Mechanism of leakage in flat seals

F. Bottiglione[1], G. Carbone[2], L. Mangialardi[2], G. Mantriota[1]
[1]*DIASS - Politecnico di Bari, Via A. De Gasperi s/n, 74100 Taranto, Italy*
E-mail: f.bottiglione@poliba.it, mantriota@poliba.it

[2]*DIMeG - Politecnico di Bari, Viale Japigia 182, 70182 Bari, Italy*
E-mail: carbone@poliba.it, luigi.magialardi@poliba.it

Keywords: Leakage, seals, percolation theory, critical path analysis.

SUMMARY. We present the results of a novel approach to estimate the fluid leakage in flat seals, based on the analogy between the seal-substrate interface and a porous medium. We assume that the interface is a random distribution of non-contact patches (the pores) and contact spots (islands): leakage may occur only through the pores. The lateral size and height of the pores are distributed according to a probability density function, that we calculate on the basis of Persson's theory of contact mechanics. Our theoretical approach is based on a novel percolation scheme that can stimulate further theoretical or experimental investigations. Within this percolation scheme we apply critical path analysis (CPA) to calculate the hydraulic conductivity of the medium. We also compare our predictions with other calculations very recently presented to the scientific community.

1 INTRODUCTION

In several mechanical devices the simplest way to prevent fluid flow under a pressure gradient is the use of a mechanical seal. A typical flat seal is usually made of a relatively soft elastic block squeezed against a stiffer body, so that the nominally flat contact interface behaves as a wall the liquid can not pass through. However, two nominally flat surfaces in contact often permit some fluid to leak because roughness makes the contact between the sealing surfaces imperfect. As a consequence, the fluid can find a path to percolate and leak between the two chambers at different pressures. Although seals are often one of most critical components in practical engineering applications only very recently some theoretical approaches have been proposed to calculate leak rate of seals [1], [2], [3] and clarify the basic physics behind the problem. These theories are based on multiscale theory of contact mechanics [4] and make use of percolation theory [5] and Critical Path Analysis (CPA), which have been developed by physicists in seventies to calculate the conductivity of a network of randomly distributed resistors [6], [7], [8], [9]. The basic physics of the phenomenon is here described. According to Ref. [4], the apparent area of contact $A(\zeta)$ between the two surfaces is a function of the magnification $\zeta = L/\lambda$ for any given applied load and material properties. Here λ is the length scale at which we observe the system, and L is the lateral size of the nominal contact area A_0. When $\zeta = 1$, the contacting bodies look like as they were perfectly smooth and the apparent area is just equal the nominal contact area. But at magnifications $\zeta > 1$ non contact patches will appear. The number and the extension of such non contact zones will rapidly increase as the magnification is increased. When the magnification reaches a critical value ζ_c (percolation threshold), one of these clusters becomes so large to connect the two opposite sides of the seal, i.e. it percolates and leakage of fluid may occur along this existing channel. Percolation theory and theory of contact mechanics allow to evaluate the threshold value ζ_c, and to determine the lateral size and height of the smallest constriction along the channel, which we assume to be responsible of the entire pressure drop. In the earliest approaches (See Refs. [1], [2]) the value of ζ_c and the size of critical constriction along the percolation channel at $\zeta = \zeta_c$ was considered to be enough to estimate the leakage of fluid and, hence, the conductivity σ_S of the seal. However, we observe that the calculation of seal conductivity needs also the distance χ between two adjacent channels to be calculated, as well as the distance l between the critical constrictions along the same channel. These two quantities in previous approaches were fixed arbitrarily. In this article we present a further step (Ref. [3]) and suggest a possible improvement of the methodology. Indeed, we first observe that percolation theory predicts that at the critical magnification the distance $\chi(\zeta_c)$ between to adjacent channels is infinitely large, and (unless also the distance $l(\zeta_c)$ is infinite) one concludes that the seal conductivity is necessarily zero. Therefore, we infer that fluid leakage will actually occur on channels which are formed at magnifications beyond the critical value ζ_c. In this case, although the constrictions of such channels are smaller (with a smaller conductance g), the distance χ between these channels is finite and the seal conductivity σ_S must result larger than zero i.e. $\sigma_S > 0$. One can then, by following the CPA technique (see Ref. [9] for details), calculate the actual value of the seal conductivity by means of an optimization procedure.

2 THE MODEL

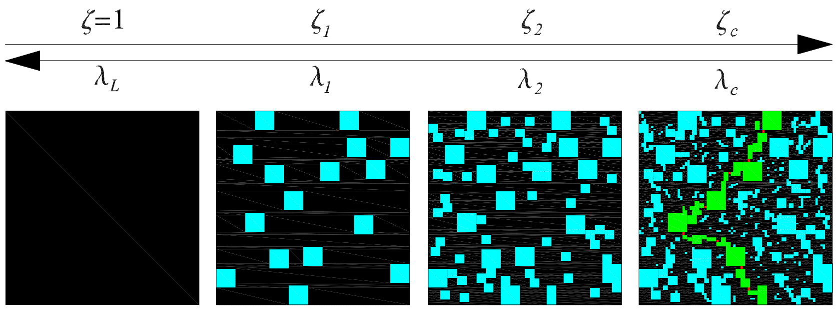

Figure 1: Sketch of the continuous percolation scheme adopted. The figure shows the apparent area of contact (black) when the magnification is increased (from left to right). Randomly distributed voids (cyan) with size $\sim \lambda$ initially make unconnected non contact areas. As magnification is increased new vacancies of smaller dimensions join the non contact area and form clusters. Only when $\zeta = \zeta_c$ one cluster of percolation (green) in an infinitely extended region connects the two opposite sides permitting the leakage. Rate limiting pores are schematically shown (red). In the proposed percolation scheme, voids are randomly distributed squares, with dimensions following a probability density function $\eta(\lambda)$ that the authors have derived from a theory of contact mechanics.

We assume that the seal is a rubber block pressed against a isotropic rough rigid surface by means a uniform distribution of pressure p_0. The nominal contact area is a rectangle of sizes l_x, l_y where l_y is the length of the seal along the liquid leakage direction and l_x the seal lateral size. We take under analysis a very large (infinite in the limiting case) square in the interior of the seal-substrate interface. We assume the square has a side of length L, and we define the magnification $\zeta = L/\lambda$, where λ is the resolution at which we observe the contact. At magnification equal to 1, the two surfaces matches perfectly at the interface, therefore leakage of liquid cannot occur at this magnification. However because of surface roughness, the squeezing pressure is not in general sufficient to guarantee complete contact, as a consequence non contact areas will appear as we reduce the length scale λ of observation, i.e. as we increase the magnification ζ. When the magnification is sufficiently large some percolation channels are formed which connect the two opposite sides of the square. The channels are separated by an average spacing χ. The size and shape of the channels

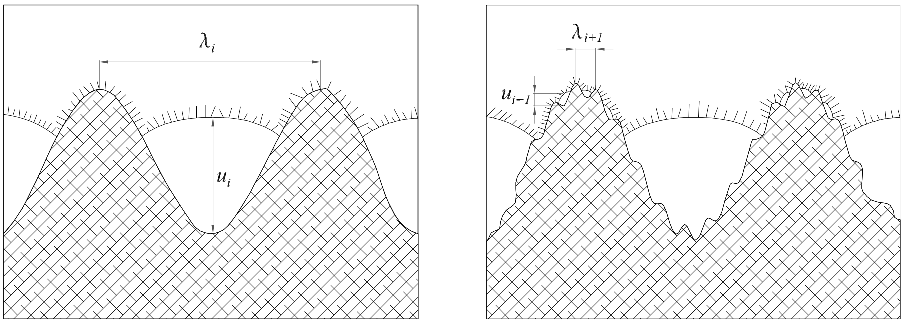

Figure 2: Sketch of a two-scale rough profile in contact with an elastic block. At scale i the voids have a width λ_i and height u_i, while at scale $i+1$ the voids have a width λ_{i+1} and height u_{i+1}. In three dimensional contact the voids have dimensions of order $\lambda_i \times \lambda_i \times u_i$. The picture is not in scale: according to theory of contact mechanics $u << \lambda$.

depend on the roughness and on the elastic deformation of the block. In particular the cross section of the channel varies along the path and one can imagine that it reaches a minimum value at some location. Assuming that the pressure drop is only caused by these strong channel restrictions, the calculation of the seal conductivity needs the distance l between these restrictions, the conductance of each restriction, and the spacing between two parallel channels. Figure 1 shows a schematic projection of the non contact areas over the *x-y* plane at different magnifications: as ζ is increased new voids are formed. Being, at each magnification ζ, the size of the new formed void of the order $\lambda = L/\zeta$, one concludes that as the magnification is increased, increasingly smaller voids may join the existing ones to form clusters of non contact areas. The size of the voids is distributed according to a continuous probability density function $\eta(\lambda)$. In particular, if $A(\lambda)$ is the apparent area of contact in units of nominal contact area at a fixed magnification $\zeta = L/\lambda$, then the probability of a square site to have a characteristic size $\lambda^* < \lambda$ is $P(\lambda^* < \lambda) = A(\lambda)$. Thus, taking the λ derivative we obtain

$$\eta(\lambda) = \frac{dA(\lambda)}{d\lambda} \tag{1}$$

where $A(\lambda)$ can be derived from numerical calculations of contact between rough surfaces or from a theory of contact mechanics. In particular we have found Persson's theory of contact mechanics [4], [10], [11] is the most convenient approach. As shown in the Figure 1 and according to [1, 2] we know that at the critical magnification ζ_c the non contact area percolates and one percolating channel connects the two opposite sides of the nominal contact area. This critical value of the magnification ζ_c is found enforcing the condition $A_{NC}(\zeta) = P_c$ where $A_{NC}(\zeta) = 1 - A(\zeta)$ is the normalized non contact area and P_c is the critical threshold probability which depends on the percolation scheme [5]. We assume $P_c \approx 0.6$ as suggested in Refs. [1] and [12]. At the critical magnification ζ_c, percolation theory shows that just one channel is formed connecting the two opposite sides of the infinite interface. For this reason, although the channel will carry a certain amount of fluid through its smallest restriction of size $\lambda_c = L/\zeta_c$, its contribution to the overall conductivity σ_S of the seal vanishes. We must conclude that the hydraulic conductivity of the seal is instead determined by channels which are formed at magnification $\zeta > \zeta_c$. In fact, if the magnification is increased beyond the critical value, additional non contact zones appear [9], so that at very large magnifications the interface will look like a large sea made of non contact areas with a distribution of small islands of contact zones. We map the sea as an ensemble of squares of random distributed lateral size $\lambda = L/\zeta$. Each square has a height $u(\zeta) \ll \lambda$, which we calculate on the basis of the theoretical approach presented in Refs. [10], [11]. Being, $u \ll \lambda$ the hydraulic conductance $g(\zeta)$ of each

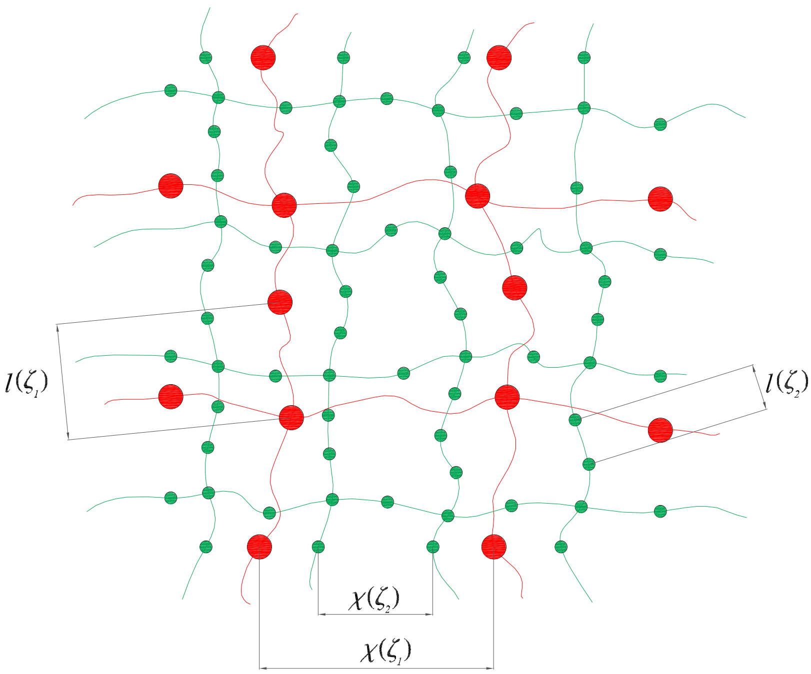

Figure 3: Sketch of the equivalent hydraulic network. When $\zeta = \zeta_1$ the percolation channels have distance $\chi(\zeta_1)$ from each other and the the distance between the rate limiting conductances (big red circles) is $l(\zeta_1)$. When $\zeta = \zeta_2$ the percolation channels have distance $\chi(\zeta_2)$ and the the distance between the rate limiting conductances (small green circles) is $l(\zeta_2)$. The two networks have different conductivity σ, that is so a function of the magnification ζ.

non-contact square is

$$g(\zeta) = \frac{u^3(\zeta)}{12\mu} \tag{2}$$

where μ is the fluid viscosity. At any given magnification $\zeta \geq \zeta_c$ the channels will be constituted of squares of size λ larger than L/ζ, i.e. we assume that at any given magnification ζ, the smallest restriction that the fluid flow encounters along the channel has lateral size L/ζ, height $u(\zeta)$ and conductance $g(\zeta)$ (it is noteworthy to observe that on each channel the number of smallest restrictions can be larger than one). We further assume that the total pressure drop on each channel is determined only by the smallest restrictions. Now, let us focus on that ensemble of channels with smallest restrictions of lateral size L/ζ, and call $\chi(\zeta)$ the average distance between the channels and $l(\zeta)$ the average distance between the smallest restrictions along the channel. A sketch of this hydraulic system is shown in the Figure 3 for two different values of ζ with $\zeta_2 > \zeta_1$. We can then easily calculate the conductivity of the ensemble of channels at magnification ζ as

$$\sigma(\zeta) = g(\zeta) \frac{l(\zeta)}{\chi(\zeta)} \tag{3}$$

According to percolation theory χ follows a power law

$$\chi(\zeta) = \chi_0 \left| A(\zeta_c) - A(\zeta) \right|^{-\alpha} \tag{4}$$

nearby the percolation threshold, with α an universal exponent that for two dimensional systems is about $4/3$ [5] and χ_0 a reference length. Eq. (3) shows that the conductivity of the ensemble of channels with smallest restrictions of lateral size L/ζ depends on the magnification ζ. In particular as ζ is increased the correlation length $\chi(\zeta)$ rapidly decreases from an infinite value at ζ_c to a finite value at $\zeta > \zeta_c$. As a consequence, the conductivity $\sigma(\zeta)$ will increase as the magnification is increased. However, increasing the magnification also determines a strong reduction of the conductance $g(\zeta)$, whereas the quantity $l(\zeta)$ may decrease or change weakly with the magnification. Therefore, an optimal value ζ_{opt} can be found at which the conductivity takes the maximum value $\sigma(\zeta_{opt}) = \sigma_{opt}$. In the spirit of CPA, one assumes that all the fluid leakage is carried only by channels with smallest restriction $\lambda_{opt} = L/\zeta_{opt}$, and, therefore, that the total conductivity σ_S of the seals is given by $\sigma_S = \sigma_{opt}$. In order to carry out a quantitative estimation of the seal conductivity we need to calculate the average distance l among the critical restrictions. Some authors argue [5] that $l = \chi$. Such an assumption simply gives: $\sigma_S = \sigma_{opt} = g(\zeta_c) = \sigma_c$, which is the same value obtained by Persson [1]. However other authors [9] assert that, in general, the quantity l may be much smaller then χ, and should depend only on the probability density function of conductances $\eta(\lambda)$. Following this second idea the estimation of the quantity $l(\zeta)$ can be carried out as shown in Ref. [3] and gives the following expression of the conductivity of the ensemble of channels with smallest restriction of size $\lambda = L/\zeta$:

$$\sigma = g\frac{\lambda}{\chi_0} \left[A(\zeta_c) - A(\zeta) \right]^{\alpha} \left[\frac{1 - A(\zeta)}{A(\lambda + \Delta\lambda/2) - A(\lambda - \Delta\lambda/2)} \right]^{1/2} \tag{5}$$

where $\Delta\lambda$ is a properly selected class breadth. Eq. (5) holds true only close to the percolation threshold provided that $\zeta > \zeta_c$. According to CPA the maximum value of σ given by Eq. (5) gives the value of the conductivity of the seal. We remark that the actual area of contact and the separation between surfaces as functions of the magnification have been calculated following Refs. [4, 10].

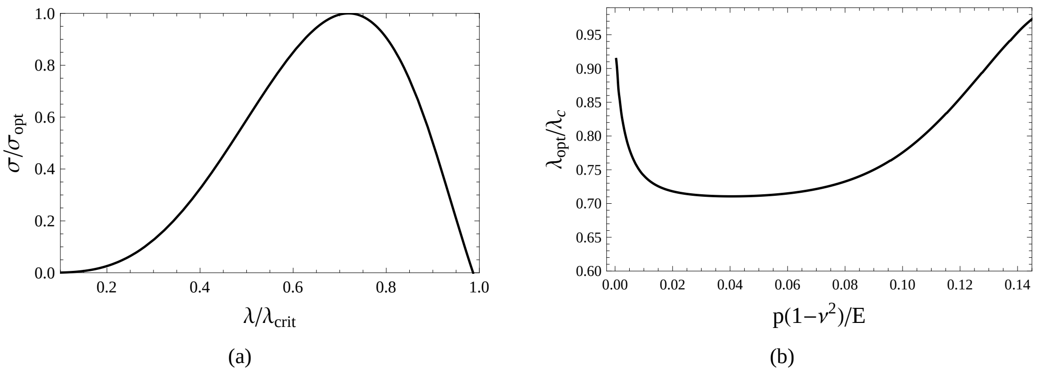

Figure 4: The ratio σ/σ_{opt} of channels having the smallest restrictions with lateral size λ as a function of the ratio λ/λ_{opt} at a given applied pressure $p_0 \sim 1$ MPa (a) and the ratio λ/λ_{opt} as a function of non dimensional load $p_0\left(1-\nu^2\right)/E$ (b). Results are shown for a surface with features described at the beginning of the section and with an $rms = 2$ μm.

3 RESULTS

In this section we present the results of our theoretical approach. We consider a flat seal made of a rubber block squeezed against a rough rigid substrate. The rough surface is a isotropic self affine fractal for $q_0 < q < q_1$, with fractal dimension $D_f = 2.2$. The quantity $q_0 = 2\pi/L$ and q_1 is the short-distance cut-off vector i.e. $q_1 = 2\pi/\lambda_1$ with λ_1 the shortest wavelength of the surface roughness. We assume that the lateral size of the non-contact squares is a multiple of the shortest length-scale λ_1, so that we can assume $\Delta\lambda = \lambda_1$. Also we use $\chi_0 \approx L$. The surface power spectrum of a self-affine surface satisfies the relation

$$C\left(q\right) = C_0\left(\frac{q}{q_0}\right)^{-2(H+1)}; \qquad q_0 < q < q_1 \tag{6}$$

where H is the Hurst exponent $H = 3 - D_f$. C_0 can be calculated as

$$C_0 = \frac{\left\langle h^2\right\rangle}{\pi}\left[\frac{q_0^{2(H+1)}}{H}\left(q_0^{-2H} - q_1^{-2H}\right)\right]^{-1} \tag{7}$$

where $\left\langle h^2\right\rangle$ is the root mean square roughness of the surface.

Numerical calculations have been carried out for $q_0 = 10^4$ m^{-1}, $q_1 = 7.8\ 10^9$ m^{-1}. The rubber has a Young's modulus $E = 10$ MPa and a Poisson ratio $\nu = 0.5$. The fluid is assumed to be incompressible and Newtonian with a viscosity $\mu = 0.001$ Ns/m^2. In Figure 4(a) the normalized conductivity σ/σ_{opt} is shown as a function of the ratio λ/λ_c at a given applied pressure $p_0 \simeq 1$ MPa and for a surface $rms = 2$ μm. As expected, the conductivity of those ensemble of channels with smallest restrictions of size λ, has a local maximum for $\lambda < \lambda_c$. Local maxima identify the wave length λ_{opt}, which is shown in Fig. 4(b) as a function of the applied load in units of λ_c. Observe that the optimal value λ_{opt} has a local minimum equal to about the 70% of the critical wave length λ_c, so that in the whole range of loads the difference between the optimal value λ_{opt} and the critical value λ_c is less than 30%. In fig. 5(a) we show the critical wave length λ_c (dashed lines) and of the optimal wave length λ_{opt} as a function of the applied load in a linear-log diagram. The figure

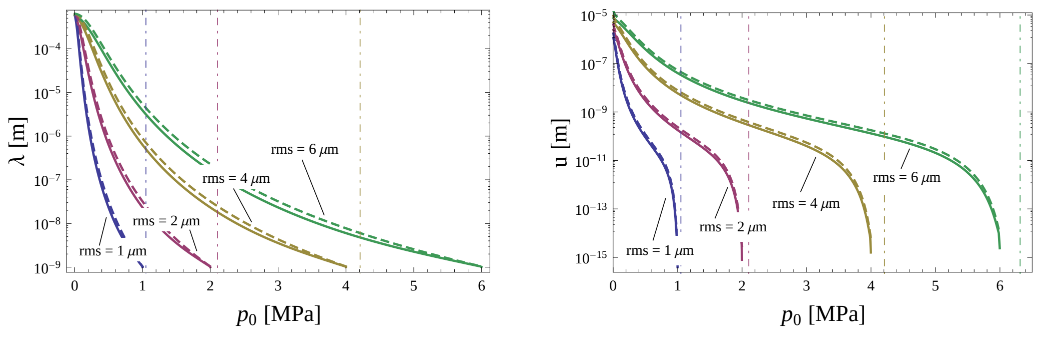

Figure 5: The lateral size λ of the smallest restriction at critical (dashed) and optimal (continuous) magnification (a) and the height u of the smallest restriction at critical (dashed) and optimal (continuous) magnification (b) as a function of the nominal applied load p_0. Results are shown for the self affine fractal surface described in the text and for values of the root-mean-square roughness $1\ \mu m$, 2 μm, $4\ \mu m$, $6\ \mu m$.

again shows that the difference between the two quantities is very limited. The same behavior is followed by the height of the smallest restriction at the critical magnification $u_c = u(\zeta_c)$ and the height of the smallest restriction $u_{opt} = u_{opt}(\zeta_{opt})$ at the optimal magnification [see Fig. 5(b)]. As expected both the lateral size and the height of the smallest restriction along the channel rapidly decrease as the load is increased. The curves are plotted for different values of the root mean square (rms) roughness of the substrate, and show how strong is the influence of this surface parameter in determining the size of the channel restriction. Being u_{opt} and u_c relatively close to each other, we expect that the optimal conductance g_{opt} turns out to be of the same order of magnitude as the critical conductance $g_c = g(\zeta_c)$. However the optimal conductivity may instead differ very much if compared to the conductivity $\sigma_c = g(\zeta_c)$, that, instead, would be obtained under the assumption that $l = \chi$. The reason of this very strong difference can be explained if one considers that following the arguments of Ref. [3] about the calculation of l, it can result several order of magnitude smaller than the correlation length χ. Finally, Figure 6 shows the seal conductivity versus the nominal applied load in a linear-log diagram assuming: (i) that $l = \chi$ (dashed line) and (ii) that l follows Ref. [3] (solid line). The former assumption gives the same result as in Persson's theory [1]. The latter, instead, according to Eq. (5) needs the definition of the two constants $\Delta\lambda$ and χ_0. As stated above, we have used $\Delta\lambda = \lambda_1$ and $\chi_0 = L$. Such an arbitrary choice of the constants makes rather difficult a clear comparison between the two different approaches. Therefore an experimental activity should be carried out to choose the best parameters $\Delta\lambda$ and χ_0 and compare the different methodologies to determine which one gives the best results.

4 CONCLUSIONS

In this paper we show the results of a novel approach to address the problem of fluid leakage in seals. The approach is based on critical path analysis and percolation theory. It makes use of a recent theory of contact mechanics between rough surfaces to calculate the contact area between the contacting solids and the separation at interface as functions of the magnification. These are, indeed, fundamental quantities to calculate the conductance of the micro channels carrying the liquid flow. We show that the conductivity of the seals is strictly related to the distance χ between two adjacent

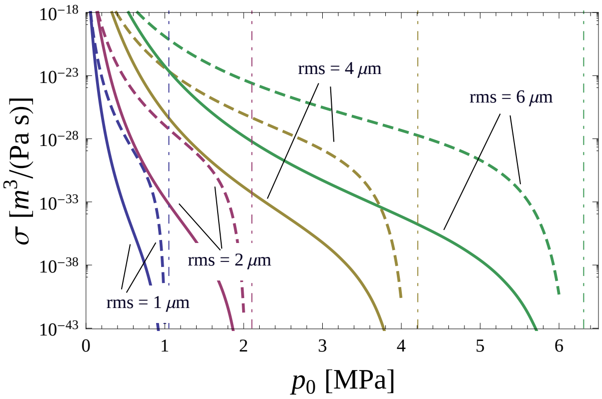

Figure 6: Seal conductivity calculated assuming that l is calculated as suggested by [3] (continuous line) and that $l = \chi$ (dashed line) as a function of the applied load p_0. Four values of the rms have been investigated. Surface parameters are those described in section III.

channels and to the distance l between adjacent smallest restrictions along the same channel. Some authors argue that $l = \chi$. In this case we obtain that the seal conductivity is just equal to the conductance of the smallest restriction that is encountered along the flow carrying channel which is formed just at the threshold magnification ζ_c (the quantity ζ_c is the value of magnification at which for the first time a percolation cluster of non contact regions connects the two side at different pressure of the seal). In this case our calculated value of the seal conductivity coincides with the one calculated with different approaches [1], [2]. However some authors argue that l may be much smaller than χ. In this case l must be calculated on the basis of a probability distribution of local conductances and may lead to a much smaller value of the conductivity of the seal. This, suggest to carry out a detailed experimental investigation or fully numerical calculations to clarify which one of the proposed assumptions gives more accurate predictions.

Acknowledgments

The Engineering Faculty of Taranto of the Technical University of Bari has financially supported the participation of the authors at the conference, using funds of the Provincia di Taranto for the support of the faculty's didactic and scientific activities.

References

[1] Persson B.N.J, Yang C., Theory of Leak Rate of Seals, *Journal of Physics: Condensed Matter* **20**, 315011, (2008).

[2] Bottiglione F., Carbone G., Mantriota G., Fluid Leakage in Seals: An Approach Based on Percolation Theory, *Tribology International* **42** 731-737, (2009)

[3] Bottiglione F., Carbone G., Mangialardi L., Mantriota G., Leakage Mechanism in Flat Seals, *Submitted to the Journal of Applied Physics*.

[4] Persson B.N.J, Theory of Rubber Friction in Contact Mechanics, *Journal of Chemical Physics*, **115**, 3840, (2001).

[5] Stauffer D., Aharony A., *An Introduction to Percolation Theory*, Taylor & Francis (1994).

[6] Ambegaokar, V. N., B. I. Halperin, and J. S. Langer, Hopping conductivity in disordered systems. *Physical Review B*, **4**, 2612–2621 (1971).

[7] Pollak, M., A percolation treatment of dc hopping conduction. *J. Non-Cryst. Solids* **11**, 1–24 (1972)

[8] Friedman, L., and M. Pollak, The Hall effect in the variable-range hopping system, *Philos. Mag. B* **44**, 487–507 (1981)

[9] Hunt A.G., *Percolation Theory for Flow in Porous Media*, Springer (2005)

[10] Persson B.N.J., Relation Between Interfacial Separation and Load: A General Theory of Contact Mechanics, *Physical Review Letters*, **99**, 125502, (2007)

[11] Yang C., Persson B.N.J., Contact Mechanics: Contact Area and Interfacial Separation From Small Contact to Full Contact, *Journal of Physics: Condensed Matter*, **20**, 215214, (2008)

[12] Sahlin F., *Lubrication, Contact Mechanics and Leakage Between Rough Surfaces*, Doctoral Thesis, (2008)