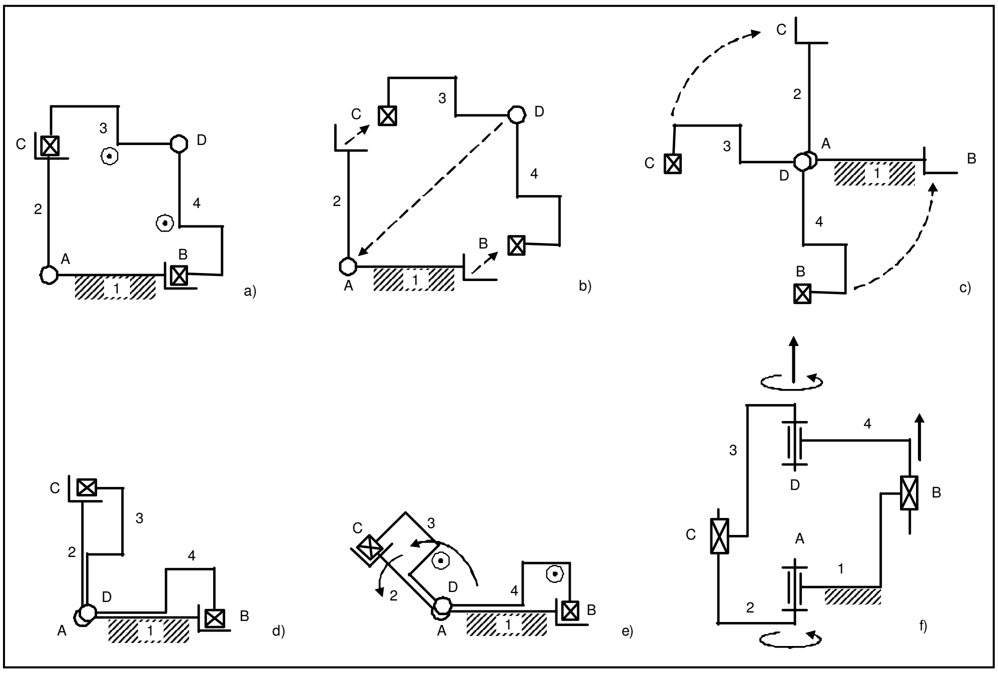

Figure 3: RPRP chain.

Table 3: Connectivities of RPRP chain in the branch with mobility 1

	1	2	3	4
1	-	0	P	P
2	0	-	P	P
3	R	P	-	0
4	P	P	0	-

The pairs C and B are disassembled (Fig. 3-b) and the two open chains are displaced until the axes of pairs A and D coincide. Both links of the subchain 3-D-4 are rotated (90°) according to Fig. 3-c. In this position pairs C and B can be assembled, Fig. 3-d. In such position the chain is closed. but now pairs A and D have coincident axes. Now the chain is formed by two cylindrical groups with coincident axes, and according to row 7 of Table 1, the chain has mobility equal to 2. It can rotate around the axis common to A and D in the plane of the figure and can translate in direction perpendicular to it, achieving a cylindrical motion. Figure 3-f shows a lateral view of the chain to make the chain displacements easier to understand. The connectivities of the chain in the branch with mobility 2 are reported in Table 4.

Table 4: Connectivities of RPRP chain in the branch with mobility 2

	1	2	3	4
1	-	C	R	P
2	C	-	P	R
3	R	P	-	C
4	P	R	C	-

3.3 *RFRF chain*

This chain is obtained by considering the change of mobility between two planar groups F according to rows 5 and 6 of Table 1. Two revolute A and D are used to assemble the sub-chains made of two planar pairs B and C. In position shown in Fig. 4-a the planes of the pairs B and C are not parallel and, according to row 6 of Table 1, their intersection allows only a translation in one direction (perpendicular to the plane π of the figure) of links 3 and 4. A further mobility arises because 2 rotations and 2 translations are possible in the plane π. Therefore, the chain performs also as a four bar linkage in the plane π, and the mobility of the chain is 2. The connectivities of the chain are reported in Table 5.

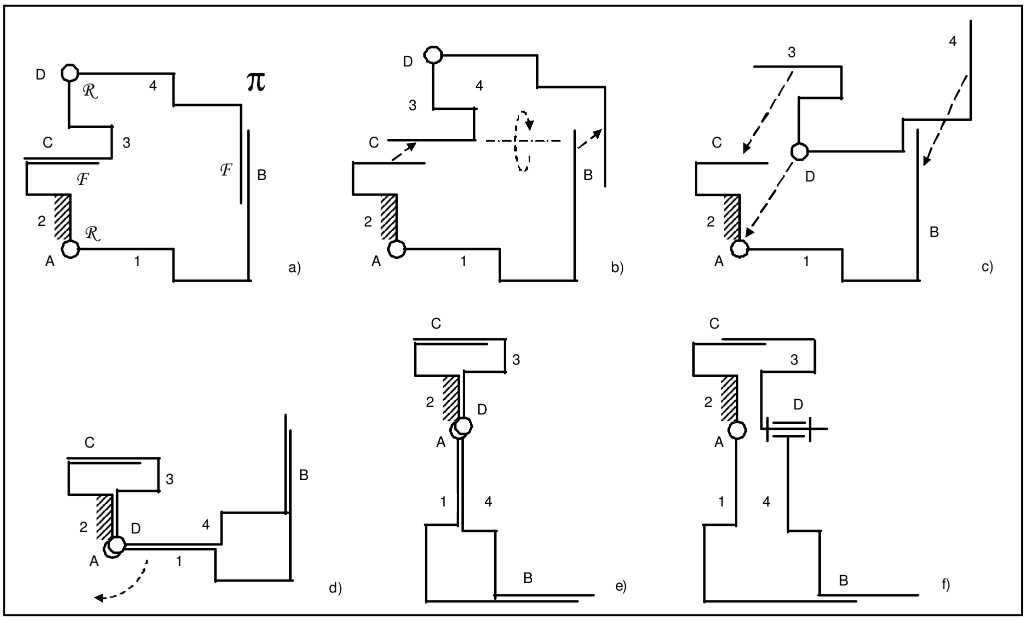

Figure 4: RFRF chain.

The displacements between links 1 and 3, and links 2 and 4, are composition of a rotational motion in π with mobile axis and with mobility 1 and a translational motion perpendicular to π. Therefore, the resulting displacements are a subset, with connectivity 2, of a Schonflies group, reported as X,2 in Table 5.

Table 5: Connectivities of RFRF chain in the branch with mobility 2

	1	2	3	4
1	-	R	X,2	P2
2	R	-	P2	X,2
3	X,2	P2	-	R
4	P2	X,2	R	-

The planar pairs C and B are disassembled (Fig. 4-b) and one of the two open chains is rotated (180°) along the dashed line in Fig. 4-b, obtaining the position in Fig. 4-c, in which the planar pairs are reassembled. Now pairs A and D have coincident axes, Fig. 4-d, and a rotation (90°) is given to links in order to make parallel the planes of the pairs B and C. According to row 5 of Table 1, the chain has mobility equal to 3 and can perform a full planar motion parallel to the common plane of the pairs. Figure 4-f shows a displacement of the chain.

The connectivities of the chain in this branch of positions are reported in Table 6.

Table 6: Connectivities of RFRF chain in the branch with mobility 3

	1	2	3	4
1	-	0	F	F
2	0	-	F	F
3	F	F	-	0
4	F	F	0	-

4 CONCLUSIONS

Using results obtained with the theory of the displacement groups we synthesized 3 B-kinematotropic four-bar mechanisms. All chains are non-planar. In the first mechanism, an RRRR chain, the mobility changes from 0 to 1; in the second one, a RPRP chain, the mobility changes from 1 to 2; in the third one, a RFRF chain, the mobility changes from 2 to 3.

All these chains embody exceptions to the rules for computing the number of degrees of freedom by equations or by algorithms.

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