# Kinematic Optimization of a Parallel Manipulator 5R 2-dof Driven by Pneumatic Cylinders

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SUMMARY. In industrial field, the need to ensure high levels of production enhancing working precision requires the development of ever more efficient robots. The behaviour of a serial or parallel manipulator can be investigated through its kinetostatic performances [1] such as repeatability, stiffness, maximum force or velocity. They all depend on the kinematics structure of the system, on its configuration inside the working space and on the kind of drive system used to operate the robot. The manipulator may have singular configurations in which the performances in some directions are extremely poor while in others are extremely good. Conversely the manipulator may have configurations where the performances are identical in all directions. This behaviour can be described through the concept of isotropy [2], [3]. Naturally, the design of an isotropic machine is desirable because it assures homogeneous performances in all the directions in terms of accuracy, repeatability, stiffness, maximum force and velocity [4]. This paper deals with the optimization of a parallel kinematics machine (PKM) from the isotropy point of view examining the case of a five-bar linkage manipulator which is a widely used mechanism in industrial applications like pick-and-place, assembly and positioning. First the geometrical dimensions of the system are optimized to achieve the best performance in term of isotropy inside a given workspace. This process consists in optimizing some indices related to the Jacobian matrix [5], [6] of the system (an example is shown in figure 1), representing the kinematical relationships between the end-effector and the actuated joints. In this contest a simple geometric condition to guarantee isotropy will be demonstrated. Secondly, the performance of the system are investigated when the drive system changes from rotational motors to linear actuators. The resulting PKM is characterized by different performance due to a different behaviour of the actuators which are no more independent by the position of the manipulator. Differences between the two manipulators are shown, evaluating how the drive system can affect the kinetostatic properties of the robot.

## 1 SYNTHESIS OF A MANIPULATOR

### 1.1 Kinetostatic duality

The kinetostatic properties of a serial or parallel manipulator in term of achievable velocity, force, stiffness and motion precision can be studied using the well known relations:

$$\dot{S} = J\dot{Q} \qquad F_q = J^T F_s \tag{1}$$

$$J = \frac{\partial S}{\partial Q} \tag{2}$$

where J is the jacobian matrix which relates the gripper velocities $\dot{S}$ with those of the actuators $\dot{Q}$, as well as the forces (or torques) $F_q$ exerted by them with the forces and the torques $F_s$ applied to the gripper. Since J appears both in the velocity and in the force/torque relations, the set of the two equation (1) is said to represent the *kinetostatic duality*.

### 1.2 Synthesis criteria

In some occasions the synthesis of a robot is led evaluating its performances through some indices related to the jacobian matrix [5], [9], [10]. In order to discuss them we remember the $i^{th}$ singular value $\sigma_i(A)$ of a matrix A is defined as the square root of the eigenvalue $\lambda_i$ of the matrix $A^T A$:

$$\sigma_i(A) = \sqrt{\lambda_i \left(A^T A\right)} \tag{3}$$

where $\lambda_i \geq 0$. Let's write $\lambda_{min} = \min\left(\lambda_i\right)$ and $\lambda_{max} = \max\left(\lambda_i\right)$.
Main indices are:

- $I_1$ - Minimum Stiffness

  $$I_1 = \sigma_{min} = \sqrt{\lambda_{min}} \tag{4}$$

  It's the minimum singular value that corresponds to the minimum stiffness;

- $I_2$ - Manipulability

  $$I_2 = \prod_{i=1}^{N} \sigma_i = |det(J)| \tag{5}$$

  It's the determinant of the jacobian matrix;

- $I_3$ - Isotropy

  $$I_3 = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} = \sqrt{\frac{\sigma_{max}}{\sigma_{min}}} = \text{cond}(J) \tag{6}$$

  It's the condition number of the jacobian matrix. When it is verified cond($J$)=1, the minimum and the maximum eigenvalues coincide and the manipulator is defined as isotropic. The same condition of isotropy can be expressed as [13]:

  $$JJ^T = kI \quad \leftrightarrow \quad J^T J = k' I \tag{7}$$

  where $k$, $k'$ are scalars and $I$ is the identity matrix. It means the robot is isotropic if the jacobian matrix is proportional to an orthogonal matrix.

### 1.3 Ellipses of manipulability

The condition of isotropy can be interpreted by defining the ellipsoids of manipulability of the robot [2], [5]. Imagine that the actuators can generally have a "total speed" $k_v$ to share between them with the constraint that the sum of the squares of the velocity is constant. We obtain:

$$\left\|\dot{Q}\right\|^2 = k_v^2 = \dot{Q}^T \dot{Q} \tag{8}$$

$$\dot{S}^T J^{-T} J^{-1} \dot{S} \leq k_v^2 \tag{9}$$

$$\dot{S}^T \left(JJ^T\right)^{-1} \dot{S} \leq k_v^2 \tag{10}$$

Once the maximum speed reached by each actuator is set, equation (10) defines an ellipse in the plane $\dot{x} - \dot{y}$ described by the matrix $\left(JJ^T\right)^{-1}$. More precisely the lengths of the principal axis of the ellipse, which correspond to the inverse of minimal and maximal eigenvalues of $JJ^T$, are considered as an image of the minimum and maximum velocity amplification factor. According to this

representation, the ellipse of manipulability in velocity represents the locus of points of maximum velocity. Isotropy is achieved when ellipses (or ellipsoids) become circles (or spheres) [5]. Moreover, since the determinant of a matrix is the product of its eigenvalues, the area of the ellipse is proportional to det($J$).
Considering the cited kinetostatic duality, similar considerations can be done for forces and torques (cfr. 1.1) [2].

## 2 GENERALIZED JACOBIAN MATRIX

Usually, in the kinematic optimization of a manipulator, the effect introduced by a non homogeneous behavior of the drive system is not considered. Anyway, when actuators are not identical (i.e. different maximum velocities), the manipulator, even if in an isotropic configuration, can't generate the same maximum velocity along all directions, deforming the ellipses of velocity.
In this case, instead of studing the jacobian matrix J, it will be necessary to consider the generalized jacobian matrix $J^*$:

$$J^* = JD \tag{11}$$

or its inverse, where D is a matrix (generally diagonal) to be defined in order to properly weigh the different contributions of $\dot{Q}$ or $F_q$.
Therefore it is essential, in the design of a robot, to evaluate the performance indices (4), (5), (6) as related to the generalized jacobian matrix $J^*$ rather than J.
The effect described by the D matrix can be better explained introducing some definitions about isotropy:

- geometrical isotropy, it's reached when the manipulator, independently by its drive system, is in an isotropic configuration. In this case:

$$\text{cond}(J) = 1 \quad \text{or} \quad JJ^T = kI \tag{12}$$

- drive system isotropy, it's achievable if the behaviour of the drive system is the same in all the configurations reached by the manipulator. It holds:

$$\text{cond}(D) = 1 \quad \text{or} \quad DD^T = k'I \tag{13}$$

- effective isotropy, it's when the robot, driven by a defined drive system, has an isotropic behaviour. In this condition, independently by the condition number of J and D, one gets:

$$\text{cond}(J^*) = 1 \quad \text{or} \quad J^*J^{*T} = JDD^TJ^T = k''I \tag{14}$$

However the concepts of geometrical isotropy and drive system isotropy sometimes don't have a physical meaning if they are considered separately. An example is the polar robot depicted in figure 1: it is actuated by a linear motor that changes the arm length and by a rotational motor that defines the direction.
Let be $S = [x, y]^T$ and $Q = [\alpha, \rho]^T$. We get:

$$\begin{cases} x = \rho cos(\alpha) \\ y = \rho sin(\alpha) \end{cases} \quad \rightarrow \quad J = \begin{pmatrix} -\rho\text{sin}\alpha & \text{cos}\alpha \\ \rho\text{cos}\alpha & \text{sin}\alpha \end{pmatrix} \tag{15}$$

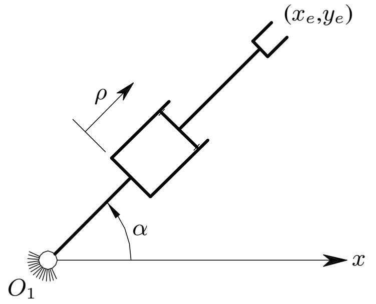


Figure 1: A polar robot

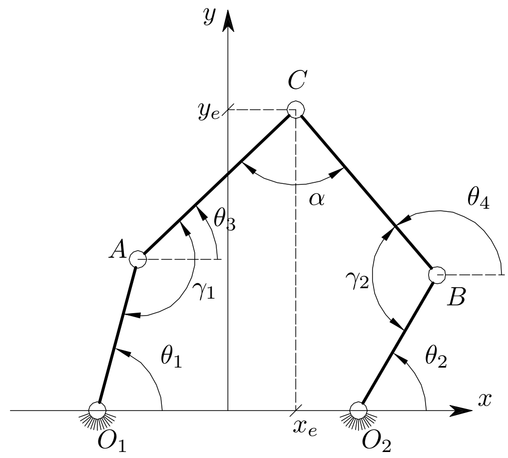


Figure 2: Layout of the robot

The condition of isotropy shown in (7) becomes:

$$JJ^T = \begin{pmatrix} \rho^2 & 0 \\ 0 & 1 \end{pmatrix} \quad \rightarrow \quad \rho^2 = 1 \tag{16}$$

This condition has no physical meaning since it depends on the units used to measure the length (i.e. $\rho$=1mm or $\rho$=1m). In this cases, the generalized jacobian matrix (11) should be analyzed where the D matrix will be defined to describe some relevant characteristics, for istance the different behaviour of the actuators.

## 3 A 5R 2DoF MANIPULATOR

To better understand the effect of the drive system, represented by the D matrix, let's approach the analysis of a kinematical project of a parallel kinematic robot. With reference to figure 2, we can analyze the planar manipulator 5R 2dof: it consists of 4 links (5 considering the ground) connected by five revolutionary joints (R) two of which are located on ground and driven by motors. The position $S = [x_e, y_e]^T$ of the joint C can be expressed as function of the actuated joints coordinates $S = [\theta_1, \theta_2]^T$ as described in [9].

The jacobian matrix can be written as [12]:

$$J = \frac{a}{\sin\alpha} \begin{pmatrix} \sin\theta_4 \sin\gamma_1 & \sin\theta_3 \sin\gamma_2 \\ \cos\theta_4 \sin\gamma_1 & \cos\theta_3 \sin\gamma_2 \end{pmatrix} \tag{17}$$

where $\gamma_1$ and $\gamma_2$ are the transmission angles of the mechanism.
Indices $I_2$ and $I_3$ related to the jacobian matrix can be graphically represented as a function of manipulator position (figg. 3, 4).

This manipulator is generally driven by electrical motors placed in the joints on the ground or by means of linear motors that impose rotations to the same links [7], [8]. For the first solution, the behaviour of the actuators is independent of the configuration assumed by the robot and then the matrix D cabe easily defined. Generally, if the two motors are identical, it coincides with the identity

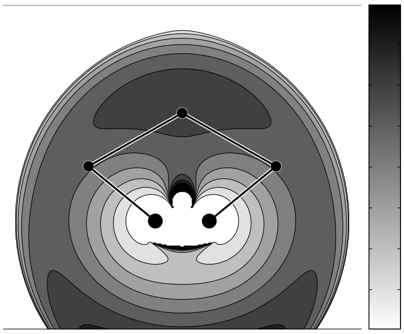


Figure 3: Values of index $I_2$ in the manipulator workspace

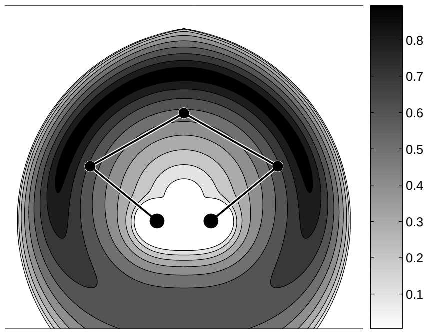


Figure 4: Values of the inverse of index $I_3$ in the manipulator workspace

matrix.
Otherwise, in the second configuration, different technologies can be used to drive the manipulator (pneumatic or hydraulic actuators, linear motors, etc..). In these cases the behaviour of the actuators is dependent on the configuration assumed by the robot and the D matrix is more complex.
For a planar manipulator actuated by linear motors (fig. 5) the "joints coordinates" can be represented by the displacements $l_1$, $l_2$. To obtain the D matrix is necessary to analyze the relationships between the new joints coordinates ($l_1$, $l_2$) and the previous ones ($\theta_1$, $\theta_2$).

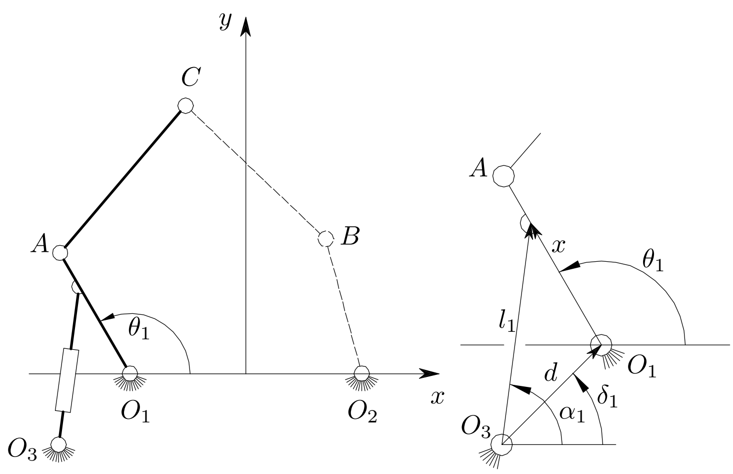


Figure 5: Model of a generic machine

Writing the equation of closure, as shown in figure 5, one gets:

$$l_1 e^{i\alpha_1} = d_1 e^{i\delta_1} + x_1 e^{i\theta_1} \tag{18}$$

Deriving with respect to time:

$$\dot{l}_1 e^{i\alpha_1} + l_1 i\dot{\alpha}_1 e^{i\alpha_1} = x_1 i\dot{\theta}_1 e^{i\theta_1} \tag{19}$$

and projection on real and imaginary axis:

$$\begin{cases} \dot{l}_1 sin\alpha_1 + l_1\dot{\alpha}_1 cos\alpha_1 = x_1\dot{\theta}_1 cos\theta_1 \\ \dot{l}_1 cos\alpha_1 - l_1\dot{\alpha}_1 sin\alpha_1 = -x_1\dot{\theta}_1 sin\theta_1 \end{cases} \tag{20}$$

one gets:

$$\dot{l}_1 = \dot{\theta}_1 x_1 sin\left(\alpha_1 - \theta_1\right) \tag{21}$$

Therefore, the first diagonal element of the D matrix is:

$$D_{11} = \frac{1}{x_1 sin\left(\alpha_1 - \theta_1\right)} \tag{22}$$

Doing the same for the second joint, one gets:

$$D_{22} = \frac{1}{x_2 sin\left(\alpha_2 - \theta_2\right)} \tag{23}$$

Finally the D matrix is:

$$D = \text{diag}\left(D_{11}, D_{22}\right) \tag{24}$$

Since the D matrix is diagonal, $D_{11}$ and $D_{22}$ coincide with the eigenvalues of D and the condition of isotropy becomes $D_{11} = D_{22}$. This condition can be guaranteed only when the manipulator is in a “symmetric configuration”.

## 4 EFFECTS OF D MATRIX ON ISOTROPY

Known the expression of the generalized jacobian matrix $J^*$, the design of the robot could be carried out by optimizing the numerical indices defined in equations (4), (5), (6). However, to highlight the contribution of the actuators evaluated through the D matrix, a geometric interpretation of this effect will be shown.

### 4.1 Geometrical isotropy

The condition of isotropy can be evaluated considering the geometry of the manipulator. It has been said that the ellipses of manipulability in velocity approximate the locus of points of maximum speed. To get that ellipse one can impose the end effector to move along a certain direction at the maximum speed, by pushing up the motors. Obviously the contribution of each motor will depend on the direction to follow. Let’s say, for example, we want to move the end-effector of the manipulator along the direction $r$ orthogonal to the link $AC$ (fig. 6). To achieve maximum speed ($v_{2,max}$) in this direction the system will be forced to hold the motor 1, while moving the motor 2 at maximum speed ($\omega_{2,max}$). The reverse should be done if we want to reach the maximum speed along a direction orthogonal to $BC$. By repeating this for all directions, the locus of points will be represented by a rectangle that, when the robot is isotropic, degenerates into a square (fig. 7). For convenience of representation, it is usual to approximate these polygons respectively with ellipses, or with circles (sec. 1.3).
Note that, depending on the configuration of the robot, the ellipse of manipulability in velocity has different shapes and sizes (fig. 8).

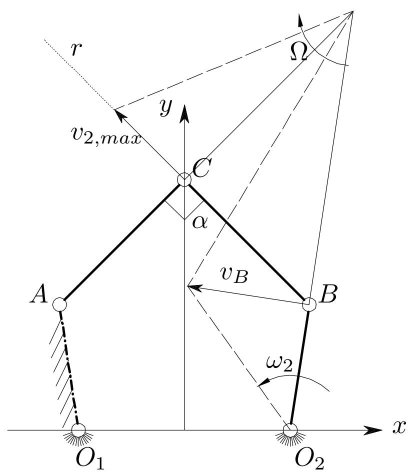


Figure 6: Maximum velocity along $r$ direction

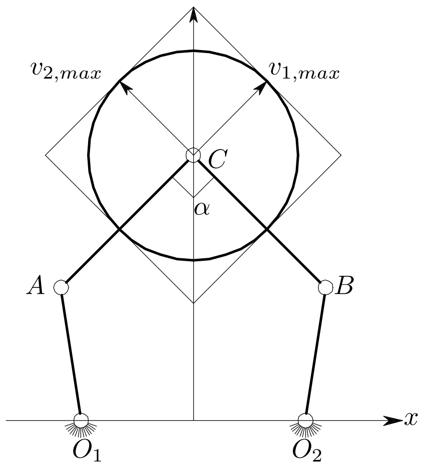


Figure 7: Ellipse of manipulability in velocity in an isotropic configuration

To give a geometrical interpretation of isotropy let's remember eq.(7): it assures that the robot is isotropic if the jacobian matrix is proportional to an orthogonal matrix. It happens when $v_{2,max}$ is normal to $v_{1,max}$ (figg. 6, 7) that means, in this particular case, that links $AC$ and $CB$ ar perpendicular and $\alpha = 90^o$. Moreover, since it should be $|v_{2,max}| = |v_{1,max}|$, transmission angles have to be identical ($\gamma_1 = \gamma_2$).

However, as discussed in section 2, this condition is not sufficient for isotropy, since it can't evaluate the behaviour of actuators that can be dependent on the configuration reached by the robot.

### 4.2 Drive system isotropy

The condition of isotropy, in fact, requires that the manipulator has the same properties (maximum force, maximum achievable speed, etc..) along all the directions. In cases where the manipulator is driven by the identical motors which are characterized by the same performance in every configuration of the robot, it is immediate to observe that geometrical isotropy ($\alpha = 90^o \wedge \gamma_1 = \gamma_2$) is sufficient to ensure effective isotropy.

Otherwise, if the robot in the same configuration is driven by two different motors (for example that can reach different maximum speeds), the condition $\alpha = 90^o$ can be meaningless (fig. 9). It is important to note that, to ensure the effective isotropy, if the manipulator is geometrically isotropic, it is necessary that the behaviour of the actuators is isotropic.

Vice versa, it may happen that a robot has a geometric configuration such that the isotropic condition is not guaranteed: figure 10 shows the case of a manipulator in which the driven links have different lengths. It's evident that, using the same motors to move the manipulator, the ellipse of manipulability in velocity can't be a circle even if $\alpha = 90^o$. However, if a reducer whose transmission ratio is $\tau = c/a$ is coupled with the motor 1, the system would be effectively isotropic, even if the manipulator is far from the geometrical isotropy. In this case the matrix D looks like:

$$D = \begin{pmatrix} \tau & 0 \\ 0 & 1 \end{pmatrix} \tag{25}$$

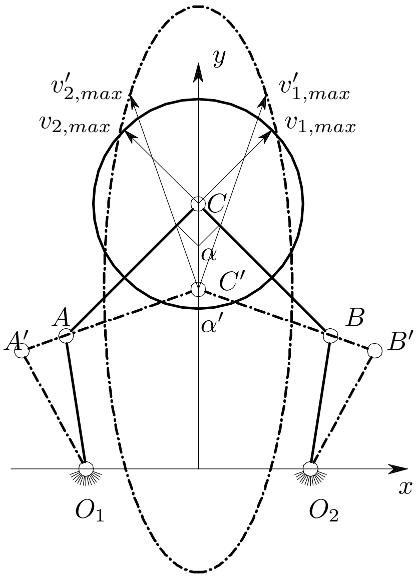


Figure 8: Ellipses of manipulability in velocity as function of manipulator configurations

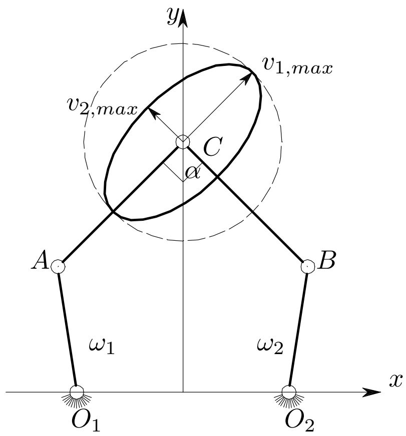


Figure 9: Ellipse of manipulability in velocity in an isotropic configuration when the motor is driven by two different motors

In the case of a robot driven by linear motors the behaviour of actuators is strongly influenced by the configuration reached by the manipulator. In particular, the D matrix (24) shows that the condition of drive system isotropy can be reached only in the symmetrical configurations of the robot.

### 4.3 Solution for a 2dof planar manipulator driven by linear actuators

To eliminate this constraint and obtain drive system isotropy in all the manipulator workspace, it is necessary that the directions of the actuators remain constant. This condition is made possible degenerating the kinematical structure of the manipulator (fig. 12): moving the joints connected to ground to an infinite distance, so that rotations can be approximated by translations. The kinematic structure of the robot can be represented as in figure 13.
This configuration allows to reach drive system isotropy in the whole workspace, making the condition $\alpha = 90^o$ sufficient to guaranty effective isotropy.

## 5 CONCLUSIONS

The paper discusses the concepts of isotropy and the influence of the drive system on the kinetostatic properties of a manipulator. Differences between geometrical and effective isotropy are explained introducing a matrix describing the actuators behaviour in the robot workspace. A case study has been discussed considering a 2 dof planar manipulator: isotropy has been analyzed as a function of drive system used to move the robot. Finally an optimized solution to reach drive system isotropy has been presented for a manipulator driven by linear motors.

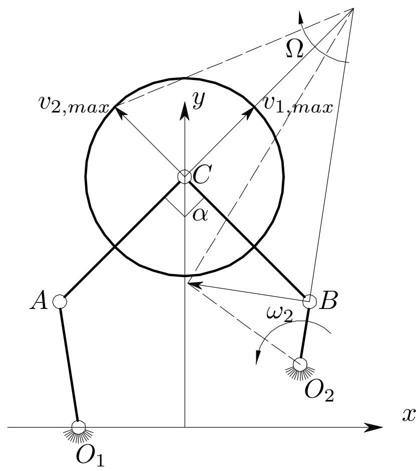


Figure 10: A condition of effective isotropy

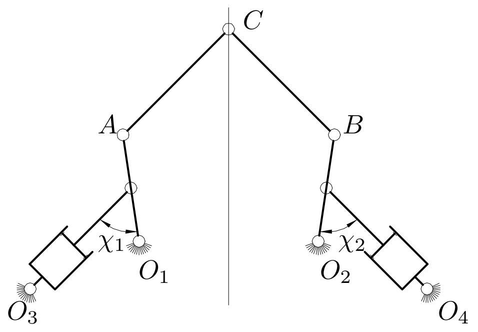


Figure 11: Planar manipulator driven by linear motors

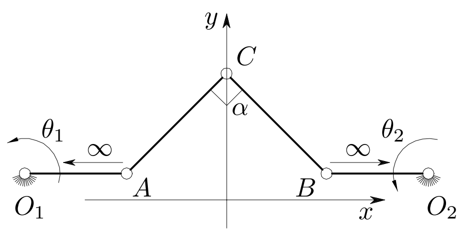


Figure 12: A change in manipulatore kinematical layout

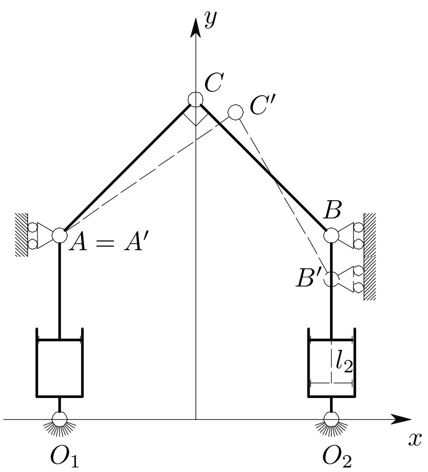


Figure 13: A planar 2dof manipulatore driven by linear actuators characterized by drive system isotropy.

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