The effects of back-reaction on particle clustering and turbulence modulation in shear flows

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SUMMARY. Small scale turbulent fluctuations induce a commonplace phenomenology on the transport of small inertial particles, known as clustering. Particles spread disunformly and form aggregates where their local concentration is much higher than it is in nearby rarefaction regions, the voids, where in extreme cases not even a single particle can be found.

The underlying physics has been exhaustively analyzed in statistically homogeneous and isotropic flows under the so called one-way coupling regime, i.e. in conditions where the momentum exchange between the carrier fluid and the disperse phase is negligible.

In this framework we addressed in a recent paper the effect the anisotropic advecting field may have on such aggregates to show how the clusters can even increase their directionality in the smallest scales, contrary to the naive expectation based on the isotropy recovery behavior of velocity fluctuations. This unexpected finding opens new issues in presence of large mass loads, when the momentum exchange between the two phases becomes significant and the back-reaction of the particles on the carrier flow cannot be neglected.

By discussing direct numerical simulations of particle laden homogeneous shear flows in the two-way coupling regime under various mass loads, we present here the new effect we have found: The energy depletion of the classical inertial scales and the amplitude increase of the smallest ones where the particle back-reaction pumps energy on the turbulent eddies dramatically altering their energy content. Overall this appears as by-pass of the classical energy cascade in the presence of an additional dissipation channel represented by the friction exerted by the particles on the flow accompanied by a substantial increase of anisotropy down to viscous dissipation.

1 INTRODUCTION

Transport of inertial particles is involved in several fields of science such as droplets growth and collisions in clouds [9, 20] or the plankton accumulation in the oceans [12]. Several technological applications are concerned, e.g. inertial particles dynamics is crucial for designing injection systems of internal combustion engines, to prevent sediment accumulation in pipelines [14] or for the appropriate dimensioning of filtering devices of various kinds.

The relevant physical aspect in particles dynamics consists in their finite inertia which prevents them from following the fluid trajectories. Consequently new interesting features emerge. The most evident is the “preferential accumulation” which, in inhomogeneous flows such as wall bounded flows, occurrs in the form of the so called “turbophoresis”, i.e. preferential localization of particles in the near wall region. An exhaustive review of the subject can be found e.g. in [16], see also the recent paper [13] for a physical explanation in terms of statistical properties of velocity fluctuations in the near wall region.

When the idealized conditions of isotropic turbulence are addressed, preferential accumulation manifests itself in the form of small scale clustering, with the disperse phase forming small scale aggregates where most particles concentrate, separated by void regions of small particle density, see e.g. [2] and references therein.

So far the effect of turbulent transport on particle dynamics has been studied extensively in many flow configurations. Much less is known about the effect the disperse phase may have on the carrier flow. It is expected that, under proper coupling conditions, the momentum exchange between the two phases may become relevant in driving the turbulent fluctuations away from their universal equilibrium state predicted by Kolmogorov in the early forties. Clearly, in contrast to the one-way coupling regime, addressing these effects calls into play the more realistic two-way coupling mechanism, where the disperse phase provides an active modulation of velocity fluctuations.

In this context, the few experimental investigation at finite mass load [11, 18] or the even less numerous two-way coupling numerical simulations [3, 17] are typically focused on the idealized conditions of isotropic turbulence. In some cases both experimental and numerical works have addressed the classical channel flow geometry [19]. In any case, the overall effect of particles back reaction on the carrier phase is reported as an attenuation of turbulence fluctuations controlled by the mass load ratio defined as the ratio of total disperse phase mass to fluid mass, see e.g. [8] for a review of the main results both in isotropic and wall bounded flows.

In this scenario important issues still need to be addressed more in depth. A first point concerns the range of turbulent scales which are directly modulated by the particles. We anticipated already that, at least for isotropic conditions, the particles lead to the attenuation of turbulent fluctuations. We do not know, however, which is the range of scales where such attenuation predominantly occurs, and how the range of affected scales depends on the characteristics of the particles and of the turbulence. We do not know either if the effect is a systematic depletion of turbulent flactuation irrespective of the scale or, to the contrary, whether there exists scales where particles pump back part of the energy on the carrier flow. To properly address this issue, preferential accumulation might become central, since the back force on the fluid will present a continous spectrum controlled by the multiscale nature of the particle accumulation process and by its significant small scale features. In shear flows in particular [7, 5] the phenomenology is expected to be particularly rich. Hence, motivated by recent findings in the context of anisotropic clustering [10, 15], we consider here the modulation of turbulence by transported particles addressing the particle laden homogeneous shear flow in the two-way couling regime, see also [1]. Such flow can be considered as a sort of bridge between the idealized conditions of isotropic turbulence and the more realistic geometries of wall bounded flows, since it preserves spatial homogeneity and retains the anisotropic features of shear flows.

For our purposes here, it is worth recalling that, under shear, turbulent fluctuations are strongly anisotropic at the largest scales due to production of turbulent kinetic energy via interaction of the mean velocity gradient and turbulent fluctuations. At smaller scales, below the so-called shear scale L_S, inertial energy transfer usually prevails (see e.g. [4] for a discussion of high accuracy wind tunnel data showing that this may not always be the case). In such conditions re-isotropization of turbulent fluctuation takes place following a route described in [7].

In such flows we have found that turbulent fluctuations in one-way coupling regime induces the anisotropic clustering of the disperse phase. Actually, in contrast to the small scale behavior of velocity fluctuations, particles aggregates do not lose their directionality. Their anisotropy even icreases down to the viscous scales where clusters still keep memory of the spatial orientiation of the large scale coherent motions [10].

Here we consider the same flow under the two-way coupling scenario. In this more complex case the first physical result we achieve consists in a new picture of multi-phase turbulent flows. As it will be shown, small scales anisotropic clusters acts as a source/sink of momentum for the turbulent motions distributed along all the range of scale. They deplete the energy from the largest inertial scales which is in part retrived in the low-inertial/dissipative range. Here the clusters keep the velcoity fluctuations to a higher exicitation state than expected on the basis of the standard Kolmogorov theory. As we will see such back energy scatter is highly anisotropic in shear flow, hence the small scales fluid motions are prevented from recovering isotropy and eventually increment their level of anisotropy due to the momentum exchange with the disperse phase, when they would have otherwise already completely recovered the equilibrium state described by Kolmogorov.

2 METHODOLOGY

Concerning the carrier fluid, the velocity field $\mathbf{v}$ is decomposed into a mean flow $\mathbf{U} = Sx_2\,\mathbf{e}_1$ and a fluctuation $\mathbf{u}$ where $\mathbf{e}_1$ is the unit vector in the streamwise direction, x_2 denote the coordinate in the direction of the mean shear S and x_3 is in the spanwise direction. Rogallo's technique is employed to rewrite the Navier-stokes equations for velocity fluctuations in a deforming coordinate system convected by the mean flow according to the transformation of variables $\xi_1 = x_1 - Stx_2; \quad \xi_2 = x_2; \quad \xi_3 = x_3; \quad \tau = t$. The resulting system

$$\nabla \cdot \mathbf{u} = 0\,; \qquad \frac{\partial \mathbf{u}}{\partial \tau} = (\mathbf{u} \times \boldsymbol{\zeta}) - \nabla \pi + \nu \nabla^2 \mathbf{u} - Su_2 \mathbf{e}_1 - \mathbf{F_p}\,, \tag{1}$$

is numerically integrated by a pseudo-spectral method combined with a fourth order Runge-Kutta scheme for temporal evolution. In equations (1) $\boldsymbol{\zeta}$ is the curl of $\mathbf{u}$, π is the modified pressure which includes the fluctuating kinetic energy $u^2/2$, ν is the kinematic viscosity and $\mathbf{F_p}$ denote the back-reaction due to the disperse phase. The latter consists of diluted particles with mass density ρ_p much larger than the carrier fluid ρ_f. The approximation of point particles can be adopted whenever the particle diameter d_p is much smaller than the typical turbulence scales. It follows that the only relevant force is the Stokes drag. Accordingly, the equations for particles position $x_i^p(t)$ and velocity $v_i^p(t)$ read

$$\frac{dx_i^p}{dt} = v_i^p\,; \qquad \frac{dv_i^p}{dt} = \frac{1}{\tau_p}\left[v_i(x^p, t) - v_i^p(t)\right] \tag{2}$$

where $v_i(x^p, t)$ is the instantaneous fluid velocity evaluated at $x_i^p(t)$ and $\tau_p = \rho_p d_p^2/(18\nu\rho_f)$ is the Stokes relaxation time. Particle velocities are decomposed as $v_i^p = U_i[x_k^p(t)] + u_i^p$ where u_i^p denotes the particle velocity deviation with respect to the local mean flow of the carrier fluid. Finally by using Rogallo's transformation eqs. (2) can be written in computational space as

$$\frac{d\xi_i^p}{d\tau} = u_i^p - S\tau u_2^p \delta_{i1}\,; \qquad \frac{du_i^p}{d\tau} = f_i^p \tag{3}$$

where $f_i^p = \frac{1}{\tau_p}\left[u_i(\xi^p, \tau) - u_i^p(\tau)\right] - Su_2^p\delta_{i1}$ is the expression of the Stokes drag acting on the p^{th} particle. In the so called two way coupling regime an equal and opposite force acts on the carrier fluid accounting for the momentum exchange between the two phases. Modeling the back reaction in numerical simulations is an issue. Actually, fluid properties are known in an Eulerian frame while particles evolve along their own Lagrangian trajectories. This requires a first interpolation when the fluid velocity is computed at the particle position, namely $u_i(\xi^p, \tau)$. An other interpolation is required when the back-reaction on the fluid is computed since f_i^p is known at the particle position. In fact, the force acting on the p^{th} particle is re-distributed via interpolation to the nearest Eulerian grid points where the fluid velocity is defined. The resulting force on the fluid is computed as

$$\mathbf{F}_p = -\frac{N_c}{N_p}\Phi \sum_{k}^{n_p(\xi)} \mathbf{f}^p \tag{4}$$

Φ	ρ_p/ρ_f	N_p	$\langle u^2\rangle/2$	$-\langle u_1\, u_2\rangle$	ϵ	ϵ_p
0	–	–	0.43	0.118	0.059	–
0.2	100	$3.35\,10^6$	0.43	0.127	0.049	0.014
0.4	100	$6.7\,10^6$	0.42	0.134	0.049	0.018
0.8	10	$4.2\,10^6$	0.41	0.133	0.046	0.020

Summary of Direct Numerical Simulation dataset. Navier-Stokes equations are integrated in a $4\pi \times 2\pi \times 2\pi$ periodic box with a resolution of $256 \times 256 \times 128$ Fourier modes for the cases $\Phi = 0 - 0.4$. The simulation at $\Phi = 0.8$ uses $384 \times 384 \times 192$ Fourier modes. The $3/2$ dealiasing rule is adopted to compute the non linear terms. The Stokes number based on the Kolmogorov time is $St_\eta \simeq 1$.

Table 1:

where the sum is extended to all the $n_p(\xi)$ particles belonging to the computational cell centered at point ξ. In eq. (4) N_c denote the number of Eulerian cells, N_p is the total number of particles and Φ denote the mass load ratio i.e. the ratio between the mass of the disperse phase $M_p = N_p \pi \rho_p d_p^3/6$ and the carrier fluid phase $M_f = \rho_f V_f$ where V_f is the volume of the computational box. Equations (3) are integrated by the same fourth order Runge-Kutta scheme used for the Navier-Stokes equations and the interpolation adopt a tri-linear scheme.

The two parameters controlling the homogeneous shear flow are the Taylor-Reynolds number $\mathrm{Re}_\lambda = \sqrt{5/(\nu\epsilon)}\langle u_\alpha u_\alpha\rangle$ and the shear strength $S^* = S\langle u_\alpha u_\alpha\rangle/\epsilon$ where ϵ the turbulent kinetic energy dissipation per unit mass. For the simulations discussed below they are $\mathrm{Re}_\lambda \simeq 100$ and $S^* \simeq 7$, corresponding to a ratio of shear to Kolmogorov scale $L_s/\eta \simeq 35$. Navier-Stokes equations are integrated in a $4\pi \times 2\pi \times 2\pi$ periodic box see table 1 for a full description of the dataset. The Kolmogorov scale is $\eta = 0.02$ which correspond to $K_{max}\eta = 3.1$ ensuring sufficient resolution at small scales in view of an accurate interpolation required in the simulations. Concerning the disperse phase the dynamics is controlled by the ratio of the particles relaxation time τ_p to a characteristic flow time scale, typically the Kolmogorov time scale $\tau_\eta = (\nu/\epsilon)^{1/2}$, i.e. the relevant control parameter is the Stokes number $St_\eta = \tau_p/\tau_\eta$. When the two-way coupling regime is considered other non dimensional parameters are required to describe the momentum exchange between the two phases namely the density ratio ρ_p/ρ_f –assumed to be much larger than unity– and the mass load fraction $\Phi = M_p/M_f$. For the simulations in table 1 particles are injected in an already fully developed turbulent flow. Their position is initialized at random homogeneous points with initial velocity matching the fluid velocity at particle position. The total number of particles is changed to achieve different values of the mass load parameter while the Stokes number is kept constant. Samples of particles statistics are collected after discarding an initial transient. To this purpose 150 statistically independent snapshots are used to compute the relevant statistical observables which characterize both the carrier fluid and the disperse phase.

3 PARTICLES CLUSTERING

A visual impression of instantaneous particles configurations is provided in figure 1 where slices of the domain in selected coordinate planes are displayed for different values of the mass load parameter Φ at $St_\eta = 1$. In all the four cases, the disperse phase is characterized by a multi-scale distribution of particles concentration and voids. From the figure the shear induced orientation of the clusters is apparent. For a quantitative description of the anisotropic features of particle clustering we adopt the Angular Distribution Functions (ADF) which measures the number of particle pairs at separation r in the direction $\hat{\mathbf{r}}$. The ADF is defined as

$$g(r, \hat{\mathbf{r}}) = \frac{1}{r^2} \frac{d\nu_r}{dr} \frac{1}{n_0} , \tag{5}$$

where $n_0 = 0.5 N_p(N_p - 1)/V_f$ is the volume density of particles pairs and $\nu_r(r, \hat{\mathbf{r}}) d\Omega$ is the numbers of particles pairs contained in the spherical cone of radius r with axis along $\hat{\mathbf{r}}$ and amplitude $d\Omega$. The spherical average of the ADF $g(r) = 1/(4\pi) \int_\Omega g(r, \hat{\mathbf{r}}) d\Omega$ is called the Radial Distribution Function (RDF) and has been already used to characterize particles clustering in isotropic conditions. The ADF extend the tool to anisotropic conditions retaining information on the directionality of the clusters. The behavior of the RDF near the origin $g(r) \propto r^{-\alpha}$ can be shown to be related to important geometrical features of the clusters. In particular $\mathcal{D}_2 = 3 - \alpha$ is the so-called correlation dimension of the multi-fractal measure associated with the particle density. A positive α indicates the occurrence of small scale clustering.

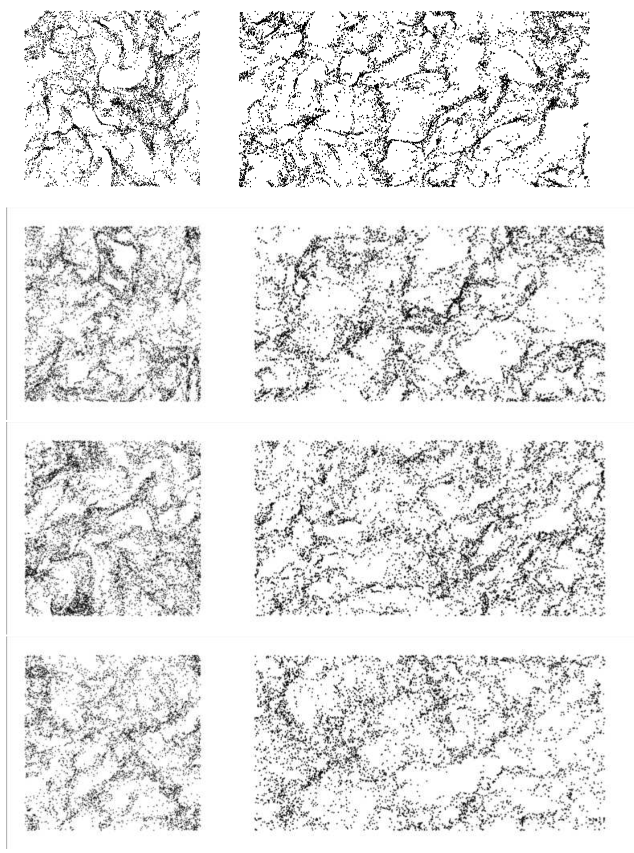

Figure 1: Snapshots of particle positions for increasing values of the mass load parameter $\Phi = 0\ 0.2\ 0.4\ 0.8$ from top to bottom. For all the datasets $St_\eta = 1$. Left column thin slice in the $y - z$ plane; right column slice in the $x - y$ plane. The slice thickness is of the order of a few Kolmogorov scales.

The RDF is shown in figure 2 for the different values of the mass load parameter Φ. As apparent from the data, the back-reaction progressively attenuates the clustering process at small scales leading to measure a smaller value of the scaling exponent α. This behavior already emerges at a qualitative level by looking at the patterns in figure 1 where the clusters are definitely less defined in the case $\Phi = 0.8$ then at $\Phi = 0$, compare bottom and top panels of the figure, respectively. In any case clustering, though partially attenuated, is a persistent feature of the two way coupling regime as measured by the exponent α. In view of turbulence modulation, the anisotropy of particles cluster will play a crucial role to be discuss in the next section devoted to the analysis of the back reaction of the disperse phase on the fluid.

We complete the discussion of particle clustering under two-way coupling by characterizing the anisotropy of the aggregates. This can be done by exploiting the directionality properties of the ADF. Its angular dependence of $g(r, \hat{\mathbf{r}})$ can be resolved in terms of spherical harmonics [10]

$$g(r, \hat{\mathbf{r}}) = \sum_{j=0}^{\infty} \sum_{m=-j}^{j} g_{jm}(r) \, Y_{jm}(\hat{\mathbf{r}}) \, . \tag{6}$$

thus achieving a systematic description both in terms of separation r, accounted for by the coefficients $g_{jm}(r)$, and in terms of directions, inherently described by the shape of the basis functions $Y_{jm}(\hat{\mathbf{r}})$. Each successive subspace, here labeled j, accounts for increasing levels of anisotropy consistently with the geometrical meaning of the spherical harmonics.

In the right panel of figure 2 we show the most energetic anisotropic projection normalized by the RDF namely g_{2-2}/g_{00}. This indicator quantifies the average level of anisotropy of particles clusters at that particular scale. The data show how, going down the scale range, the anisotropy of the clusters is increased also in the two-way coupling regime. The behavior is in clear contrast with the isotropy recovery of turbulent velocity fluctuations. Here, when clustering happens to occurs at scales where velocity fluctuations are almost isotropic, the large scales anisotropic coherent motions still imprint on particles aggregates. We observe that the back-reaction on the carrier fluid partially reduces

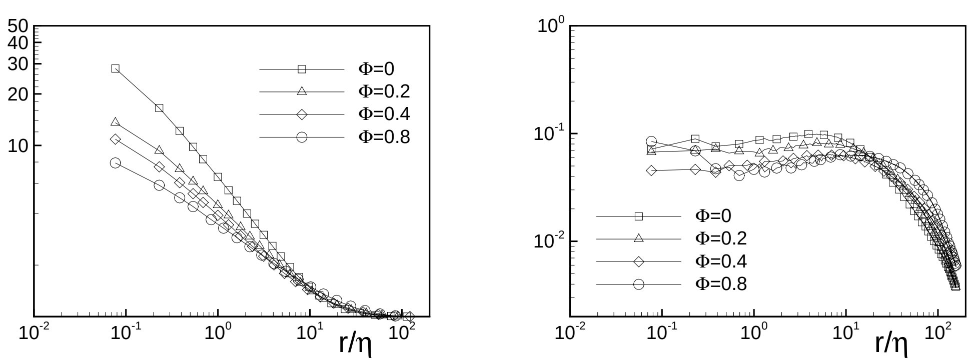

Figure 2: Left: RDF vs separation, for different mass load ratios Φ at $St_\eta = 1$. Right: ratio between the most energetic anisotropic sector $(2, -2)$ normalized by isotropic sector as a function of separation. Same data as in the left panel.

the small scale anisotropy of the disperse phase. The overall behavior is however not substantially altered from we already described in previous papers on the one-way coupling regime.

4 TURBULENCE MODULATION

The discussion addressed in the previous section concerning anisotropic clustering becomes now fundamental for its implications for the back reaction of the transported phase on the carrier fluid. As shown, particle clusters are spatially organized in multi scales sets whose orientation is controlled by the large scales anisotropic motions in conjunction with the inertial of the suspended particles. It should be reminded that the classical Kolmogorov like energy cascade, leading to small scales isotropy recovery of velocity fluctuations, is ineffective in achieving isotropization of particles aggregates.

Clearly, in presence of two-way coupling, the support of the reaction field on the fluid is provided by those sets where most of the disperse phase concentrate. We conclude that the fluid is stirred by an highly anisotropic, spectrally non-compact forcing, quite an unusual circumstance in turbulence. In these conditions, the back-reaction of the disperse phase is expected to deeply alter the classical scenario of turbulence. Energy extraction/injection is now strongly anisotropic and active down to the smallest scales of the flow leaving no room for setting-up a classical inertial range.

Observe that, in the homogeneous shear flow, the mean velocity profile is imposed both in the one way and two way coupling regime. The simplest observables which characterize the response of turbulent fluctuations are the turbulent kinetic energy, the energy dissipation rate and, as always for shear flows, the Reynolds shear stresses see table 1. Increasing the mass load ratio, turbulence fluctuations are progressively attenuated. In the classical homogeneous shear flow in statistically statistically steady state, the production term $\mathcal{P}_0 = -S\langle u\,v\rangle_0$ balances viscous dissipation ϵ_0, where the subscript refers to case $\Phi = 0$ (no back-reaction). When the balance is addressed in the two-way coupling regime a new term enters the balance, $\mathcal{P} - \epsilon - \epsilon_p = 0$ where $\epsilon_p = \langle \mathbf{F_p} \cdot \mathbf{u} \rangle$ is the amount of energy the fluid looses in part to the particles by accelerating them though the Stokes force and part by dissipation in the relative friction, see [6] for a similar mechanism in the context of polymer laden flows. ϵ_p is reported in table 1 for several mass load ratio Φ. As anticipated, the energy

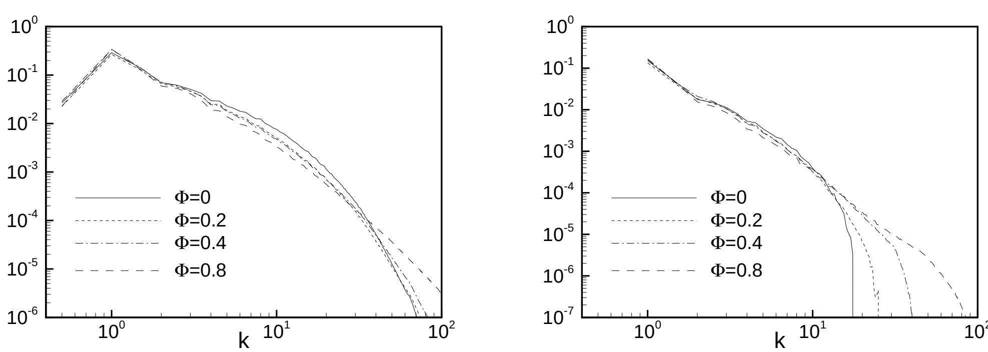

Figure 3: Left: energy spectra vs wavenumber, for different mass load ratios Φ at $St_\eta = 1$. Right: energy co-spectrum. Same data as in the left panel.

injected into turbulent fluctuations by their interaction with the mean flow is only partially dissipated by fluctuating velocity gradients. As Φ is increased an increasing part of energy is transfered to the particles. A clear signature of this effect is found in the velocity energy spectra of figure 3. As Φ increases turbulent fluctuations are attenuated in an intermediate range of scale they are strongly enhanced at small scales. In other words, the Stokes drag intercepts energy from the classical cascade at intermediate scales where velocity fluctuations are attenuated to pumps part of the intercepted energy back in the fluid at small scales.

The emerging picture of turbulence in presence of a disperse phase consists of standard transfer across the inertial range via non linear interactions, partial removal of energy from the cascade by the Stokes drag and partial re-injection of energy in the small scales. Under many respect, the conceptual picture is similar to the one operating in polymeric solutions.

In addition to the alteration of the cascade, in essence associated with the existence of an alternative dissipation channel, in shear flows we observe a definite change in the spectral distribution of the turbulent shear stress, figure 3.

It provides the scale-by-scale energy production and allows to identify the range of scales directly affected by the anisotropic forcing its integral amounts to the Reynolds stress). As the mass load ratio increases, the range of scales affected by anisotropic production is progressively enlarged. In fact the Stokes drag removes energy from the normal stresses reducing velocity variances to force the anisotropy. This is the effect of the highly directional small scales clusters. In these conditions serious doubts are cast on the small scale isotropy assumption in multiphase flow under sensible mass loads. Quantification of the isotropy recovery rate is provided by the cospectrum to energy spectrum ratio, identically vanishing in a purely isotropic state. This ratio, shown in figure 4, indicates the enhancement of anisotropy at small scales at finite mass load in contrast to the nearly isotropic state achieved at $\Phi = 0$. In the two way coupling regime E_{12}/E decays much more slowly and eventually increases at small scales in our most severe loading, $\Phi = 0.8$.

This gives reason of the substantial anisotropy measured at the level of velocity gradients by means of the non-vanishing component $\epsilon^d_{\alpha\beta}$ of the deviatoric contribution of the pseudo-dissipation

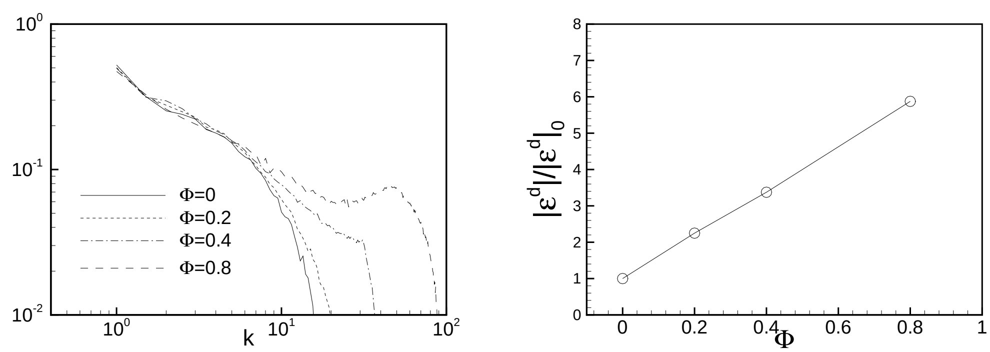

Figure 4: Ratio between the energy co-spectrum and the energy spectra vs wavenumber, for different mass load ratios Φ at $St_\eta = 1$. Right: norm of the deviatoric part of the pseudo-dissipation tensor normalized with its value at $\Phi = 0$ as a function of the mass load ratio.

tensor, $\epsilon_{\alpha\beta} = 2\nu\langle\partial_\gamma u_\alpha\, \partial_\gamma u_\beta\rangle$, shown in figure 4 after normalization with its value at $\Phi = 0$. The data show a dramatic increase in the anisotropy level of the velocity gradients.

5 FINAL REMARKS

In conclusion, as already discussed in a recent paper for passively advected particles, also in the two-way coupling regime anisotropy imprints on the particle clusters at small scales. The clustering process is found to be essentially controlled by the Stokes number as in the one-way coupling regime. In the two-way coupling regime, however, the back-force existing at finite mass load Φ controls the the scaling exponent of the RDF and the level of anisotropy observed in the particles aggregates.

The momentum exchange between the disperse phase and the carrier fluid has a dramatic effect and our data indicate that the process of energy cascade typical of turbulent flows is substantially altered by the particles back-reaction on the carrier fluid. Small scales velocity fluctuations are affected by the directionality of the clusters via the Stokes drag which, under strong coupling, prevails over the energy transfer mechanisms. In these conditions small scale velocity fluctuations are driven by an anisotropic, spectrally un-compact forcing operated by particle clusters down to viscous scales. Consistently, the statistics of the velocity gradient field is found to develop substantial anisotropic features.

In our feeling, these findings are bound to have major impact on turbulence modeling of multiphase flow which, both in the context of Reynolds averaged (RANS) and filtered equations (LES), heavily rely upon the concepts of inertial energy cascade and of a presumed universal small scale statistics. Actually, we have shown beyond doubt, we believe, that the cornerstone Kolmogorov theory no longer safely applies, since the classical energy cascade is overewhelmed by the anisotrpy-enhancing back-reaction of the particles.

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