The contact patch test for linear contact pressure distributions

Giorgio Zavarise, Laura De Lorenzis

Department of Innovation Engineering, University of Salento, Italy
E-mail: *giorgio.zavarise@unisalento.it, laura.delorenzis@unisalento.it*

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SUMMARY. It is well known that the classical one-pass node-to-segment algorithms for the enforcement of contact constraints fail the contact patch test. This implies that solution errors may be introduced at the contacting surfaces, and these errors do not necessarily decrease with mesh refinement. The previous research has mainly focused on the Lagrange multiplier method, but the situation is even worse with the penalty method. In a recent study, the authors proposed a modified one-pass node-to-segment algorithm which is able to pass the contact patch test also in conjunction with the penalty method. In a general situation, the pressure distribution transferred across a contact surface is non-uniform. Hence, even for a contact element which passes the contact patch test under a uniform distribution of the contact pressures, the transfer of a non-uniform state of stress may give rise to disturbances related to the discretization, which affect the accuracy of the analysis. This paper, following up to the previous study, develops an enhanced node-to-segment formulation able to pass a modified version of the contact patch test whereby a linear distribution of pressures has to be transmitted across the contact surface. The proposed formulation is illustrated and some numerical examples demonstrate the good patch test performance of the enhanced contact element.

1 INTRODUCTION

Several investigations have shown that the classical one-pass node-to-segment (NTS) algorithm for the enforcement of contact constraints fails the contact patch test proposed by Papadopoulos and Taylor (Figure 1a) [1-4]. This implies that solution errors may be introduced at the contacting surfaces, and these errors do not necessarily decrease with mesh refinement. The previous research has mainly focused on the Lagrange multiplier method, to exactly enforce the contact geometry conditions. The situation is even worse with the penalty method, due to its inherent approximation which yields a solution affected by a non-zero penetration.

In a recent study, the authors analyzed and improved the contact patch test behavior of the one-pass NTS algorithm used in conjunction with the penalty method for 2D frictionless contact [4]. The proposed formulation was a modified one-pass NTS algorithm which is able to pass the contact patch test also if used in conjunction with the penalty method. More in detail, this algorithm is able to correctly reproduce the transfer of a constant contact pressure with a constant proportional penetration.

In a general situation, the pressure distribution transferred across a contact surface is non-uniform. Hence, even for a contact element which passes the contact patch test under a uniform distribution of the contact pressures, which will be called “uniform contact patch test” henceforth (Figure 1a), the transfer of a non-uniform state of stress across the contact surface may give rise to disturbances related to the discretization. Such disturbances introduce local solution errors and ultimately affect the accuracy of the analysis.

This paper, following up to the previous study [4], develops an enhanced NTS formulation able to pass the contact patch test depicted in Figure 1b, here termed "linear contact patch test". In this modified version of the test, a linear distribution of pressures has to be transmitted across the contact surface. When the continuum is discretized with linear elements, a contact element passing the linear contact patch test achieves the highest possible level of accuracy. This paper illustrates the proposed formulation, and presents some numerical examples demonstrating the good linear patch test performance of the enhanced contact element. All the details are reported in [5].

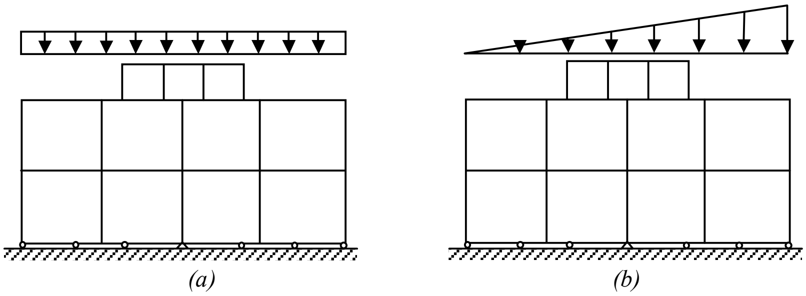


Figure 1: Patch test for constant (a) and linear (b) contact pressure distributions.

2 NTS FORMULATION

In the context of the finite element method and using the classical NTS algorithms, the geometrical non-penetration condition is enforced on the discretized geometry at each slave node. The penalty method consists in a local approximate enforcement of the geometrical non-penetration condition, obtained by minimizing a suitable modification of the potential

$$\overline{W} = W + \bigcup_{A} \frac{1}{2} \varepsilon_N g_N^2 \tag{1}$$

where W and $\overline{W}$ are, respectively, the unmodified (contactless) and the modified potential functionals of the problem; $\bigcup_A$ stands for the summation extended to all the slave nodes where the non-penetration condition has been violated; g_N is the distance between the contacting surfaces, i.e. the measure of the penetration (normal gap function, see Figure 2); ε_N is the penalty parameter. This approach corresponds to locating linear discrete springs, of zero initial length and stiffness ε_N, at those slave nodes for which the evaluation of the normal gap, g_N, detects a penetration state. The elongation of these springs corresponds to the value of the penetration [4].

3 NTS WITH AREA REGULARIZATION (NTS-AR)

Prior to any other consideration, a specific issue of the penalty method in the classical NTS algorithm has to be solved. The use of discrete springs at the slave nodes prevents the contact patch test (uniform or linear) from being passed, due to the non-uniform contact areas and the constant penalty parameter associated to the various slave nodes. This problem can be solved by considering the stiffness of each nodal spring as the result of an integration over the "area of competence" of the slave node. This area can be defined as the sum of the half-lengths of the two segments adjacent to the slave node. Details about this geometry are provided in Figure 2, where S is the slave node, 1 and 2 are the end nodes of the master segment, $\mathbf{t}$ and $\mathbf{n}$ are, respectively, the tangent and normal unit vectors, l_{12} is the master segment length and ξ is the normalized projection of the slave node onto the master segment, $0 \le \xi \le 1$ [6]. The discrete spring stiffness located at each slave node is then the resultant of the distributed stiffness of the springs located along the area of competence (Figure 3), i.e. $\varepsilon_N = \hat{\varepsilon}_N \left(l_{AS} + l_{SB}\right)$, where $\hat{\varepsilon}_N$ is the stiffness per unit length of the bed of springs, and $l_S = \left(l_{AS} + l_{SB}\right)$ is the area of competence of the slave node (Figure 2). Such equivalence is computed for each slave node. In this case the input parameter of the analysis is the stiffness per unit length, $\hat{\varepsilon}_N$. This algorithm is named penalty method with Area Regularization (AR), or NTS-AR. For more details, see [4].

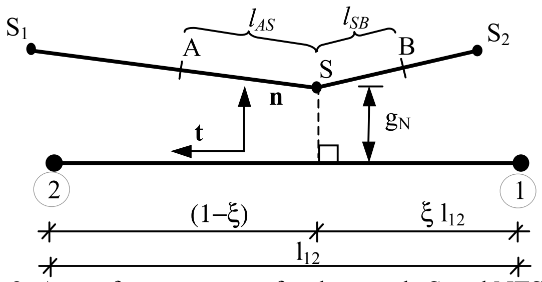
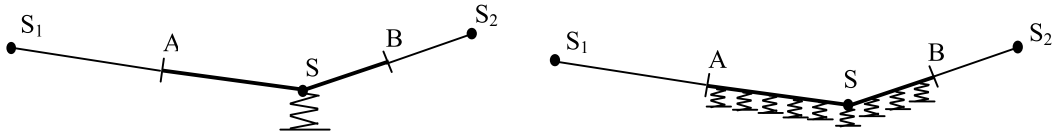


Figure 2: Area of competence of a slave node S and NTS geometry.



(a) Spring of stiffness ε_N *(b) Distributed springs of stiffness* $\hat{\varepsilon}_N$

Figure 3: Penalty with area regularization.

4 THE BASIC IDEA

In order to discuss the patch test performance of the NTS algorithm, we have first to look not at a single slave node, but rather at the two end nodes of a slave segment. Then, two main categories of cases can be introduced. The first category includes cases where the projection of each slave segment is contained within a single segment on the master surface. These are indicated as “normal” cases (Figure 4a). The second category includes all the remaining cases, where the projection involves more than one segment on the master surface. These cases are indicated in the following as “pathologic” cases (Figure 4b).

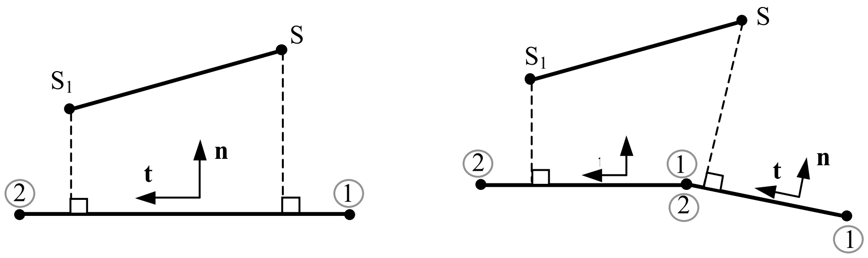
The basic idea underlying the proposed enhancements to the NTS formulation is the following: in order for the uniform (linear) contact patch test to be passed, there has to be equivalence of forces and moments between any uniform (linear) distribution of contact pressures acting on the slave segment and the concentrated forces transmitted to the corresponding master nodes by using the NTS algorithm. Such equivalence has to hold *locally*, i.e. for each slave segment (or for the area of competence of each slave node). Once the aforementioned equivalence holds, with the additional use of area regularization when the penalty method is adopted, the contact patch test is passed.

In “normal” cases, it can be easily demonstrated that the aforementioned equivalence is automatically satisfied for each slave segment, both for a uniform and for a linear contact pressure distribution. Therefore, in these cases, the NTS-AR algorithm passes both the uniform and the linear contact patch test.

For a general discretization, i.e. in presence of “pathologic” cases, this equivalence does no longer hold for each slave segment, while it continues to hold at the global level. As a result, the patch test is not passed by the NTS-AR algorithm, and additional enhancements are needed. To deal with these cases with a NTS strategy, we have to look again at a single slave node. In this context, two distinct ideal steps are involved in the transfer of a uniform (linear) contact pressure between two discretized contacting bodies:

- the *first step* is the slave nodal force computation, i.e. the transformation of a uniform (linear) contact pressure over the slave surface into concentrated forces at the slave nodes;
- the *second step* is the slave nodal force transmission, i.e. the ideal transformation of the concentrated forces at the slave nodes into a uniform (linear) contact pressure acting on the master surface. This has to be transformed into equivalent forces on the master nodes.

In order for the uniform (linear) patch test to be passed, equivalence of forces and moments between the concentrated forces and a uniform (linear) contact pressure must hold at each contact element during both the above phases.



(a) "Normal" case *(b) "Pathologic" case*

Figure 4: Projection cases.

5 REVIEW OF THE CONTACT ELEMENT PASSING THE UNIFORM CONTACT PATCH TEST

In the following, we briefly review the improvements of the NTS-AR algorithm which enable it to pass the uniform contact patch test. For all the details, see [4].

5.1 *Step 1: ensuring local equilibrium during the slave nodal force computation (VTS-ME)*

The virtual slave node technique (Virtual To Segment or VTS), proposed in [7], consists in changing the integration scheme usually adopted in NTS contact elements. An arbitrary number of points is specified inside each contact element, and each of these points is treated by the NTS algorithm as a (virtual) slave node.

The authors [4] showed that this technique can be employed to satisfy equivalence of forces and moments between a uniform contact pressure and the contact forces transferred at each slave node. For this purpose, it is sufficient to place one virtual slave node at the centroid of each of the two half-segments adjacent to the generic slave node S, as shown in Figure 5a. This implies that each element of the slave surface contains two virtual slave nodes located at its quarter points (Figure 5b). A modified virtual slave node technique results, whereby the NTS-AR strategy is applied to the virtual slave nodes, and these are located at the quarter points of each slave segment. This technique is such that Momentum Equilibrium (ME) is locally satisfied, hence it is indicated as VTS-ME.

5.2 *Step 2: ensuring local equilibrium during the slave nodal force transmission (VTS-PPT)*

The VTS-ME algorithm passes the contact patch test provided that the area of competence of each virtual slave node, when projected to the master surface, falls within a single segment on the master surface. If the previous condition is not satisfied, the concentrated contact forces situated at the virtual slave nodes are incorrectly transformed into forces at the master nodes [4].

In order to solve this problem, care must be taken while transferring the contact forces acting at the virtual slave nodes to the elements of the master surface. More in detail, the following steps have to be followed for each virtual slave node:

- projection of the area of competence of the virtual slave node over the master surface;
- transformation of the concentrated contact force acting at the virtual slave node into a uniform contact pressure acting over this projected area;
- transformation of the uniform contact pressure acting over the projected area into equivalent concentrated contact forces acting at the end nodes of all the elements on the master surface involved by the projected area.

Following the above strategy, a contact element passing the uniform patch test was devised in [4]. The corresponding algorithm was indicated as VTS-PPT.

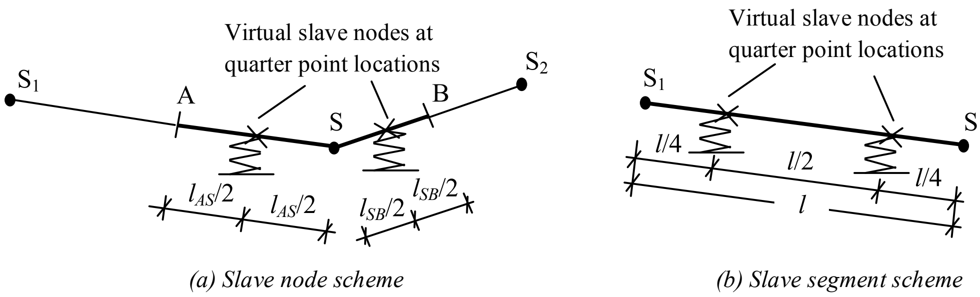


Figure 5: Modified virtual slave node technique.

6 A NEW CONTACT ELEMENT PASSING THE LINEAR CONTACT PATCH TEST

As recalled in the previous section, the VTS-PPT algorithm is based on a piecewise constant contact pressure interpolation across the contacting surface. As a result, it cannot be able to pass the linear patch test. In the following, suitable modifications to the NTS-AR algorithm are conducted, with the final goal to develop an enhanced algorithm able to pass the linear patch test. The same approach used for the uniform patch test will be maintained, i.e. incremental improvements will be made to the basic formulation in order to achieve the goal for the most general discretization cases.

As mentioned earlier, in "normal" cases the NTS-AR algorithm passes the linear contact patch test. Therefore, the attention will be directly focused on "pathologic" cases. In such cases, two types of errors occur during the slave nodal force computation and the slave nodal force transmission. Both problems, solved in the earlier work for the uniform case, will be solved henceforth for the linear case.

6.1 *Step 1: ensuring local equilibrium during the slave nodal force computation (VTS-MEL)*

The VTS-ME algorithm is no longer successful in case of a linear contact pressure, as in this case the quarter points of a slave segment do no longer coincide with the centroids of the contact pressure distributions acting on the two halves of the slave segment. In order to solve the problem, there are different possible strategies. For instance, the location of the virtual slave nodes could be changed, in order to place them at the centroids of the linear contact pressure distributions.

Obviously, the resulting location would no longer be fixed but rather depend on the pressure (i.e. on the normal gap) distribution. Another possible method would consist in adopting a larger number of virtual slave nodes per slave segment.

In order to keep the formulation as simple as possible, the strategy adopted as follows still adopts two virtual slave nodes at the quarter points of each slave segment. The next consideration is that any linear distribution of contact pressures along the area of competence of each virtual slave node (Figure 6a) can be thought of as the superposition of a constant average component (Figure 6b) and a linear antimetric distribution (Figure 6c). The centroid of the constant component is located at the virtual slave node, therefore it is correctly transformed into slave nodal forces. For the linear antimetric distribution, the computation of the slave nodal forces has to be conducted appropriately. In order to remain within a NTS framework (as opposed to a segment-to-segment one), this distribution can be treated as two opposite concentrated forces located at the centroids of the two triangles. Under this assumption, the contact problem can be formulated correctly. The entire formulation, including the computation of the residual and its consistent linearization to obtain the tangent stiffness matrix, is reported elsewhere [5].

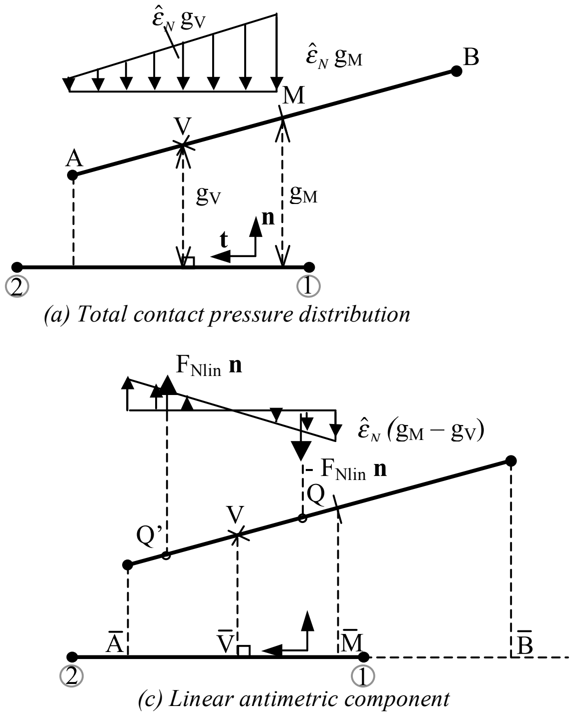
The resulting algorithm, which guarantees Moment Equilibrium during step 1 also for a Linear contact pressure distribution, will be indicated as VTS-MEL. It is important to note that, in cases where the area of competence of a virtual slave node is projected entirely within a single master segment, the VTS-MEL algorithm conducts to the exact computation not only of the slave but also of the master nodal forces. Therefore, in such cases this algorithm passes the linear patch test.

6.2 *Case test A*

In Case test A, shown in Figure 7a, two sets of springs at non-matching locations are pressed into each other with a linear pressure distribution. The upper surface is the slave and the lower is the master one. The use of 1D springs instead of 2D blocks is due to the fact that, under a linear applied pressure distribution, a bending stress regime would arise in 2D blocks. The modeling of such regime by linear quadrilateral elements is affected by the shear locking effect, and the resulting inaccuracies prevent a clear examination of the role of the contact algorithm in the transfer of stresses across the contact interface.

Figure 7b shows the expected correct values of the contact forces, obtained moving to the nodes on both surfaces the externally applied linear pressure distribution. Figure 7c shows how the upper set of forces at the slave nodes is transmitted to the master nodes with the classical NTS algorithm. It is obvious that the resulting distribution is incorrect, i.e. it is not compatible with a linear pressure over the master surface. Hence, in this case the penalty method, either with or without AR, cannot pass the patch test. This is shown in Figure 8a, which plots the reaction forces in the lower springs. A large discrepancy is visible between the exact reactions and those predicted by the NTS-AR algorithm, the latter coinciding with those depicted in Figure 7c.

For the discretization in Figure 7a, the area of competence of each virtual slave node (located at the quarter points of each slave segment) falls within one master segment. Therefore, the VTS-ME algorithm would pass the uniform patch test (see case test B in [4]). However, it does not pass the linear one, as visible in Figure 8. Nevertheless, it is evident that a significant improvement is achieved with respect to the NTS-AR algorithm. Finally, the VTS-MEL algorithm carries out a proper transformation of the linear contact pressures into slave and master nodal forces. As a results, the linear patch test is passed (see Figures 7d and 8).

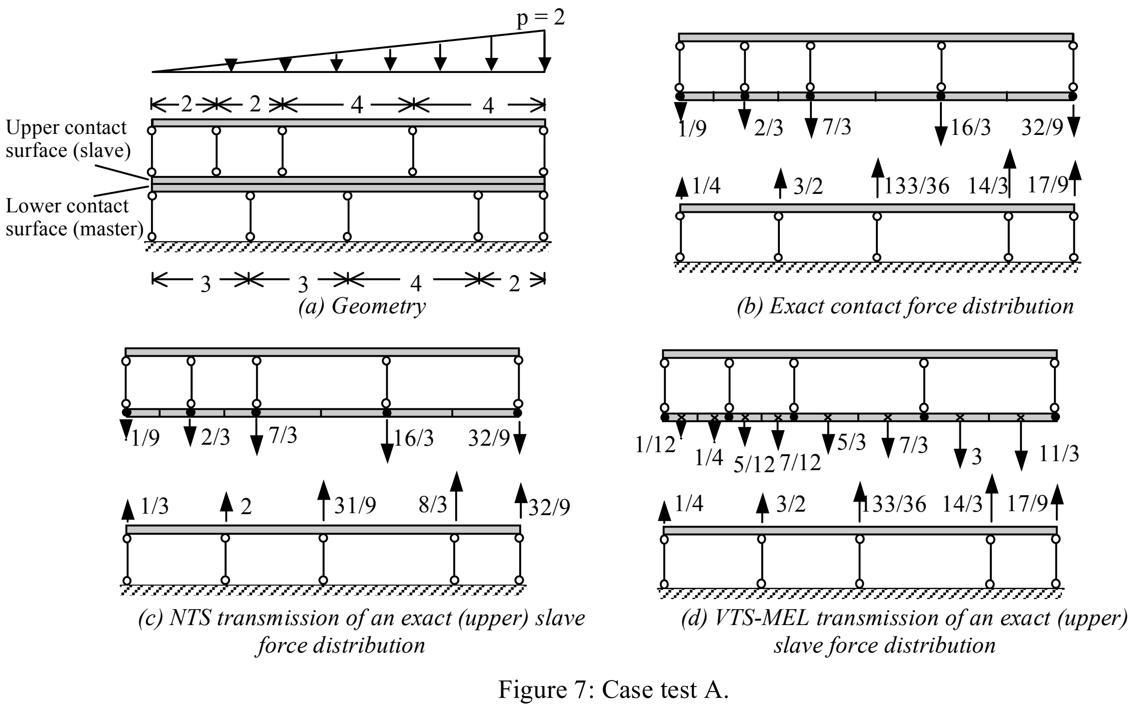


(a) Total contact pressure distribution

(b) Constant component

(c) Linear antimetric component

Figure 6: Slave nodal force computation.



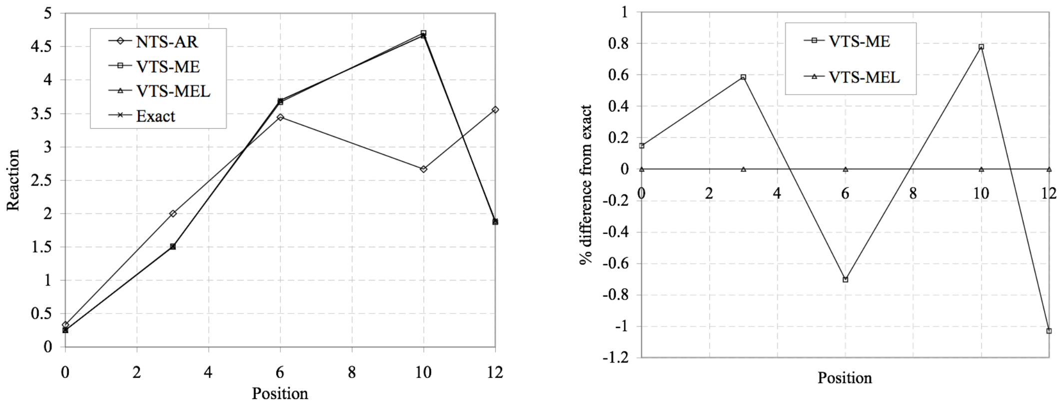
(a) Geometry

(b) Exact contact force distribution

(c) NTS transmission of an exact (upper) slave force distribution

(d) VTS-MEL transmission of an exact (upper) slave force distribution

Figure 7: Case test A.



(a) *(b)*

Figure 8: Case test A – contact force distributions.

6.3 *Step 2: ensuring local equilibrium during the slave nodal force transmission (VTS-PPTL)*

In the most general discretization case, the projection of the area of competence of a virtual slave node onto the master surface involves more than one master segment (Figure 9). As a result, the VTS-MEL algorithm yields the exact computation of the slave but not of the master nodal forces. In order to solve this problem, care must be taken while transferring the contact forces acting at the virtual slave nodes to the elements of the master surface. More in detail, the following steps have to be followed for each virtual slave node:

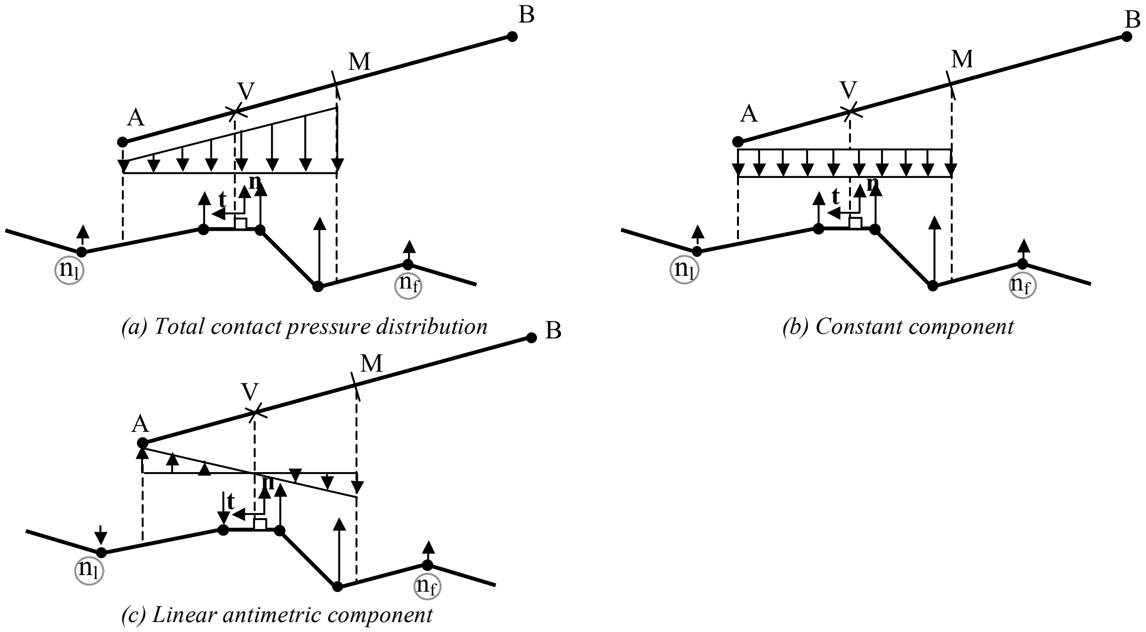
- projection of the area of competence of the virtual slave node over the master surface;
- transformation of the concentrated contact force acting at the virtual slave node into a linear contact pressure acting over this projected area;
- transformation of the linear contact pressure acting over the projected area into equivalent concentrated contact forces acting at the end nodes of all the elements on the master surface involved by the projected area.

Following the above strategy, a contact element Passing the Patch Test also for Linear contact pressures (PPTL) can be devised. The corresponding algorithm will be indicated as VTS-PPTL. Its detailed description is reported elsewhere [5]. Once again, the linear pressure distribution (Figure 9a) is considered as the sum of a constant component (Figure 9b) and a linear antimetric component (Figure 9c). Both have to be appropriately transformed into forces at the master nodes.

6.4 *Case test B*

The geometry of Case test B is depicted in Figure 10a. As usual, the upper surface is the slave and the lower is the master one. Figure 10b reports the expected correct values of the contact forces, whereas Figure 10c shows the contact forces obtained by assuming the correct distribution at the slave nodes, and computing the distribution of forces at the master nodes resulting from application of the NTS algorithm. As in Case test A, the latter distribution is obviously incorrect (see also Figure 11a), hence the patch test cannot be passed with the standard formulation.

Unlike for Case test A, using the VTS-MEL technique does not completely solve the problem (Figure 11), although it improves results with respect to the NTS-AR algorithm. The reason is that, for this geometry, the area of competence of one of the virtual slave nodes intercepts two segments on the master surface. Hence the transfer of contact forces to the master nodes is incorrect. Also the VTS-PPT algorithm, which is able to pass the uniform patch test, is unsuccessful for the linear patch test (Figure 11). The problem is solved by using the strategy summarized above (and described in detail in [5]), as shown in Figures 10d and 11.

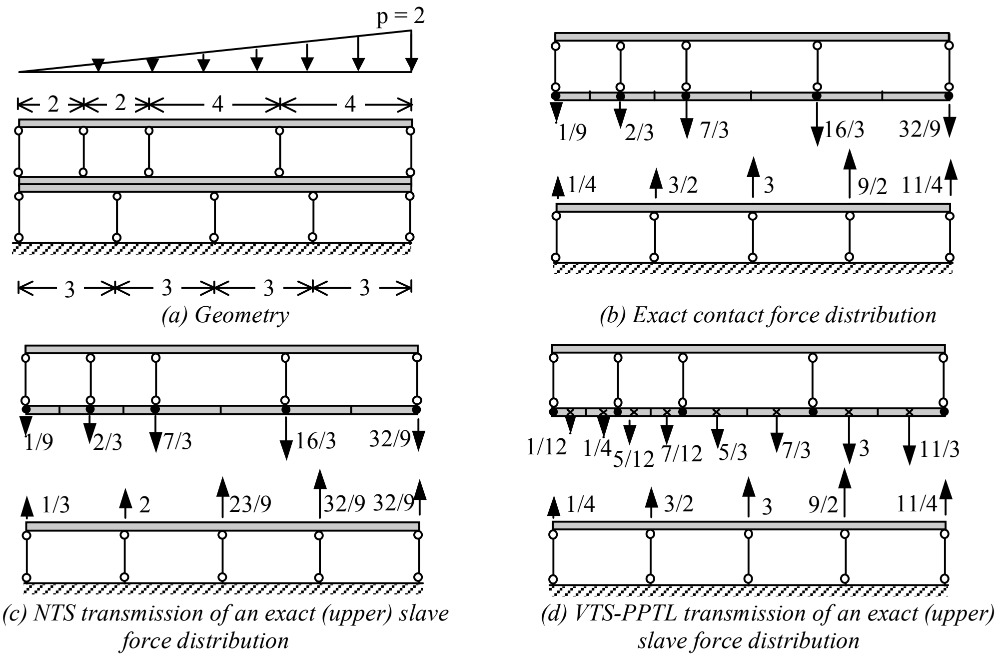


(a) Total contact pressure distribution

(b) Constant component

(c) Linear antimetric component

Figure 9: Contact force transmission to more than one segment on the master surface.



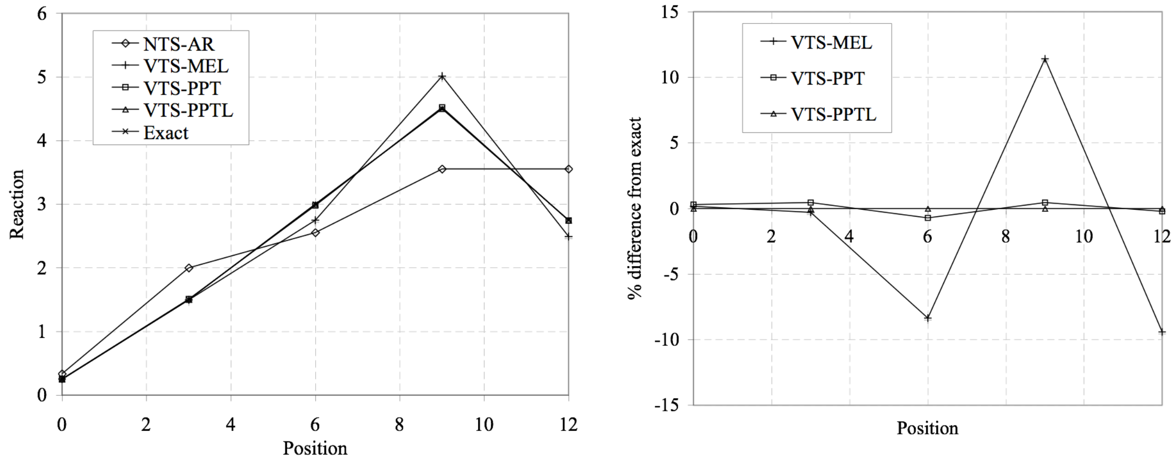
(a) Geometry

(b) Exact contact force distribution

(c) NTS transmission of an exact (upper) slave force distribution

(d) VTS-PPTL transmission of an exact (upper) slave force distribution

Figure 10: Case test B.



(a) *(b)*

Figure 11: Case test B – contact force distributions.

7 CONCLUSIONS

This study has illustrated several sequential modifications of the basic NTS formulation for 2D frictionless contact, used in conjunction with the penalty method. These modifications yield incremental improvements in results of the linear contact patch test. In particular, the last proposed formulation (VTS-PPTL) is a modified one-pass NTS algorithm which is able to correctly transfer a linear contact pressure distribution from the slave to the master surface, hence it passes the linear contact patch test. The differences between the proposed algorithm and the standard NTS one are: i) the use of the modified virtual slave node technique, with virtual slave nodes located at the quarter points of each slave segment and a proper computation of the slave node forces; ii) the use of a specific procedure to correctly compute the contact forces at the master nodes. All the details on the algorithm and additional examples on its effectiveness including 2D continuum elements are reported elsewhere [5].

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