EHL modeling of hip implants based on a ball-on-plane configuration

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**SUMMARY.** An elastohydrodynamic lubrication model of hip joint replacements was investigated, under quasi-static operating conditions, for both hard-on-hard and soft-on-hard implants. A simplified ball-on-plane configuration was adopted to represent the articulation between the femoral head and the acetabular cup. The governing equations, including the Reynolds and the elasticity equations, were solved, respectively, using MLMG and MLMI methods, which showed a low computational cost. On the base of many simulations, the validity of the model was discussed; it can reliably evaluate film thickness and pressure maps in the case of hard-on-hard implants, while it introduces an error for soft-on-hard materials. The model was used to estimate lubrication regimes of various hip implants under simplified walking conditions: a vertical load and flexion-extension motion were adopted according to ISO 14242-1. The results evidenced the better behavior of hard-on-hard bearing couples, explaining the renewed interest in these implants.

1 INTRODUCTION

The hip joint is the synovial joint between the femoral head and the acetabulum; the term synovial means that it’s wrapped in a capsule that contains the synovial fluid (SF), a biological lubricant that acts also like shock-absorber. SF is a pseudoplastic fluid; its viscosity decreases with the shear rate, with an asymptotic value of 0.002 Pa s at the shear rate of $10^5$ 1/s \([1]\). Thanks to the SF presence and the ball-in-socket geometry, hip joint can transmit a large dynamic load (7-8 times the body weight) and accommodate a wide range of movements. When the hip joint is affected by chronic pain and diseases such as osteoarthritis, rheumatoid arthritis or bone tumors, the hip arthroplasty is necessary. It is a surgical procedure in which the hip joint is replaced by the hip implant, preserving the joint capsule: hip implant restores the anatomy and the function of the natural joint, with the aims of pain relief and improvement in hip function. In Figure 1 the main prosthesis components are showed: femoral stem (stainless steel), femoral neck, femoral head, acetabular cup and, eventually a metal back. Hip implants are classified, on the base of cup and head materials, in metal on metal (MoM), metal on plastic (MoP) and ceramic on ceramic (CoC) (Table 1) \([2]\). In the case of a plastic cup, often a metal back is used to fix the cup to the pelvis bone. Total hip replacement has proven to be the greatest achievement in orthopaedic surgery in the last decades. Its early success began with the metal-on-plastic materials combination in the 1960s. Currently the majority of hip prostheses uses a metallic or ceramic head coupled with a plastic acetabular cup (MoP) in ultra high molecular weight polyethylene (UHMWPE).
Figure 1: Hip joint implants: main components (a) and head-cup bearing geometry (b).

### Table 1: Bearing couples available for hip implants.

<table>
<thead>
<tr>
<th>Femoral Head</th>
<th>Acetabular Cup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UHMWPE</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>✓</td>
</tr>
<tr>
<td>CoCrMo</td>
<td>✓</td>
</tr>
<tr>
<td>Al2O3</td>
<td>✓</td>
</tr>
<tr>
<td>ZrO2</td>
<td>✓</td>
</tr>
<tr>
<td>Alumina composite</td>
<td></td>
</tr>
</tbody>
</table>

These soft-on-hard implants can normally survive in human body up to 15 years [2]. Although this service life is suitable for use in patients over 65 years, it is not satisfactory when used in younger patients demanding a life expectancy in excess of 20 or 25 years. This is primarily due to the osteolysis of the bone and consequent loosening of both the acetabular cup and femoral stem, caused by an adverse biological reaction to polyethylene wear debris. Therefore, a revival of alternative bearing forms of hip implants has been prompted; some hard-on-hard material combinations have been introduced, consisting in MoM and CoC hip implants. Comparing to the polyethylene wear of 100-300 $\mu$m/year, hard-on-hard bearings generally have extremely low wear rates of about 1-20 $\mu$m/year [2].

Nowadays tribological studies of hip joint replacements play an important role in the optimization of bearing materials and design of hip implants: the ultimate goal consists in minimizing adhesive and abrasive wear and promoting favorable lubrication conditions at the articulating surfaces. In the literature theoretical lubrication modeling has been extensively investigated in order to predict lubrication regimes under transient load and motion conditions (e.g. during cyclic walking), which can span from boundary to fluid film including mixed lubrication. Over the years many lubrication models were developed separately for both hard-on-hard and soft-on-hard hip implants, calling the solution of Reynolds equation and elasticity equation (elastohydrodynamic lubrication (EHL) is typically assumed). Initial attempts to estimate film
thickness in quasi-static condition for all implant typologies, were made by Jin et al. [3] using an equivalent ball-on-plane configuration, by means of the well-known Hamrock-Dowson formulae for isoviscous-elastic lubrication [4]. For both MoM and CoC implants the contact area is markedly smaller than curvature radii of the ball-in-socket configuration; hence the assumption of semi-infinite solids (i.e. ball-on-plane) is readily justified. However for MoP implants the contact dimension can become similar to or greater than cup-head radius, as well as than cup wall thickness. In these cases it is necessary to model a layered surface geometry; in particular when the pressure distribution extends over a significant portion of acetabular cup surface, the curvature has to be considered by means of the ball-in-socket geometry. The first attempt to address this problem was made by Goenka and Booker [5], who formulated and solved Reynolds equation in spherical coordinates, by means the finite element approach. A complete description of EHL for MoP hip implants, assuming a ball-in-socket geometry, is due to Jalali-Vahid et al. [6-7], who solved simultaneously the Reynolds equation, using Newton-Raphson (N-R) method, and the elasticity equation, based on the simple column model [8]. A comparison of steady state [6] and transient [7] simulation results showed that the average of the predicted transient film thickness was remarkably close to that estimated by the quasi-static condition based on the average load and speed. In parallel Jagatia and Jin [9] presented a general methodology for steady state EHL analysis of MoM hip implants, based of the combination of N-R method to solve Reynolds equation and finite element analysis to obtain displacement coefficients for calculating the elastic deformation. This advancement allowed to examine EHL in various MoM implants, including tapered connection between the acetabular cup and the shell, a polyethylene backing for metal cup, MoM hip implants resurfacing and Mckee-Farrar MoM hip replacements [2].

Transient EHL modeling of hard-on-hard implants was firstly investigated by Chan [10], assuming a ball-on-plane geometry, based on the linear superposition of the load carried by the squeeze-film and the entraining actions. A similar but more complete study was carried out by Williams [11] and Jalali-Vahid and Jin [12]; a significant variation of the film thickness with dynamic load and speed was observed with respect to the average quasi-static predictions, unlike Chan's results [10]. Significant advancement has been made in the last years by Wang and Jin, who proposed an efficient numerical method in order to calculate the elastic deformation of spherical bodies, called Spherical Fast Fourier Transform (SFFT) technique [13]. Both computing time and storage space required for this method are very reduced compared to traditional summation technique [9], thus it is possible to consider a large number of nodes. This method was firstly applied to simulate EHL of MoP in steady state; moreover, it was recently used to investigate the minimum film thickness of MoM implants under realistic 3D load and motion of the walking cycle [14], and the effect of non spherical bearing geometry [15]. Nevertheless, limitations of the N-R method have been recognized in the simulation of more realistic conditions such as topography and micro-texture of the bearing surfaces, particularly under transient conditions [16]. This explains the recent interest in the numerical technique initially introduced by Venner and Lubrecht [17] and widely used to solve EHL point contact problem. Gao et al. in [16] applied this method to study of artificial joint lubrication and demonstrated it is fast and robust.

In this work a steady state EHL model of hip implants was developed on the base of an equivalent ball-on-plane geometry and solved by means MLMG and MLMI. The validity and the accuracy of a simple geometric model equivalent to the spherical bearing were discussed for both hard-on-hard and soft-on-hard hip implants. Their lubrication regimes were predicted under quasi-static load and velocity conditions of a walking cycle (according to ISO 14242-1), in order to compare their performance. Moreover the suitability and the stability of the multi-level method for EHL hip implants were confirmed.
2 EHL MODELING

A custom EHL transient model for a point contact, originally developed for investigations on non-conformal bearings [18], was modified in order to simulate the lubrication of hip implants. According to the theory of the semi-elastic half space, the hip spherical joint (Figure 2a) can be simplified into an equivalent ball-on-plane model (Figure 2b). The anatomical position of the acetabular cup (inclined at 45°) is not relevant, since it causes just a translation of pressure and contact area over the domain [19]. The cup wall thickness was not modeled. Only the acetabular cup and the head were modeled like an isolated system. The metal back of UHMWPE plastic head and the fixation system composed of cement and bone, can be excluded without accuracy loss [9]. The input geometrical (femoral head radius \( R_1 \), acetabular cup internal radius \( R_2 \), clearance \( c = R_2 - R_1 \)) and mechanical (elastic modulus \( E \) and Poisson's ratio \( \nu \)) parameters, required for lubrication analysis, are summarized Figure 3a.

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>( M_{\text{CoC}} )</th>
<th>( M_{\text{CoP}} )</th>
<th>SIZE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetabular cup radius (( R_2 ))</td>
<td>14.03 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Femoral head radius (( R_1 ))</td>
<td>14.00 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clearance</td>
<td>30 ( \mu )m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>Elastic modulus ( E )</th>
<th>Poisson's ratio ( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CoCrMo</td>
<td>210 GPa</td>
<td>0.3</td>
</tr>
<tr>
<td>UHMWPE</td>
<td>10 GPa</td>
<td>0.4</td>
</tr>
<tr>
<td>Alumina</td>
<td>3000 GPa</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(a) Abduction/Adduction
(b) Internal-External Rotation

\( W_y \) Vertical load
\( \omega_x \) Flexion-Extension

![Figure 2: Geometries of EHL model of hip implants and simulated conditions. (a) Anatomical ball in socket model subjected to 3D load \((W_{x,y,z})\) and motion \((\omega_{x,y,z})\) conditions. (b) Equivalent ball on plane model simulating the vertical load \((W_y)\) and the flexion-extension velocity \((\omega_x)\).](image)

(a) Figure 3: Simulated conditions: different types of hip implants (a) are simulated during the walking cycle, assuming a simplified vertical load \((W_y)\) and flexion-extension angular velocity \((\omega_x)\), according to ISO 14242-1 (b).
At the high shear rates experienced in hip joint during walking, SF asymptotically behaves as Newtonian ($\mu = 0.002$ Pa s) [1,9]; moreover, considering the low maximum pressures reached during walking ($<100$ MPa), piezoviscous effect and density variation are negligible. Therefore the lubricant is assumed Newtonian, isoviscous and incompressible. Lubricant model is also suitable for the bovine serum (diluted at 25%), used in joint simulators, since it is a Newtonian fluid ($\mu = 0.0009$ Pa s) [9]. The model can simulate a viscosity range of 0.0009–1 Pa s. The operating conditions representative of normal walking were adopted according to ISO 14242-1, which considers the hip joint subjected to a 3D time-dependent load ($W_{x,y,z}$) and motions, referred to three euler angular velocities of the flexion-extension ($\omega_x$), internal-external rotation ($\omega_y$) and abduction-adduction rotation ($\omega_z$) (Figure 2a). However, only the major vertical load and the dominant flexion-extension were solved in the present lubrication problem (Figure 2b, Figure 3b).

2.1 Governing equations

The main governing equations were expressed in the Cartesian frame $\{O,x,y,z\}$, reported in Figure 2b: the centre $O$ was located in the centre of the contact, corresponding to the central offset film thickness $h_{00}$ in the absence of elastic deformation; the $x$ axis direction overlapped with the flexion extension direction. The set of equations for the present EHL problem, in steady state, is:

Reynolds equation:

$$\frac{\partial}{\partial x} \left( \frac{\rho h^3 \partial p}{12\mu \partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\rho h^3 \partial p}{12\mu \partial z} \right) - u \frac{\partial (\rho h)}{\partial x} = 0 \quad (1)$$

Film thickness equation:

$$h(x,z) = h_{00} + \frac{x^2 + z^2}{R'} + \delta(x,z) \quad (2)$$

Elasticity equation:

$$\delta(x,z) = \frac{2}{\pi E'} \iint_{\Omega} \frac{p(x',z')dx'dz'}{\sqrt{(x-x')^2 + (z-z')^2}} \quad (3)$$

Load balance equation:

$$W_y = \iint_{\Omega} p(x,z) dxdz \quad (4)$$

where:

- $p(x,z)$ pressure
- $h(x,z)$ film thickness
- $h_{00}$ film thickness in the centre of the contact (central offset film thickness)
- $\delta(x,z)$ elastic displacements
- $\rho$ density of the lubricant
- $u$ entraining velocity, calculated as $\omega_y R'/2$
- $\mu$ lubricant viscosity
- $E'$ equivalent elastic modulus, calculated as $2 \times (1 - v_1^2)/(E_1 + (1 - v_2^2)/E_2)$
- $R'$ equivalent radius, calculated as $R_2 R'/c$
- $W_y$ external force (vertical load)

The computational domain adopted was the hemispherical femoral cup: $\Omega = \{(x,z) \mid -\pi/2 \leq x/R_1 \leq \pi/2, -\pi/2 \leq z/R_1 \leq \pi/2\}$. The following boundary conditions were applied to the present problem:
\[ p = 0 \quad \text{at the edge of spherical cup} \quad (5) \]
\[ \frac{\partial p}{\partial x} = \frac{\partial p}{\partial z} = 0 \quad \text{cavitation condition} \quad (6) \]

### 2.2 Numerical methods

The set of the governing equations (1-4) was solved using the multi-level method which allows to maintain computational costs low. In the EHL analysis the equations and the domain were expressed in the dimensionless form. The dimensionless domain was obtained normalizing it with the half-width of the Hertzian contact \( \frac{\sqrt{3WR}}{2E} \); it was assumed quadrangular and asymmetric with respect to \( x \) axis, with an inlet region longer than the outlet one, for convergence reason (e.g. dimensionless domain 8x8 correspond to the dimensional domain 8x8 mm², i.e. \(-4a\leq x\leq 4a, -5a\leq z\leq 3a\)). MLMG was used to solve the Reynolds equation, while MLMI was used to solve the elasticity equation. A detailed description of these numerical algorithms is reported in [17, 18]; here only a brief introduction related to EHL hip implants problem, is given.

The calculation domain is divided into \( m \) levels with different size of grid mesh (\( n \)). There are \( n_1 \times n_1 \) nodes on each grid (\( k = 1,2, \ldots, m \)), where \( n_k = n_1 \cdot 2^{(k-1)} \) (e.g. \( m=5, n_1=32, n_5=512 \)): the coarsest grid has \( n_1 \times n_1 \) points, while the finest grid \( n_m \times n_m \). Hence the points number of a certain grid depends on both the number of levels and the number of the coarsest grid points. Equations are solved on a certain level and the solution is projected to the following level, after a correction by means of the previous level solution. Proceeding in this way a cycle, across all levels, is completed. In this problem, where the load is lower than the traditional mechanical bearing one, V cycle was preferred respect to W cycle. Full Multi Grid (FMG) process was adopted so that preliminary relaxations were implemented, from the coarsest grid to the finest grid, before starting V cycles. In each grid Reynolds equation was solved using line relaxation methods: Gauss Seidel line relaxation (GS), for low film, and Jacobi distributive line relaxation (JD), for high film. The selection of the correct method to use was defined by the dimensionless film thickness value of 0.3 [17]. Relaxation factors were chosen, according to [17], respectively of 0.8 and 0.6 for GS and JD, given the low load conditions. V cycles continued to be performed until two convergence criteria were satisfied simultaneously: the percentage difference between the current and previous approximation of the pressure distribution has to be lower than 0.1%; moreover the percentage difference between the load sustained by the current pressure and the external applied load, has to be lower than 0.1%.

### 3 RESULTS AND DISCUSSION

#### 3.1 Numerical accuracy and computational cost

Multi-Level input parameters like the maximum level, the number of the finest grid points and the calculation domain can significantly affect the accuracy of the results and computational cost, given their correlation with the mesh size. Consequently many simulations were conducted in order to evaluate the optimum parameters combination: trends of the minimum film thickness and maximum pressure as a function of the calculation domain, the number of levels and maximum grid points number, as well as the relative computational cost (Figure 4) were analyzed.

Firstly hard-on-hard hip implants are considered. Since MoM and CoC implants investigations provided similar results, in the following only the former are discussed. Simulated conditions were: load of 1500 N, angular velocity of 2 rad/s. Figure 4a shows the variation of the minimum film thickness (in \( z \) direction) with the finest grid points number (\( m=5 \), variation of \( n_1 \)), in case of
different calculation domains (8x8, 7x7, 6x6, 4x4). The numerical convergence improves with the number of finest grid points, while it is independent of the calculation domain. The computational cost increases with the finest grid points and it is weakly lower in the case of larger domains (8x8) (Figure 4b). In Figure 4c-d the effect of the number of levels is showed, when a fixed domain is used (8x8). Two trends of the minimum film thickness are compared: the first was evaluated using 5 levels and increasing the finest grid points; the second was obtained using a fixed number of coarsest grid points (32x32), and varying the number of levels. The results show that the numerical convergence is similar in the two cases (Figure 4c), but a high number of levels combined with a low number of coarsest grid points (first case) allows to maintain computational cost low in confront of the other case (Figure 4d). The optimal parameters combination for the examined condition is the following: 5 levels, 1024x1024 finest grid points and 8x8 domain. Several simulations were executed in order to understand the effect of the Hertzian shape (i.e. load, \( R' \) and \( E' \)) on numerical convergence. In general when the contact dimension \( a \) is higher, assuming the above mentioned numerical parameters, the mesh size is lower and consequently a higher mesh density is needed. It was demonstrated that in the case of high loads (e.g. 2500 N), high femoral head radius (e.g. 16 mm), smaller clearances (20 \( \mu \)m), and more compliant materials, 2048x2048 finest grids points are necessary. On the other hand, when the contact dimension \( a \) is lower, the above parameters continue to be an optimal combination.

![Figure 4: Effect of the calculation domain dimension (top) and levels number (bottom) on numerical convergence (a,c) and computational cost (b,d).](image)

The numerical convergence of MoP implants simulations was more critical. The high elasticity of UHMWPE caused very large contact area; in certain conditions the optimum calculation domain of hard-on-hard hip implants could result bigger than the physical domain. As a consequence, in MoP simulations different dimensionless domains were adopted in function of the
operating conditions, and sometimes also 6 levels and 4096x4096 finest grid points were necessary. MoP simulations had a computational cost at least tenfold higher than MoM ones.

These simulations evidence that the multi-level method is stable and robust to solve EHL ball-on-plane model of hip implants, confirming literature results [16]. It was found to be very efficient in MoM and CoC simulations, particularly in critical conditions, such as high load, low velocity and low viscosity. Moreover the computational costs result twofold lower than those reported in [16], where multi-level method was used; this is probably due to the application of different parameters of the numerical methods (V cycle, FMG).

3.2 Hard-on-Hard hip implants lubrication

The validation of the presented EHL model for MoM hip implants was assessed. Simulations results were compared with that one reported in [20], where a ball-on-plane model of MoM hip implants was developed: the predicted film thickness and pressure profiles, along the centre line (z and x direction), for the same simulated conditions ($W_y=1500$ N, $\omega_x=2$ rad/s, $\mu=0.001$ Pa s), were in good agreement (Figure 5). Moreover simulation results were also comparable to EHL steady state simulations, based on a ball in socket geometry; that demonstrated the validity of the simplified ball-on-plane geometry for the hip spherical MoM implants [9], as later discussed.

![Figure 5: Film thickness and pressure profiles along the centre-line, in z e x direction ($W_y=1500$ N, $\omega_x=2$ rad/s, $\mu=0.001$ Pa s).](image)

The reliability of the presented model was also confirmed by means of several investigations on the effect of the load and motion conditions (assuming typical value of walking cycle), and the model parameters ($R_1$, $R_2$, $c$, $\mu$): they showed reliable results from a physical point of view and in agreement with literature data (when available). First of all the MoM hip implants model predicted a decrease of the minimum film thickness with an increase of the load ($W_y=500\div2500$ N, $\mu=0.01$ Pa s), coherent with [9]: Figure 6 shows that the contact region became larger, the film thickness profile was more deformed and the maximum pressure higher. The increase of the angular velocity $\omega_x$ produced an increase of the minimum film thickness, while the maximum pressure and the contact dimension remained identical. Similar effects were observed when the viscosity increased; in both cases the variation of the minimum film thickness was in agreement with [4]. The importance of the geometrical parameters on the MoM hip implants lubrication is well known in literature [2]; femoral head radius is related to the entraining velocity, to the sliding distance and, combined with the clearance, to the equivalent radius. The variation of the clearance was simulated separately from the variation of the head radius ($W_y=1500$ N, $\omega_x=2$ rad/s, $\mu=0.002$ Pa s). When the clearance decreased ($c=30\div90$ $\mu$m) the contact became more conformal and the
lubrication conditions improved: the minimum film thickness increased ($h_{\text{min}}=52\div80$ nm). Actually if the clearance is too tight, the edge contact of the bearings surfaces may occur elevating contact stress and restricting the lubricant entry, causing lubrication starvation. If it is too high, the bearing contact stress can increase causing boundary lubrication [21]. Also higher head radius improved the lubrication condition: large femoral head ($R=16\div18$ mm) produced higher minimum film thickness ($h_{\text{min}}=81\div130$ nm), according to [22]. Since head radius and clearance are defined together in the hip implant designing, it’s clear that the optimal geometrical parameters depend on the bearing system [2].

CoC hip implants simulations produced realistic results. The high rigidity of the ceramics, such as alumina, caused lower film thickness and higher pressure in confront of MoM ones, in the same simulated conditions. Nevertheless literature lacks of specific CoC EHL simulations and consequently a direct validation was not possible. It can be hypothesized that given the high hardness and the very low surface roughness of the ceramics, as well as their limited and recent use, CoC implants are less subjected to wear and CoC EHL simulations are less important.

![Figure 6: The effect of load variation on the film thickness and the pressure profiles, along the centre-line ($z$ direction) ($\omega_x=2$ rad/s, $\mu=0.001$ Pa s).](image)

3.3 Soft-on-Hard hip implants lubrication

The developed model for MoP hip implants seemed to be unreliable, as it was expected. Because of the presence of the plastic component, significant deformations involved region of the same dimension of head radius and cup wall thickness, hence the theory of the semi-elastic half space could not be applied. Many simulations of MoP were conducted with the aim of evaluate the error introduced by the ball-on-plane geometry. Minimum film thickness was predicted assuming the operating conditions of [23] ($\omega_x=2$ rad/s, $\mu=0.01$ Pa s), in which a ball-in-socket model was developed. The effect of a sinusoidal load (amplitude of 1000 N, minimum value of 50 N, frequency of 1 Hz) was investigated and in Figure 7a the obtained result was compared with [23]: the predicted minimum film thickness is higher during almost the walking cycle; however the percent error was not constant and assumed a maximum value of 15%. Further simulations showed error dependency from the load. In Figure 7b the ball-on-plane model was compared with the ball-in-socket model presented in [24]; the load increase, from 350 to 2500 N, was simulated, assuming a constant velocity ($\omega_x=2$ rad/s, $\mu=1$ Pa s). In this case the error increased with the load ranging between 15% and 65%. The error trends in the simulations are not coherent and further analysis would be needed. The ball-on-plane model appeared to overestimate the film thickness, evaluating a region contact bigger than the ball-in-socket one [6]. However the ball-on-plane geometry is not appropriate for MoP EHL simulations.
3.4 Prediction of lubrication regimes during the gait

The developed EHL model was used to predict the lubrication regimes of both hard-on-hard and soft-on-hard hip implants, during walking cycle. As explained above, results simulations of MoP implants can give only an idea of the realistic situation. The geometrical parameters, the load and the velocity, resumed in Figure 3, were simulated in quasi-static conditions. Figure 8a shows the obtained results and compares the minimum film thicknesses of MoM, MoC, CoC implants, during walking cycle. MoP implants had film thickness higher than hard-on-hard implants, thanks to their major deformations (as confirmed in [2,6]); MoM and CoC implants exhibited similar behavior, but CoC film thickness was reduced respect to MoM one. However the variation of the minimum film thickness during walking cycle was characterized, in all three cases, by: 1) an initial zero value due to the lack of the squeeze term in Reynolds equation (1); 2) an increment due to an increase of the velocity that was able to compensate for the increase of the load; 3) a zero thickness when, at 0.5s, the motion reversed and the velocity was annulled; 4) a new increase, due to the load decrease (up to a constant value of 300 N) and to a relatively high velocity; 5) a final decrease until zero value, caused by the reduction of the velocity up to zero. MoM quasi-static prediction of the minimum film thickness was also compared to the transient simulations, taken from [19] (Figure 8b), where a ball-in-socket transient model of MoM hip implants was developed. Excluding the instants at which the velocity is zero, the two plot are similar; the predicted transient film thickness was remarkably close to that estimated by the quasi-static condition based on the average load and speed [2]. Moreover it was also possible to estimate the lubricating regimes during walking cycle. Considering the composite roughness of unworn bearing surfaces and the highest values of the minimum film thickness during walking cycle, the $\lambda$ ratios ($k_{1}R_{b} + k_{2}R_{c}$) were calculated for various hip implants and the following lubrication regimes were predicted: boundary-to-mixed for MoP, boundary-to-mixed MoM and fluid-film for CoC (Table 2). This explained the renewed interest in hard-on-hard bearings.

4 CONCLUSIONS

The following conclusions were drawn from the present study.
- The ball-on-plane configuration can be used in EHL problems of hard-on-hard hip implants for which the presented model provided accurate results in agreement with the literature.
- In EHL modeling of MoP implants the ball-on-plane geometry appears not proper and, consequently, the presented model is unreliable; nevertheless it can be used in order to obtain a large estimation of the realistic behaviour, in certain operating conditions.
- Lubrication regimes were predicted under simplified walking conditions. A mixed and a fluid-film lubrication of hard-on-hard implants, despite the predominant boundary lubrication of MoP implants, explains the renewed interest in MoM and CoC implants.
- MLMG and MLMI numerical methods have found to be particularly efficient in this model, reducing significantly the computational cost respect to the traditional numerical methods used in this field (N-R, SFFT).

The transient version of this model is ongoing. The combination of the ball-on-plane geometry and the multi-level methods seems to be optimal for lubrication investigations on hard-on-hard hip implants, taking benefit from a simplified accurate model and a fast and robust numerical methods, necessary for more realistic simulations such as topography of the bearing surfaces and their evolution due to wear.

![Figure 8](image_url)

(a) Minimum film thickness prediction during walking cycle: (a) comparison of MoM, CoC and MoP hip implants, (b) comparison between MoM quasi static and transient prediction [19].

<table>
<thead>
<tr>
<th>Hip Implants</th>
<th>Minimum Film Thickness (nm)</th>
<th>Composite Roughness (nm)</th>
<th>( \lambda ) ratio</th>
<th>Lubrication Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>MoP</td>
<td>200</td>
<td>200-1000</td>
<td>0.68-1.1</td>
<td>boundary to mixed</td>
</tr>
<tr>
<td>MoM</td>
<td>23.5</td>
<td>14-28</td>
<td>0.84-1.68</td>
<td>boundary to mixed</td>
</tr>
<tr>
<td>CoC</td>
<td>17.5</td>
<td>7</td>
<td>2.5</td>
<td>fluid-film</td>
</tr>
</tbody>
</table>

Table 2: Estimation of the lubrication regimes experienced in various hip implants, during walking.

**References**


