Workspace computation in parallel manipulators with three translational degrees of freedom

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SUMMARY - One of the weak aspects of the parallel robots is ratio between the workspace and the dimension of the mechanical structure, if compared with the serial one. Hence the shape of the workspace of a parallel manipulator is one of the most important aspects to reflect its working capacity. This paper introduces two innovative methods to analyze the workspace of some family of parallel robots, with the aim of highlighting its shape.

INTRODUCTION

The very first theoretical studies on mechanisms with parallel kinematics go back to two hundred years ago and were made by English and French mathematicians. In particular, the first article appeared in 1813. In that work Cauchy examined the motion possibilities and stiffness aspects of an articulated octahedron. The first parallel spatial mechanism was invented by James E. Gwinnet in 1928. Some years later Willard L.V. Pollard realized the first industrial parallel robot. This manipulator, now called Tripod, was made up of three kinematics chains featured with two links interconnected by universal joints and actuated by the same number of motors placed on the base of the robot.

In 1947 Eric Gough presented the first articulated octahedron: the Exapod, a mechanism with six extensible links for tyre orientation during wear tests [1]. In 1965 D. Steward studied the possibility of employing a platform with six degrees of freedom for the realization of a flight simulator [2]. These two scholars proposed several solutions for novel industrial applications. The most famous among these is the Gough-Stewart platform. In practical applications these solutions did not achieve the expected results, because of the complexity in the kinematics relations. As a matter of fact, it was not possible to obtain a real-time solution of the kinematic problem given the limited calculation resources available at that time.

In 1989 Raymond Clavel developed the first Delta Robot [3]. This novel architecture featured with three degrees of freedom leaded to a second generation of parallel mechanisms. Industrial realizations of such a manipulator architecture allowed achieving outstanding performances (e.g. a maximum speed of 10m/s and a maximum acceleration of approximately 10g) and are suitable for those applications where quick and small movements are required.

Several methods [4, 5, 6, 7] have been proposed in literature for the workspace determining of parallel manipulators. This paper presents two methods for this purpose. Both are based on the position inverse kinematics. This because, in presence of parallel architecture, the inverse kinematics is easier to be identified with respect to the direct one.

The first method takes into account a spatial region and it distinguishes, by means of the inverse kinematics, between the points that belong to the workspace and the ones that do not. Afterwards, the same region is sliced by horizontal planes. This way it is possible to graphically highlight the frontier of the workspace by drawing the points in which the planes pass from the first to the second region. This approach is very duty from an computational point of view, and does not achieve an high accuracy level, but it gives a graphical representation of the workspace directly from the kinematic equations.

On the other hand, the second method requires to elaborate the inverse kinematic. Let us consider the kinematic in terms of $[\theta_1, \theta_2, \theta_3] = f(x, y, z)$. This novel approach consists in subdividing the workspace in region by analyzing the symmetry of the mechanical structure. For each of this region it is possible to find which actuator limits the movement of the robot. This way, in the inverse kinematic, a known terms could be changed with an unknown. As a matter of fact it is possible to find the equation of the frontier by means of the equation $[z, \theta_j, \theta_k] = f(x, y, \theta_i)$ by changing the value of i, j and k for each region. In these regions, the i-value is fixed so as to find separately the highest and the lowest z coordinate, that can be reached by the end-effector, given the x and the y coordinates ($z_h = f_h(x, y)$ and $z_l = f_l(x, y)$).

1 THE 3DOF TESTBED MANIPULATORS

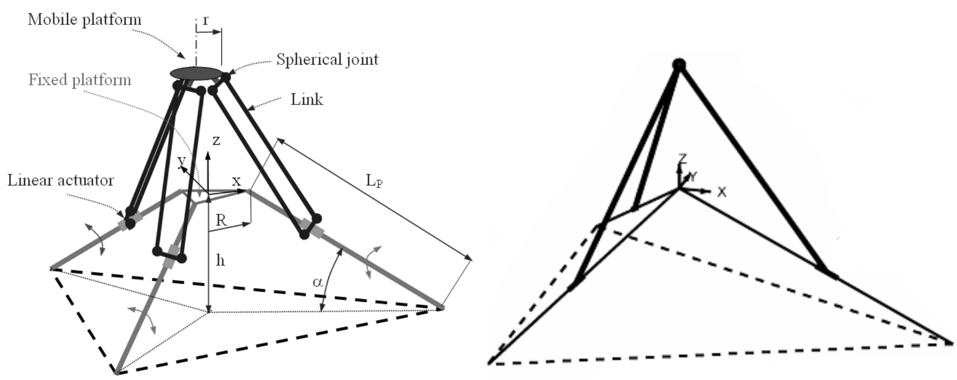

Figure 1: The 3dof parallel linear manipulator with its simplified model

Figure 1 shows a parallel manipulator, used as testbed, with its simplified model. This manipulator has some features in common with Cheope [8] and is well described in [9]. Here follows the equation of the inverse kinematics for the three dof parallel linear manipulator :

$$q_1 = x \cdot \cos(\frac{\pi}{6}) - z \cdot \sin(\frac{\pi}{6}) + \sqrt{(z \cdot \sin(\frac{\pi}{6}) - x \cdot \cos(\frac{\pi}{6}))^2 - x^2 - y^2 - z^2 + l^2} \quad (1)$$

$$\begin{aligned} q_2 &= y \cdot \cos^2(\frac{\pi}{6}) - x \cdot \sin(\frac{\pi}{6}) \cdot \cos(\frac{\pi}{6}) - z \cdot \sin(\frac{\pi}{6}) + \\ &+ \sqrt{(z \cdot \sin(\frac{\pi}{6}) + x \cdot \sin(\frac{\pi}{6}) \cdot \cos(\frac{\pi}{6}) - y \cdot \cos^2(\frac{\pi}{6}))^2 - x^2 - y^2 - z^2 + l^2} \quad (2) \\ q_3 &= -y \cdot \cos^2(\frac{\pi}{6}) - x \cdot \sin(\frac{\pi}{6}) \cdot \cos(\frac{\pi}{6}) - z \cdot \sin(\frac{\pi}{6}) + \\ &+ \sqrt{(z \cdot \sin(\frac{\pi}{6}) + x \cdot \sin(\frac{\pi}{6}) \cdot \cos(\frac{\pi}{6}) - y \cdot \cos^2(\frac{\pi}{6}))^2 - x^2 - y^2 - z^2 + l^2} \quad (3) \end{aligned}$$

The other testbeds are a classic Delta Robot [3] and the Adept Quattro manipulator [10].

2 THE FAST METHOD

The first approach takes into account the set of points near the robot, and it verify if these points can be reached by the manipulator. As the matter of fact, by means of the inverse kinematics, given the coordinates x y and z of each point, it is possible to calculate the value of θ_1 θ_2 and θ_3. If these values exist and are inside a proper range, the considered point can be reached, and so it belongs to the workspace. This way, given the inverse kinematics of a manipulator, it is possible to highlight directly the workspace. On the other hand this method is computationally complex, because the inverse kinematic should be calculated of each point. Dense points lead simultaneously to a very accurate representation and to an high numbers of calculation.

Interesting results in representation are reached by means of a certain number of horizontal planes in which are the points that belong to the workspace in contact with points that do not belong to. In figure 2 is represented the frontier of the parallel linear manipulator with horizontal planes at different distance. Figure 3 shows the workspace frontier of a classic delta robot from different points of view. It is often interesting to analyze the workspace from the top in order to highlight the shape and the symmetries.

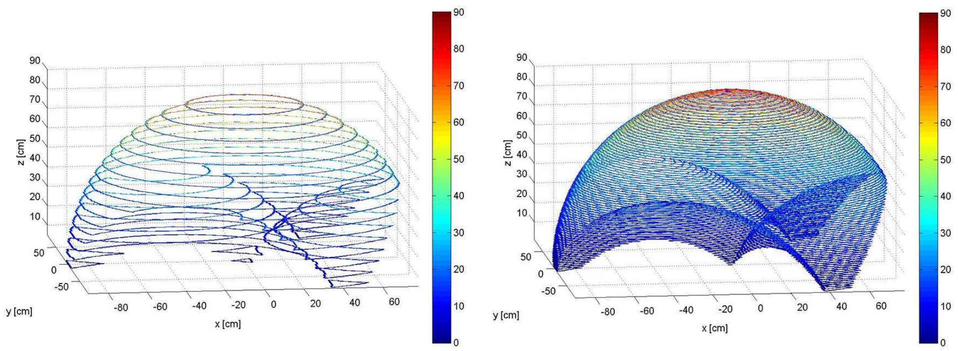

Figure 2: The first method with 5cm and 1cm division

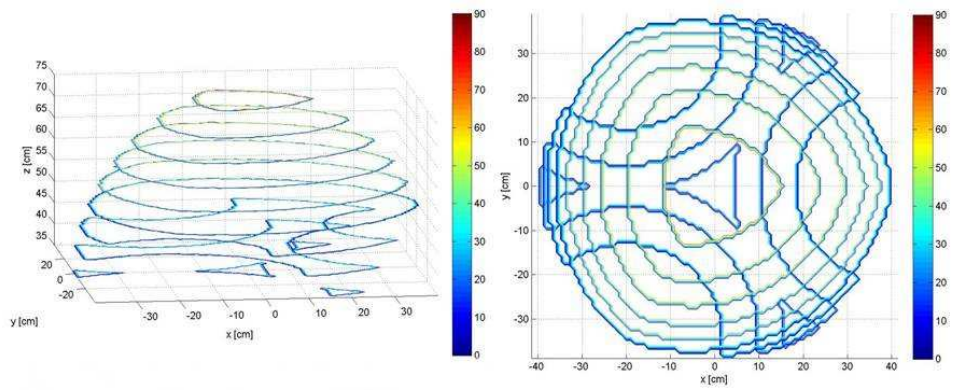

Figure 3: The first method applied to the ABB kinematics

3 THE ACCURATE METHOD

The main goal of the second method consists in achieving the equations of the manipulator workspace frontier. The first step of this method consists in finding in which area of the xy plane each actuator limits the the upper and the lower movements of the robot link becomes the upper or the lower limit of the workspace. This method is still based on the analysis of the inverse kinematics, but it requires an elaboration. As the matter of fact, the inverse kinematics allows to fing θ_1, θ_2 and θ_3 given the x y and z values. In this algorithm the z value becomes an unknown, while one by one the free coordinates becomes a known value. The $theta_i$ known is set to the value which makes the corresponding actuator to limit the workspace. The other θ_j and θ_k are used to figure out if the considered point belongs to the workspace or not. This way the inverse kinematics results modified in two functions z=$f_u(x,y)$ and z=$f_l(x,y)$.

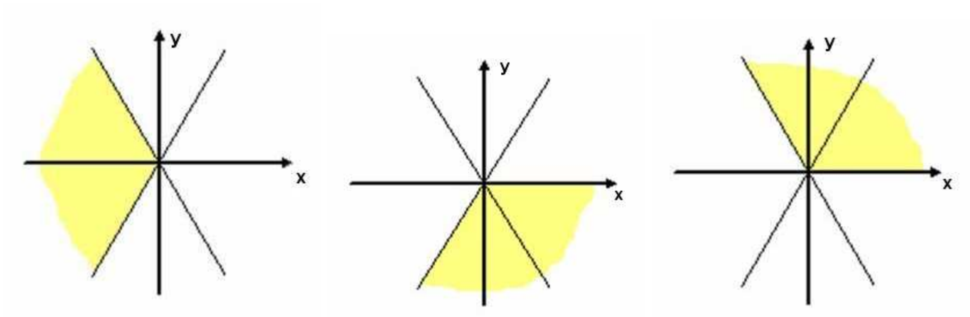

Figure 4: Relationship between regions and link limits

Taking into account the two delta robots (the linear one and the classic one), the figure 4 shows the xy areas in which each link upper limits the workspace. These areas can be written as follows:

$$\begin{aligned} \{y \geq x \cdot \sqrt{3}\} &\quad \cap \quad \{y \leq -x \cdot \sqrt{3}\} \\ \{y \leq 0\} &\quad \cap \quad \{y \leq x \cdot \sqrt{3}\} \\ \{y \geq 0\} &\quad \cap \quad \{y \geq -x \cdot \sqrt{3}\} \end{aligned}$$

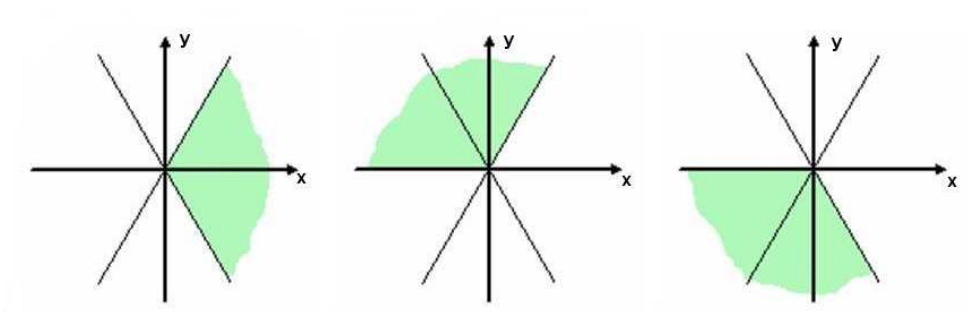

Figure 5: Relationship between regions and link limits

Besides the figure 5 shows the areas in which each link lower limits the workspace. This time these areas can be written as follows:

$$\begin{array}{rcl} \{y \leq x \cdot \sqrt{3}\} & \cap & \{y \geq -x \cdot \sqrt{3}\} \\ \{y \geq 0\} & \cap & \{y \geq x \cdot \sqrt{3}\} \\ \{y \leq 0\} & \cap & \{y \leq -x \cdot \sqrt{3}\} \end{array}$$

3.1 THE LINEAR DELTA MANIPULATOR

This manipulator, depicted in figure 1, has upper limit of each slider in the origin of the reference frame. This way each actuator limits the upper movements of the robot when it reaches this position. Hence the inverse kinematic gives the same equation for each actuator in order to understand if the considered point belongs to the workspace.

$$x^2 + y^2 - l^2 \leq 0 \tag{4}$$

Moreover the same equation is given in order to find the upper frontier of the workspace, for example with $q_1 = 0$:

$$z = \sqrt{-x^2 - y^2 - l^2}$$
$$\{0 \leq q_2 \leq l\} \cap \{0 \leq q_3 \leq l\}$$

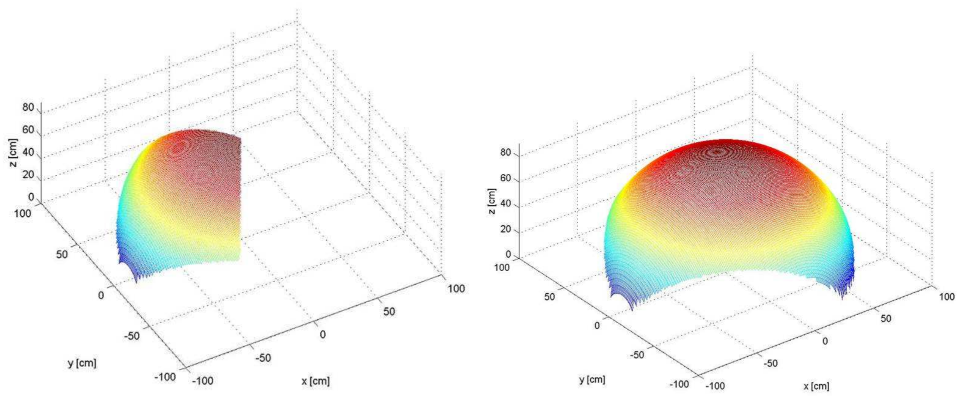

Figure 6: Upper Limit: Single and Whole Links

On the other hand, taking into account that the movement of the slider is equal to the lenght l of the link, the lower movement of the manipulator are limited when one slider reaches the lower position. The rug inclination is equal to 30^0. In this case three different equation, one for each region, allow finding the lower frontier. Hence when $q_1 = l$

$$z = -\frac{1}{2} + \sqrt{-(x - l \cdot \frac{\sqrt{3}}{2})^2 - y^2 + l^2}$$
$$\{0 \leq q_2 \leq l\} \cap \{0 \leq q_3 \leq l\}$$

In the second region the limits is given by $q_2 = l$ and the equations are:

$$z = -\frac{1}{2} + \sqrt{-(x - l \cdot \frac{\sqrt{3}}{2} \cdot \cos(\frac{2\pi}{3}))^2 - (x - l \cdot \frac{\sqrt{3}}{2} \cdot \sin(\frac{2\pi}{3}))^2 + l^2}$$
$$\{0 \leq q_1 \leq l\} \cap \{0 \leq q_3 \leq l\}$$

The last case, when $q_3 = l$, is handled by the equations that follow:

$$z = -\frac{1}{2} + \sqrt{-(x - l \cdot \frac{\sqrt{3}}{2} \cdot \cos(\frac{4\pi}{3}))^2 - (x - l \cdot \frac{\sqrt{3}}{2} \cdot \sin(\frac{4\pi}{3}))^2 + l^2}$$
$$\{0 \leq q_1 \leq l\} \cap \{0 \leq q_2 \leq l\}$$

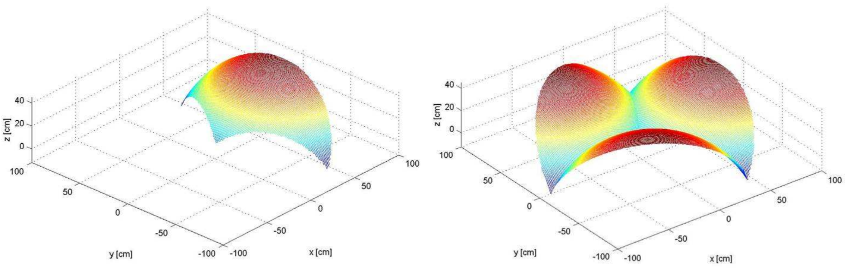

Figure 7: Lower Limit: Single and Whole Links

The results of each area should be joined in order to find the representations of the upper workspace frontier (see figure 8) and of the lower one (see figure 9). In these figure the workspace are depicted with an high resolution, and from different view point. On the right of each figure it is possible to easy identify the regions and the symmetries.

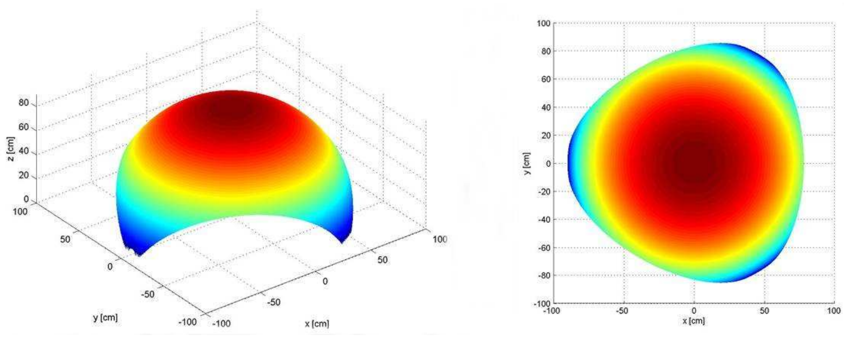

Figure 8: Two views of the upper frontier of the linear delta manipulator

3.2 THE CLASSIC DELTA MANIPULATOR

The areas of the delta robot are the same of the previous one, and are depicted in figures 4 and 5. As a consequence the study of the delta robot has similar equation. Let us consider r_0 as the distance

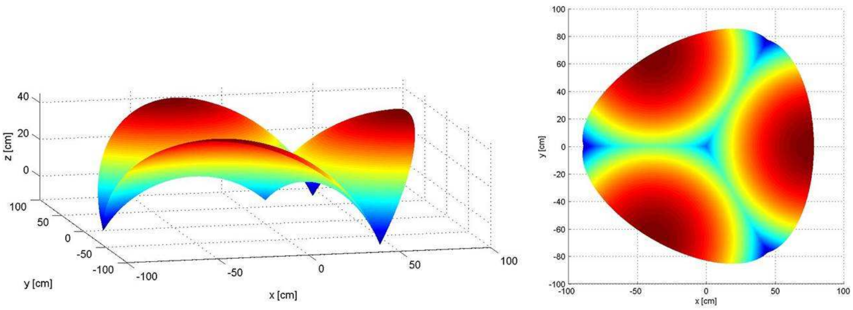

Figure 9: Two views of the lower frontier of the linear delta manipulator

between the origin of the reference frame and the actuators position, r_1 as the length of the crank, and r_2 as the length of the link connected to the end-effector. The upper limits are given when a crank takes an angle equal to $\pi/2$.

This way the first region of figure 4 is upper limited when $q_1 = \frac{\pi}{2}$ and the relationships are:

$$z = r_1 + \sqrt{-(x - r_0)^2 - (y)^2 + r_2^2}$$
$$\{0 \le q_2 \le \tfrac{\pi}{2}\} \cap \{0 \le q_3 \le \tfrac{\pi}{2}\}$$

In the second region of the same figure the upper limit is given by $q_2 = \frac{\pi}{2}$ and so:

$$z = r_1 + \sqrt{-(x - r_0 \cdot \cos(\tfrac{2\pi}{3}))^2 - (y - r_0 \cdot \sin(\tfrac{2\pi}{3}))^2 + r_2^2}$$
$$\{0 \le q_1 \le \tfrac{\pi}{2}\} \cap \{0 \le q_3 \le \tfrac{\pi}{2}\}$$

In the end, with $q_3 = \frac{\pi}{2}$, it is possible to find the following equation:

$$z = r_1 + \sqrt{-(x - r_0 \cdot \cos(\tfrac{4\pi}{3}))^2 - (y - r_0 \cdot \sin(\tfrac{4\pi}{3}))^2 + r_2^2}$$
$$\{0 \le q_1 \le \tfrac{\pi}{2}\} \cap \{0 \le q_2 \le \tfrac{\pi}{2}\}$$

This time, the link that upper limit the workspace does not reach the $x = 0$, $y = 0$ position, hence the center of the sphere is placed at a distance equal to r_0. This way the workspace has not a perfect spherical aspect, as shown in the first picture of figure 10.

Let consider the lower limits of the motor q_i equal to 0. The workspace frontier is lower limited by three spheres, one for each region of figure 5. In the first one q_1 assumes a value equal to 0 and the following equation is gathered:

$$z = \sqrt{-(x - (r_0 + r_1))^2 - (y)^2 + r_2^2}$$
$$\{0 \le q_2 \le \tfrac{\pi}{2}\} \cap \{0 \le q_3 \le \tfrac{\pi}{2}\}$$

With the same procedure when q_2 is equal to 0 it is possible to obtain the following equation:

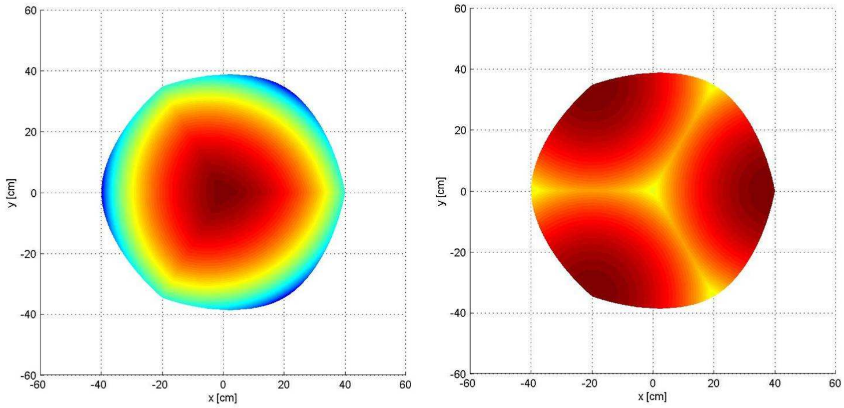

Figure 10: Upper and lower limit of the classic delta robot

$$z = \sqrt{-(x - (r_0 + r_1) \cdot \cos(\tfrac{2\pi}{3}))^2 - (y - (r_0 + r_1) \cdot \sin(\tfrac{2\pi}{3}))^2 + r_2^2}$$
$$\{0 \leq q_1 \leq \tfrac{\pi}{2}\} \cap \{0 \leq q_3 \leq \tfrac{\pi}{2}\}$$

In the end when $q_3 = 0$ the lower limit of the third region is given by:

$$z = \sqrt{-(x - (r_0 + r_1) \cdot \cos(\tfrac{4\pi}{3}))^2 - (y - (r_0 + r_1) \cdot \sin(\tfrac{4\pi}{3}))^2 + r_2^2}$$
$$\{0 \leq q_1 \leq \tfrac{\pi}{2}\} \cap \{0 \leq q_2 \leq \tfrac{\pi}{2}\}$$

3.3 THE ADEPT QUATTRO MANIPULATOR

The Adept Quattro is four degree-of-freedom robot. The method proposed can be easily applied to this manipulator if the rotation of the end-effector is not considered. Nonetheless this robot has four actuated leg, so the regions in which is divided the xy plane for the analysis are four. Figure 11 represents the regions of the upper limits, while figure 12 represents the regions of the lower ones. By observing these two figures, it is possible to notice that the first area of the first figure is equal to the third area of the second one; the second area of the first figure is equal to the fourth one of the second one, end so on. This means, for example, that if q_1 gives the upper limits in a certain region, the lower limit in the same region is given by q_3.

Let us consider the same notation for r_0 r_1 and r_2 of the last manipulator. The upper limit of each region is given by: $q_i = \frac{\pi}{2}$. The corresponding four equations can be assembled as follows:

$$z = r_1 + \sqrt{-(x - r_0 \cdot \cos(i \cdot \tfrac{\pi}{2}))^2 - (y - r_0 \cdot \sin(i \cdot \tfrac{\pi}{2}))^2 + r_2^2} \quad i = 0, 1, 2, 3$$
$$\{0 \leq q_j \leq \tfrac{\pi}{2}\} \cap \{0 \leq q_k \leq \tfrac{\pi}{2}\} \cap \{0 \leq q_l \leq \tfrac{\pi}{2}\} \qquad i \neq j \neq k \neq l$$

On the other hand the lower limit of each region is given by $q_i = 0$. The corresponding four equations can be written as follows:

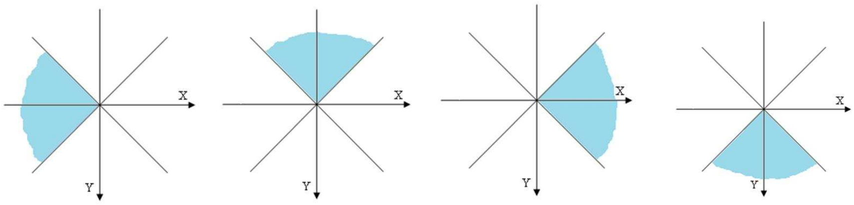

Figure 11: The Adept Quattro upper regions

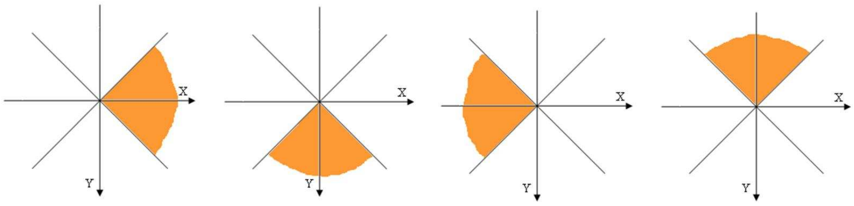

Figure 12: The Adept Quattro lower regions

$$z = \sqrt{-(x-(r_0+r_1)\cdot\cos(i\cdot\tfrac{\pi}{2}))^2-(y-(r_0+r_1)\cdot\sin(i\cdot\tfrac{\pi}{2}))^2+r_2^2} \quad i=0,1,2,3$$
$$\{0\le q_j\le\tfrac{\pi}{2}\}\cap\{0\le q_k\le\tfrac{\pi}{2}\}\cap\{0\le q_l\le\tfrac{\pi}{2}\} \qquad i\ne j\ne k\ne l$$

In the first picture of figure 13 is represented the upper and the lower limits of the Quattro in a single region. In the second picture the whole frontier of the manipulator is depicted.

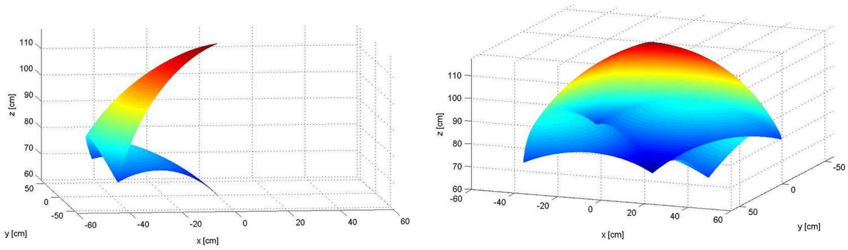

Figure 13: The Adept Quattro partial and full workspace frontier

CONCLUSIONS

This paper has presented two simplified method of frontier workspace computation of three degree of freedom manipulators. The first one requires high computation, but it can represent the frontier workspace of the manipulator directly from the inverse kinematics. A more accurate method is derived from the inverse kinematics by subdividing the workspace in regions with respect to its symmetry. Moreover for each of this region it is possible to find which actuator limits the movement of the robot. This way, in the inverse kinematic, a known terms could be changed with an unknown. As a matter of fact it is possible to find the equation of the frontier by means of the equation $[z, \theta_j, \theta_k] = f(x, y, \theta_i)$ by changing the value of i, j and k for each region. In these regions, the i-value is fixed so as to find separately the highest and the lowest z coordinate, that can be reached by the end-effector, given the x and the y coordinates ($z_h = f_h(x, y)$ and $z_l = f_l(x, y)$).

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