Design Flow-Chart of Slider-Crank Mechanisms and Applications

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Keywords: Automatic machinery, kinematic analysis and synthesis, motion geometry, coupler curves, Berard's curves.

SUMMARY. This paper deals with the formulation of a suitable algorithm for the kinematic analysis of slider-crank mechanisms for automatic machinery, in terms of shape and characteristics of the coupler curves with the aid of the cubic of stationary curvature and the inflection circle.

Moreover, a practical method for the kinematic synthesis of a slider-crank mechanism as path generator, is also proposed for the generation of a symmetrical egg shape path with an approximate straight line. In particular, starting from the knowledge of the shape and the overall sizes of the required coupler curve of the slider-crank mechanism, lengths of the coupler-link and crank-link can be obtained from the proposed practical method and design flow-chart.

1 INTRODUCTION

Planar linkages have wide applications as mechanisms in the engineering practice, since applied to perform prescribed non-uniform motions of several mechanical devices and automatic machinery, as in the case of "Function Generators" from the input to the output links, "Rigid Body Guidance" through the study of the rigid coupler motion and "Path Generators" by referring to the coupler curve, as reported extensively in [1-7]. The typical procedure to develop a systematic mechanism design consists of three main steps: task definition, type synthesis and dimensional synthesis. The task definition, which comes from the analysis of the practical application, lets to obtain the design specifications for the development of the next steps. In fact, the second step consists to choose the best topology for the required task and the last step regards the dimensional synthesis, which consists to determine the size of each mechanical element that perform the required movement and the mechanical power flow from the driving to the driven link. The dimensional synthesis of path generator mechanisms can be carried out through classical indirect methods or by means of direct methods, which consists of several optimization methods.

This paper deals with the formulation of a suitable algorithm for the kinematic analysis of slider-crank mechanisms for automatic machinery, in terms of shape and characteristics of the coupler curves with the aid of the cubic of stationary curvature and the inflection circle, as first introduced in [8-11]. Moreover, a practical method for the kinematic synthesis of a slider-crank mechanism as path generator, is also proposed for the generation of a symmetrical egg shape path with an approximate straight line. In particular, starting from the knowledge of the shape and the overall sizes of the required coupler curve of the slider-crank mechanism, lengths l and r of the coupler-link and crank-link, respectively, can be obtained from the proposed design flow-chart.

A variety of slider-crank coupler curves can be obtained to meet several practical requirements, but the designer should select, first of all, the shape of the required path in order to define the adequate initial mechanism dimensions, which can be optimized through suitable optimization algorithms when a specific continuous path is required. In fact, usually, in the industrial practice, the generation of a specific continuous path is not required because, very often, only the shape and the overall size of a coupler curve is sufficient to satisfy the design specifications.

2 POSITION ANALYSIS

Referring to the kinematic sketch of Fig.1, the position analysis of the slider-crank mechanism *ABC* is carried out by developing the following closure equation

$$\mathbf{r}+\mathbf{l}=\mathbf{m}\,. \tag{1}$$

Vectors **r**, **l** and **m** are given in the fixed frame $\mathcal{F}(O, X, Y)$ by

$$\mathbf{r}=r\begin{bmatrix}\cos\delta & \sin\delta\end{bmatrix}^T, \qquad \mathbf{l}=l\begin{bmatrix}\cos\varphi & \sin\varphi\end{bmatrix}^T \quad \text{and} \quad \mathbf{m}=\begin{bmatrix}X_C & 0\end{bmatrix}^T \tag{2}$$

where r and l are the crank and coupler lengths, respectively.

Angle φ is expressed as function of the driving crank angle δ in the form $\varphi=\sin^{-1}[(r/l)\sin\delta)]$. The position vector $\mathbf{r}_{\Omega P}$ of a generic point *P* of the coupler link *BC*, with respect to the moving frame $\ell(\Omega, x, y)$ that is attached to it, can be expressed in homogeneous coordinates as

$$\mathbf{r}_{\Omega P}=r_{\Omega P}\begin{bmatrix}\cos\alpha & \sin\alpha & 1\end{bmatrix}^T. \tag{3}$$

Thus, the position vector $\mathbf{r}_{OP}$ of the same point *P* in the fixed frame $\mathcal{F}(O, X, Y)$ is given by

$$\mathbf{r}_{OP}=\mathbf{T}_{O\Omega}\,\mathbf{r}_{\Omega P} \qquad \text{where} \qquad \mathbf{T}_{O\Omega}=\begin{bmatrix}\cos\varphi & -\sin\varphi & r\cos\delta \\ \sin\varphi & \cos\varphi & r\sin\delta \\ 0 & 0 & 1\end{bmatrix} \tag{4}$$

is the transformation matrix from the moving frame $\ell(\Omega, x, y)$ to the fixed frame $\mathcal{F}(O, X, Y)$.

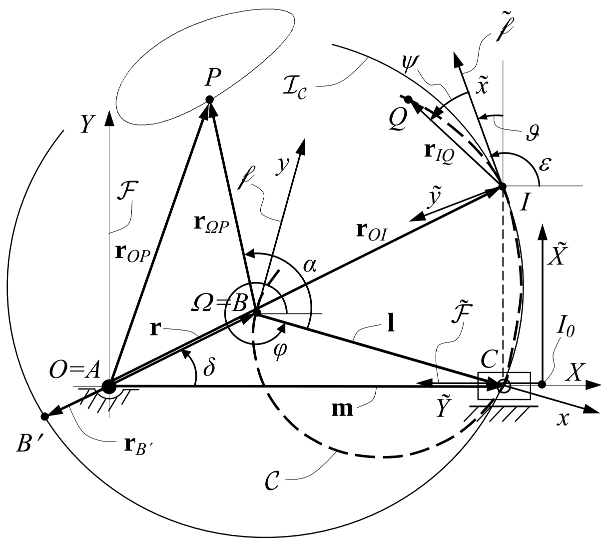


Figure 1: Kinematic sketch of a slider-crank mechanism

3 INSTANTANEOUS GEOMETRIC INVARIANTS

As reported in [6] and referring to Fig.1, the instantaneous geometric invariants b_2, a_3 and b_3 can be expressed in the form

$$b_2 = \sqrt{\left(\frac{\mathrm{d}^2 X_\Omega}{\mathrm{d}\varphi^2} + \frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi}\right)^2 + \left(\frac{\mathrm{d}^2 Y_\Omega}{\mathrm{d}\varphi^2} - \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right)^2} \tag{5}$$

$$a_3 = \frac{1}{b_2}\left[\left(\frac{\mathrm{d}^3 X_\Omega}{\mathrm{d}\varphi^3} + \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right)\left(\frac{\mathrm{d}^2 Y_\Omega}{\mathrm{d}\varphi^2} - \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right) - \left(\frac{\mathrm{d}^3 Y_\Omega}{\mathrm{d}\varphi^3} + \frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi}\right)\left(\frac{\mathrm{d}^2 X_\Omega}{\mathrm{d}\varphi^2} + \frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi}\right)\right] \tag{6}$$

$$b_3 = \frac{1}{b_2}\left[\left(\frac{\mathrm{d}^3 X_\Omega}{\mathrm{d}\varphi^3} + \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right)\left(\frac{\mathrm{d}^2 X_\Omega}{\mathrm{d}\varphi^2} + \frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi}\right) + \left(\frac{\mathrm{d}^3 Y_\Omega}{\mathrm{d}\varphi^3} + \frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi}\right)\left(\frac{\mathrm{d}^2 Y_\Omega}{\mathrm{d}\varphi^2} - \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right)\right], \tag{7}$$

where φ, X_Ω and Y_Ω are, respectively, the oriented angle and the Cartesian-coordinates of the origin Ω of the moving frame $f(\Omega, x, y)$ that is attached to the coupler link BC, with respect to the fixed frame $\mathcal{F}(O, X, Y)$. The convenient starting configuration with $\vartheta = 0$ has been chosen to have the instantaneous geometric invariants $a_0 = a_1 = a_2 = b_0 = b_1 = 0$.

The position vector $\mathbf{r}_{O\Omega}$ of Cartesian-coordinates X_Ω and Y_Ω of the Eqs.(5) to (7), coincides with vector $\mathbf{r}$, which is expressed through the first of Eqs.(2), while the oriented angle φ is expressed in the form $\varphi = \sin^{-1}(r\sin\delta / l)$, as function of the crank angle δ.

Therefore, the first, second and third derivatives of X_Ω and Y_Ω with respect to angle φ for the Eqs.(5) to (7), can be expressed in the following form

$$\begin{aligned} &\frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi} = -r\sin\delta\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} \\ &\frac{\mathrm{d}Y_\Omega}{\mathrm{d}\varphi} = r\cos\delta\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} \end{aligned}, \quad \begin{aligned} &\frac{\mathrm{d}^2 X_\Omega}{\mathrm{d}\varphi^2} = -r\left[\left(\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\right)^2\cos\delta + \sin\delta\frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2}\right] \\ &\frac{\mathrm{d}^2 Y_\Omega}{\mathrm{d}\varphi^2} = -r\left[\left(\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\right)^2\sin\delta - \cos\delta\frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2}\right] \end{aligned} \quad \text{and}$$

$$\begin{aligned} &\frac{\mathrm{d}^3 X_\Omega}{\mathrm{d}\varphi^3} = r\left[\left(\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\right)^3\sin\delta - 3\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2}\cos\delta - \sin\delta\frac{\mathrm{d}^3\delta}{\mathrm{d}\varphi^3}\right] \\ &\frac{\mathrm{d}^3 Y_\Omega}{\mathrm{d}\varphi^3} = -r\left[\left(\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\right)^3\cos\delta + 3\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2}\sin\delta - \cos\delta\frac{\mathrm{d}^3\delta}{\mathrm{d}\varphi^3}\right] \end{aligned}. \tag{8}$$

In particular, the first, second and third derivatives of angle δ with respect to φ are given by

$$\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} = \frac{l\cos\varphi}{r\cos\delta}, \quad \frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2} = \frac{-lr\sin\varphi\cos\delta + lr\cos\varphi\sin\delta\dfrac{\mathrm{d}\delta}{\mathrm{d}\varphi}}{r^2\cos^2\delta} \quad \text{and} \quad \frac{\mathrm{d}^3\delta}{\mathrm{d}\varphi^3} = \frac{\dfrac{\mathrm{d}n}{\mathrm{d}\varphi}d - n\dfrac{\mathrm{d}d}{\mathrm{d}\varphi}}{d^2}, \tag{9}$$

where n and d are the numerator and denominator of the last of Eq.(9), respectively, whose first derivatives with respect to the angle φ are expressed by

$$\frac{\mathrm{d}n}{\mathrm{d}\varphi} = -lr\cos\varphi\cos\delta + lr\sin\varphi\sin\delta\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} - lr\sin\varphi\sin\delta\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} + lr\cos\varphi\cos\delta\left(\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}\right)^2 + lr\cos\varphi\sin\delta\frac{\mathrm{d}^2\delta}{\mathrm{d}\varphi^2}$$

$$\frac{\mathrm{d}d}{\mathrm{d}\varphi} = -2r^2\cos\delta\sin\delta\frac{\mathrm{d}\delta}{\mathrm{d}\varphi}. \tag{10}$$

Therefore, the instantaneous geometric invariants b_2, a_3 and b_3 of Eqs.(5) to (7) can be calculated with the aid of Eqs.(8) to (10) because these derivatives are invariants for any pair of moving and fixed frames.

4 CUBIC OF STATIONARY CURVATURE

For a given configuration of a moving coupler link, the cubic of stationary curvature is the locus of the coupler points whose paths have, at least, four contact points with their osculating circles, which means to have a stationary curvature. In particular, points B and C of the slider-crank mechanism of Fig.1 have stationary curvature because they trace a circular path and a straight segment path, respectively.

In particular, the geometric locus of the points of the moving plane having a stationary curvature can be expressed in the following third order algebraic implicit form

$$3b_2\tilde{x}(\tilde{x}^2 + \tilde{y}^2 - b_2\tilde{y}) + (\tilde{x}^2 + \tilde{y}^2)(a_3\tilde{x} + b_3\tilde{y}) = 0. \tag{11}$$

Referring to Fig.1 and with the aim to express this geometric locus with respect to the fixed frame $\mathcal{F}(O, X, Y)$, which can be more important than the local frame for the mechanism understudy, a generic point Q of the cubic of stationary curvature can be referred to the local moving frame $\tilde{\mathcal{L}}(I, \tilde{x}, \tilde{y})$ by means of the position vector

$$\mathbf{r}_{IQ} = h\begin{bmatrix}\cos\psi & \sin\psi & 1\end{bmatrix}^T, \tag{12}$$

where h and ψ are, respectively, the magnitude and the oriented angle of $\mathbf{r}_{IQ}$.

Thus, the $\tilde{x}$ and $\tilde{y}$ Cartesian-coordinates of $\mathbf{r}_{IQ}$ can be substituted in the Eq.(33) to give the parametric form of Eq.(34) in the local frame $\tilde{\mathcal{L}}(I, \tilde{x}, \tilde{y})$ as

$$\frac{1}{h} = \frac{1}{N\sin\psi} + \frac{1}{M\cos\psi}, \tag{13}$$

where

$$M = \frac{3b_2^2}{b_3} \quad \text{and} \quad N = \frac{3b_2^2}{a_3 + 3b_2}. \tag{14}$$

The instantaneous geometric invariants b_2, a_3 and b_3 are given by Eqs. (5) to (7).

Thus, the position vector of point Q of the cubic of stationary curvature can be expressed in the fixed frame $\mathcal{F}(O, X, Y)$ as

$$\mathbf{r}_{OQ} = \mathbf{T}_{OI}\ \mathbf{r}_{IQ}\,, \tag{15}$$

where the position vector $\mathbf{r}_{IQ}$ is given by Eq.(34) and $\mathbf{T}_{OI}$ is the transformation matrix from the local moving frame $\tilde{f}(I, \tilde{x}, \tilde{y})$ to the fixed frame $\mathcal{F}(O, X, Y)$.
In particular, the transformation matrix $\mathbf{T}_{OI}$ is given by

$$\mathbf{T}_{OI} = \begin{bmatrix} \cos\varepsilon & -\sin\varepsilon & X_I \\ \sin\varepsilon & \cos\varepsilon & Y_I \\ 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

where $X_I = r\cos\delta + l\cos\varphi$ and $Y_I = r\cos\delta + l\cos\varphi)\tan\delta$, while the oriented angle ε can be expressed as

$$\cos\varepsilon = \frac{1}{b_2}\left[\left(\frac{\mathrm{d}^2 Y_\Omega}{\mathrm{d}\varphi^2} - \frac{\mathrm{d}X_\Omega}{\mathrm{d}\varphi}\right)\right]. \tag{17}$$

Therefore, h as function of ψ is obtained from Eq. (13) and then substituted in the Eq. (12) in order to give $\mathbf{r}_{IQ}$. Finally, Eq. (15) allows the determination of the cubic of stationary curvature as referred to the fixed frame $\mathcal{F}(O, X, Y)$.

5 INFLECTION CIRCLE

The inflection circle $\mathcal{I}_C$ is the geometric locus of the coupler points, which show an inflection point in their paths and is always tangent to the moving and fixed centrodes in the instant center of rotation I. In particular, point C of the slider-crank mechanism belongs always to $\mathcal{I}_C$.

Referring to Fig.1, the algebraic equation of the inflection circle $\mathcal{I}_C$ can be obtained by still referring to the instantaneous geometric invariants b_2, a_3 and b_3 of the Eqs. (5) to (7), respectively. In fact, this geometric locus is referred to the coupler points, which paths have a zero curvature for a given mechanism configuration. This geometrical condition can be expressed by

$$\frac{\mathrm{d}\tilde{X}_I}{\mathrm{d}\vartheta}\frac{\mathrm{d}^2\tilde{Y}_I}{\mathrm{d}\vartheta^2} - \frac{\mathrm{d}^2\tilde{X}_I}{\mathrm{d}\vartheta^2}\frac{\mathrm{d}\tilde{Y}_I}{\mathrm{d}\vartheta} = 0\,, \tag{18}$$

where the first and the second derivatives of the Cartesian coordinates of the instant center of rotation I with respect to the fixed frame $\tilde{\mathcal{F}}$ are given by Eqs. (8), (9) and (10), since invariants. Thus, developing Eq. (18) for the starting configuration with $\vartheta = 0$ and from Eq. (5), one has

$$\tilde{x}^2 + \tilde{y}^2 - b_2\tilde{y} = 0\ , \tag{19}$$

which is the algebraic equation of the inflection circle $\mathcal{I}_C$ with respect to the local frame $\tilde{f}(I, \tilde{x}, \tilde{y})$. Finally, Eq. (19) can be transferred to the fixed frame $\mathcal{F}(O, X, Y)$ though the Eqs. (16) and (17).

6 EXAMPLES: COUPLER CURVES

The proposed algorithm has been implemented in a Matlab program in order to trace the inflection circle $\mathcal{I}_C$ and the cubic of stationary curvature $\mathcal{C}$ for a given mechanism configuration. Moreover, the paths of significant coupler points, as the instantaneous center of rotation I, the inflection pole J, the Ball's point $\mathcal{B}$, which is the intersection point between the cubic of stationary curvature and the inflection circle, along with the paths of the coupler points $\mathcal{P}_C$ and $\mathcal{P}_{\mathcal{I}C}$, which are chosen on the geometric loci $\mathcal{I}_C$ and $\mathcal{C}$, are calculated and shown in Fig. 2a), 2b), 2c) and 2d) for the slider-crank mechanism configurations given by the crank angles $\delta = 0°, 40°, 90°$ and $320°$, respectively.

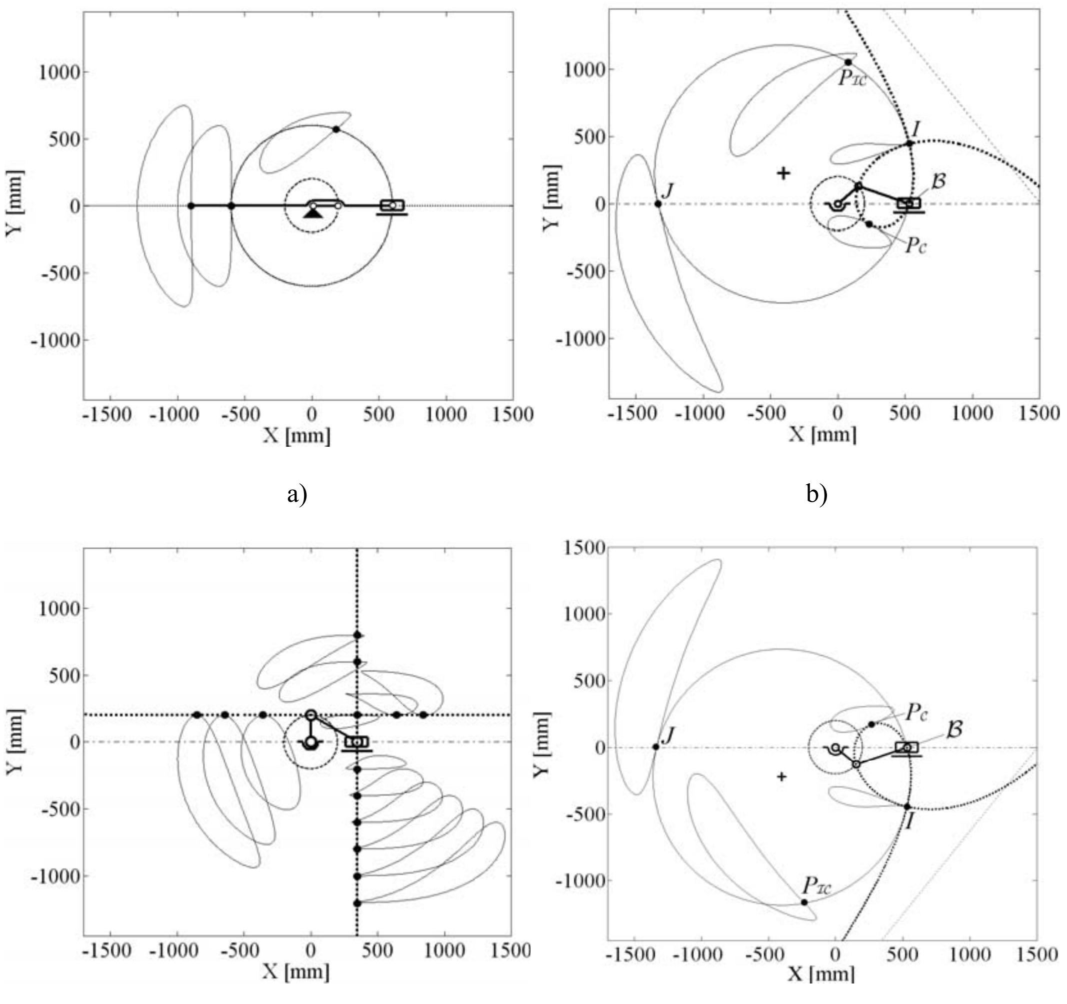


Figure 2: Coupler curves of the significant points I, J, $\mathcal{B}$ and the points $\mathcal{P}_C$ and $\mathcal{P}_{\mathcal{I}C}$ on the cubic of stationary curvature $\mathcal{C}$ and the inflection circle $\mathcal{I}_C$ for the crank angles $\delta = 0°, 40°, 90°$ and $320°$.

7 COUPLER CURVES OF BERARD AND THE QUARTIC OF BERNOULLI

Referring to Fig.3, the algebraic equations of the coupler curves of Berard can be obtained from Eqs. (3) and (4). In fact, the Berard's points P_{B} of Fig.3a can be referred to the moving frame $\mathscr{f}(\Omega,x,y)$ through the position vector $\mathbf{r}_{\Omega P\mathrm{B}}$ having magnitude w and oriented angle $\alpha = \pi$ or $\alpha = 0$ for the points which are located along the negative or the positive x-axis, respectively. Moreover, the position vector $\mathbf{r}_{OP\mathrm{B}}$ with respect to the fixed frame $\mathcal{F}(O, X, Y)$ can be expressed as

$$\mathbf{r}_{OP_{\mathrm{B}}} = \left[r\cos\delta + w\sqrt{\left(1-\frac{r^2}{l^2}\sin^2\delta\right)}, \quad r\sin\delta + \frac{rw}{l}\sin\delta \right]^T , \tag{20}$$

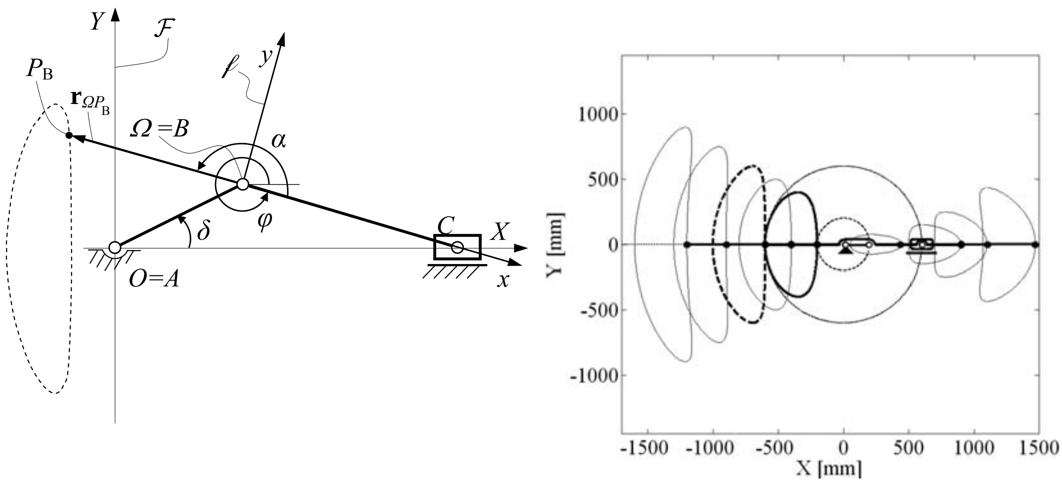
which takes the following algebraic form

$$X_{P_{\mathrm{B}}} = \frac{l}{l+w}\sqrt{\left(\frac{(rl+rw)^2}{l^2} - Y_P^2\right)} + \frac{w}{l+w}\sqrt{\left(\frac{(rl+rw)^2}{r^2} - Y_{P_{\mathrm{B}}}^2\right)} \tag{21}$$

In particular, Eq. (21) takes the form of the Quartic of Bernoulli for $w = l$, which means to choose a coupler point P_{Br}, as opposite to C with respect to B. This curve is expressed as

$$X_{P_{\mathrm{Br}}} = \frac{1}{2}\left[\sqrt{(2r)^2 - Y_{P_{Br}}^2} + \sqrt{(2l)^2 - Y_{P_{Br}}^2}\right], \tag{22}$$

which gives the path in gross line of Fig.3b, while that in dashed line refers to the inflection pole.



a) b)

Figure 3: Coupler curves: a) kinematic sketch; b) Berard's curves and quartic of Bernoulli.

8 DESIGN FLOW-CHART AND APPLICATIONS

According to the above algorithm for the kinematic analysis of slider-crank mechanisms in terms of shape and characteristics of the coupler curves, a practical method for the kinematic synthesis of a slider-crank mechanism as path generator, is proposed.
In particular, referring to Fig.4, a symmetrical egg shape path with given overall sizes b_1 and h_1, which is characterized to have a coupler point with stationary curvature, can be generated by a specific slider-crank mechanism, when a suitable Berard's points P_B is chosen along the coupler link BC, as belonging to the cubic of stationary curvature $\mathcal{C}$. In fact, for the crank angle $\delta = 0°$, the cubic of stationary curvature degenerates in a Φ-curve, as stated by Dijksmann in his book [2] and shown in the previous examples of Figs.2a and 3b.
Moreover, the coupler points that are located on the line along BC generate a family of Berard's curves, which have a point with stationary curvature since belonging to $\mathcal{C}$ and, in particular, when this point is chosen as coincident with the inflection pole J on the inflection circle $\mathcal{I}_C$, an approximate straight line is also obtained, as shown in Fig.3b in dashed line.
A practical method and a design flow-chart for the kinematic synthesis of slider-crank mechanisms as generator of a symmetrical egg shape path, which is characterized to have a coupler point with stationary curvature or, in addition, to trace an approximate straight line, is proposed by referring to the mechanism configuration for $\delta = 90°$, as shown in Fig.4. Thus, referring to Fig.4a, one has

$$\begin{aligned} & r = \frac{h_1}{2} \\ & l^2 + r(a_p - l) = 0 \\ & \frac{r}{l} = \frac{b_1}{2a_p} \end{aligned} \tag{23}$$

where b_1 and h_1 are the sizes of the required path, r and l are the dimensions of the crank link AB and coupler link BC of the slider-crank mechanism, respectively, and a_P gives the position of the particular Berard's point P_B that is coincident with the inflection pole J.

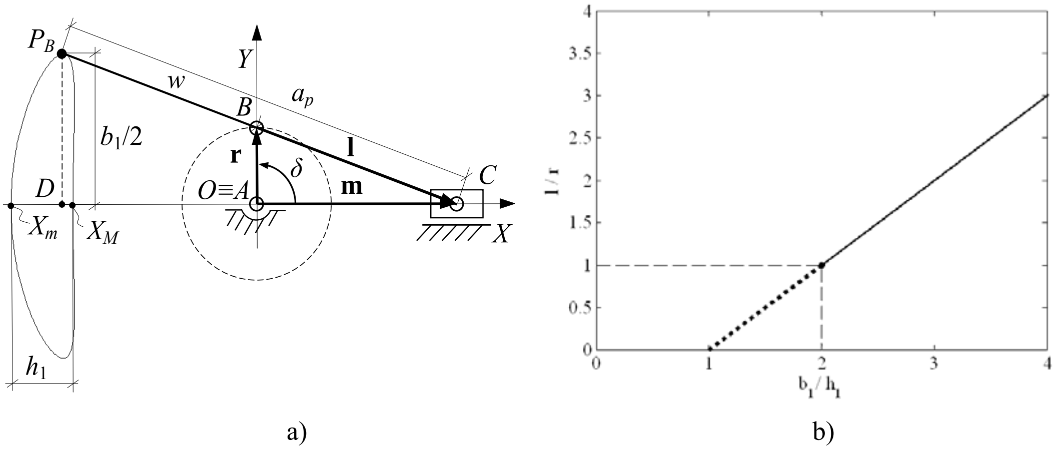


Figure 4: Slider-crank mechanism: a) kinematic sketch; b) design diagram.

Tab.1 - Dimensional synthesis of slider-crank mechanisms: numerical examples

b_1 [mm]	h_1 [mm]	b_1/h_1 [mm]	l/r [mm]	r [mm]	l [mm]	w [mm]	a_p [mm]
240	100	2.4	1.4	50	70	98	168
1200	600	2	1	300	300	300	600
800	200	4	3	100	300	900	1200
359.66	98	3.67	2.67	49	131	349.77	480.77

Therefore, developing the Eqs.(23), one has

$$\frac{l}{r} = \left(\frac{b_1}{h_1} - 1\right) \tag{24}$$

which can be used for the dimensional synthesis of a slider-crank mechanism with lengths r and l of the crank and coupler links, when the overall sizes b_1 and h_1 of the required path are given. Equation (24) gives the design diagram of Fig.4b and numerical examples are reported in Tab.1 along with the proposed design flow-chart of Fig.5. According to the proposed algorithm for the kinematic analysis of slider-crank mechanisms in terms of shape and geometrical characteristics of the coupler curves, along with the proposed practical method for the kinematic synthesis of the same mechanisms for generating egg shape paths showing an approximate straight line, two applications are reported. Thus, referring to Fig.6, the first application regards an automatic wood-feeder (Fig.6a), while the second regards a leg mechanism for a walking robot (Fig.6b). In particular, the second mechanism has been designed by taking into account the input data:

$$h_p = 90\ [mm],\ b_p = 300\,[mm],\ \chi = \frac{H_pC_p}{A_pH_p} = 3,\ B_pC_p = 400\ [mm] \text{ and } A_pB_p = 300\ [mm],$$

which have given for the pantograph:

$$G_pC_p = \frac{\chi}{\chi+1} B_pC_p = 300\ [mm],\ B_pG_p = B_pC_p - G_pC_p = I_pH_p = 100\ [mm]$$

$$\text{and} \quad I_pB_p = A_pB_p - A_pI_p = H_pG_p = 225\ [mm].$$

Likewise, for the slider-crank mechanism, one has:

$$r = \frac{h_1}{2} = 15\ [mm] \quad , \quad \frac{l}{r} = \left(\frac{b_1}{h_1} - 1\right) = 2{,}33 \quad , \quad l = 35\ [mm] \quad \text{and} \quad a_p = 116{,}67\ [mm].$$

Of course, changing the position of point P_B through the length a_P, any other Berard's curve can be obtained by the same designed slider-crank mechanism.

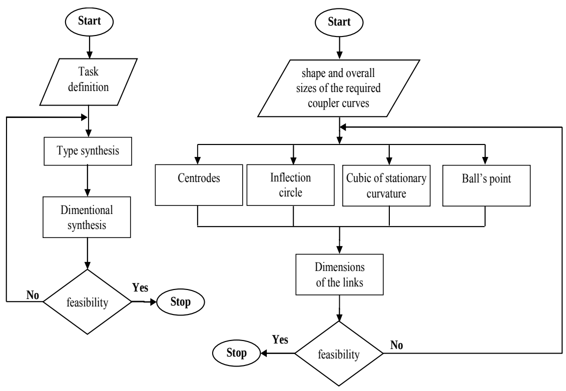
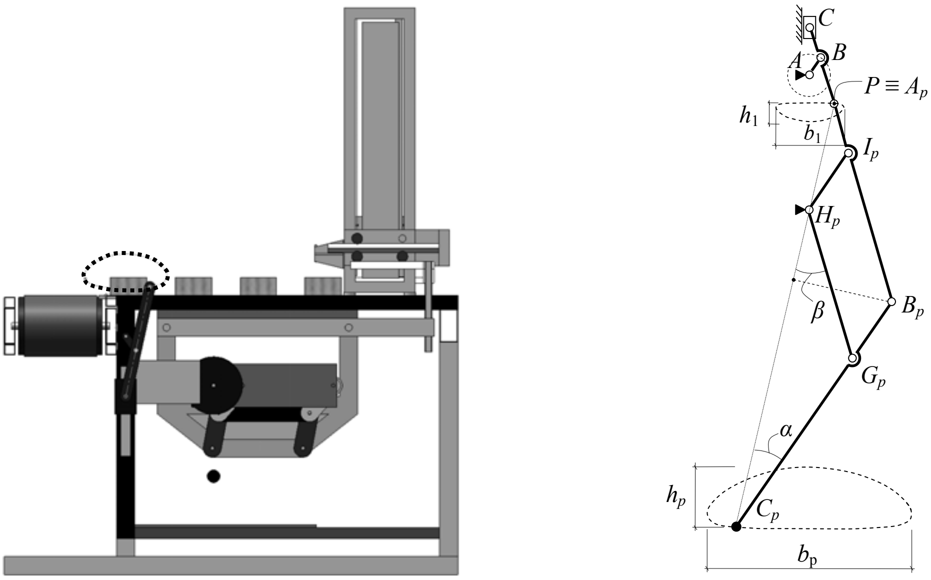


Figure 5: The proposed design flow-chart



a) b)

Figure 6: Applications: a) automatic wood-feeder; b) leg mechanism of a walking robot.

9 CONCLUSIONS

The analysis and synthesis of slider-crank mechanisms for automatic machinery, which are aimed to give symmetrical egg shape path with an approximate straight line, have been presented. A practical method for the dimensional synthesis of slider-crank mechanisms has been proposed along with a design flow-chart and two practical applications.

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