Towards fully-adaptive wavelet transform-based numerical simulation of turbulence

Giuliano De Stefano[1], Oleg V. Vasilyev[2]
[1]*Dipartimento di Ingegneria Aerospaziale e Meccanica, Seconda Università di Napoli, Italy*
E-mail: giuliano.destefano@unina2.it

[2]*Department of Mechanical Engineering, University of Colorado at Boulder, USA*
E-mail: oleg.vasilyev@colorado.edu

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SUMMARY. With the recent development of wavelet-transform based numerical techniques, adaptive numerical simulations of turbulent flows have become feasible. The use of the adaptive wavelet collocation solver induces a built-in separation mechanism between resolved more energetic eddies and residual less energetic flow that is dictated by the wavelet threshold filtering level used to control the numerical errors. By varying the thresholding level different approaches with different fidelity are obtained, from the highly accurate wavelet-based direct numerical simulation to the stochastic coherent adaptive large eddy simulation that needs modeling. In this work, a study of the effect of varying the wavelet thresholding level on the accuracy and computational efficiency of wavelet-based solutions is carried out. This study enhances our knowledge about the strong interactions between wavelet-compression and modeled turbulent dissipation in wavelet-based numerical simulations of turbulence.

1 INTRODUCTION

Though there has been considerable progress in the development of the Large Eddy Simulation (LES) method, nevertheless it is not considered yet a predictive tool for turbulent flows of engineering interest, e.g. [1]. Furthermore, following [2], one can recognize that current LES methods are not "complete", since the constituent equations are not free from flow-dependent specifications. The inherent problem with the standard LES approach is that one solves for the large-scale flow structures, which are those ones of a certain size, instead of solving for the coherent energetic eddies, which can be of any size from the characteristic length scale of the physical domain down to the Kolmogorov length scale.

In recent years, in order to detect and simulate the evolution of the coherent energetic structures in a turbulent flow field, decomposition techniques based upon the wavelet transform have been successfully applied to study turbulence. In fact, the coherent energetic eddies have been demonstrated to be mostly responsible for the turbulence evolution and the wavelet decomposition thus provides an efficient physically-based method for modeling turbulent flows. This property is exploited by the Coherent Vortex Simulation (CVS) method – introduced by Farge and co-authors [3] – where the evolution of the coherent eddies is explicitly simulated by completely discarding the effect of the residual incoherent part, which consists of a Gaussian white noise that does not provide any turbulent dissipation. The Stochastic Coherent Adaptive Large Eddy Simulation (SCALES) method – introduced by Goldstein and Vasilyev – is an evolution of the classical LES approach based upon the same wavelet-transform concept [4]. It addresses some shortcomings of LES by exploiting a dynamic grid adaptation that allows to resolve and track the most energetic structures during the numerical simulation. The use of the adaptive wavelet collocation solver [5] induces a built-in separation mechanism between resolved energetic eddies and residual less energetic flow that is governed by the wavelet threshold filtering level used to control the numerical errors. The basic idea behind SCALES is thus to solve for the most energetic coherent motions, while modeling the effect of the less energetic coherent/incoherent background flow. In fact, varying the wavelet threshold, different approaches with different fidelity are obtained. In particular, for very low thresholds, the corresponding highly accurate method can be referred to as Wavelet-based Direct Numerical Simulation (WDNS).

In this work, the systematic study of the effect of varying the wavelet thresholding level on the accuracy and computational efficiency of wavelet-based simulations and the trade-off between turbulence modeling and numerical issues is conducted. This preliminary study focuses on examining the possibility to develop an adaptive eddy capturing approach that is capable of performing variable fidelity numerical simulations based upon time-variable threshold filtering level.

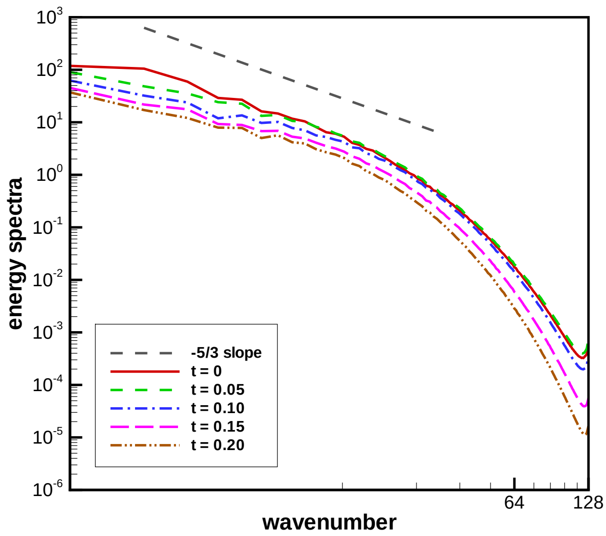

Figure 1: SDNS energy spectra at different times during the decay.

2 WAVELET-FILTERED NAVIER-STOKES EQUATIONS

The SCALES equations, which describe the space-time evolution of the energetic coherent eddies in the turbulent flow field, can be formally obtained by applying the Wavelet Threshold Filtering (WTF) procedure to the continuity and Navier-Stokes equations, e.g. [6, 7]. Disregarding the commutation error between wavelet-filtering and differentiation, the SCALES governing equations for incompressible turbulence are written as

$$\frac{\partial \overline{u_i}^{>\epsilon}}{\partial x_i} = 0\,, \tag{1}$$

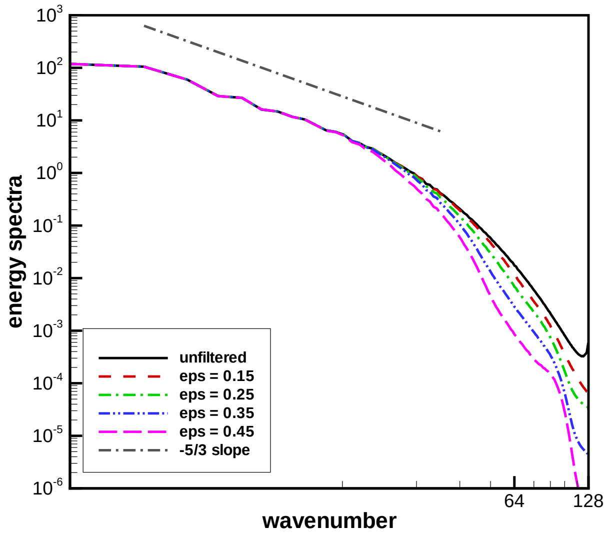

Figure 2: Energy spectra for initial filtered velocity field at different WTF levels.

$$\frac{\partial \overline{u_i}^{>\epsilon}}{\partial t} + \overline{u_j}^{>\epsilon} \frac{\partial \overline{u_i}^{>\epsilon}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}^{>\epsilon}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}^{>\epsilon}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} . \tag{2}$$

As a result of the filtering process, the unresolved quantities

$$\tau_{ij} = \overline{u_i u_j}^{>\epsilon} - \overline{u}_i^{>\epsilon}\, \overline{u}_j^{>\epsilon} , \tag{3}$$

commonly referred to as SGS stresses, are introduced. The SGS stresses can be thought of representing the effect of unresolved less energetic background flow on the dynamics of the resolved energetic eddies. In order to close the filtered equation (2), a SGS model is required to express the unknown stresses (3) as a given function of the resolved velocity field $\overline{u_i}^{>\epsilon}$. In practice, the isotropic part of the SGS stress tensor can be incorporated by a modified filtered pressure variable, so that only the deviatoric part, hereafter noted with a star, $\tau^*_{ij} = \tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij}$, must be actually modeled, and the filtered momentum equation to be solved becomes

$$\frac{\partial \overline{u_i}^{>\epsilon}}{\partial t} + \overline{u_j}^{>\epsilon} \frac{\partial \overline{u_i}^{>\epsilon}}{\partial x_j} = -\frac{\partial \overline{P}^{>\epsilon}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}^{>\epsilon}}{\partial x_j \partial x_j} - \frac{\partial \tau^*_{ij}}{\partial x_j} . \tag{4}$$

In this work, the SGS stress tensor is approximated by the following eddy-viscosity Dynamic Smagorinsky Model (DM in the following)

$$\tau^*_{ij} \cong -2C_S \Delta^2 \epsilon^2 \left|\overline{S}^{>\epsilon}\right| \overline{S_{ij}}^{>\epsilon}, \tag{5}$$

where $\overline{S_{ij}}^{>\epsilon} = 1/2\left(\partial \overline{u_i}^{>\epsilon}/\partial x_j + \partial \overline{u_j}^{>\epsilon}/\partial x_i\right)$ is the resolved rate-of-strain tensor, $\left|\overline{S}^{>\epsilon}\right| = \left(2\overline{S_{ij}}^{>\epsilon}\overline{S_{ij}}^{>\epsilon}\right)^{1/2}$, and Δ is the wavelet-filter characteristic length scale. Following [8],

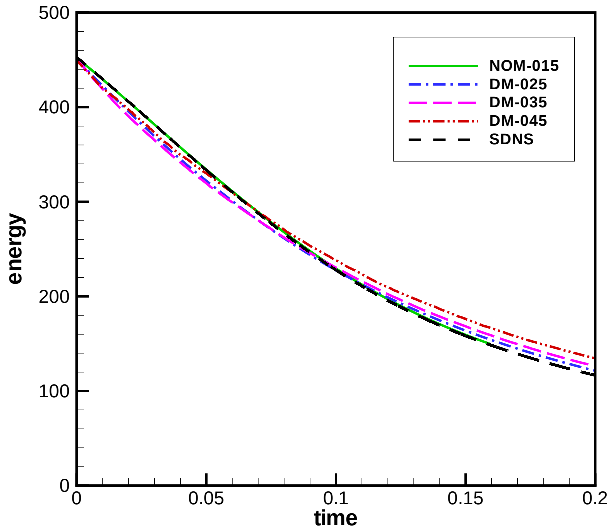

Figure 3: Resolved energy for different WTF levels, compared to SDNS.

the model coefficient C_S is dynamically evaluated as a function of time according to a Germano-like procedure adapted to wavelet-filtering.

3 NUMERICAL EXPERIMENTS

To make some experiments, let us consider the numerical simulation of incompressible homogeneous decaying turbulence at $Re_\lambda = 126$, λ being the Taylor microscale. The initial velocity field is a realization of a statistically steady turbulent flow provided by a pseudo-spectral 256^3 DNS solution obtained by solving the unfiltered Navier-Stokes equations supplied with the random forcing scheme of Eswaran & Pope [9]. The same pseudo-spectral code with the same Fourier modes is used to produce a reference DNS for the present decaying case (SDNS in the following) [10]. In Figure 1, the SDNS solution is illustrated by reporting the energy spectra at different times during the decay. It is worth noting that the initial Reynolds-number is moderately high so to allow a clear inertial scaling in the energy spectrum, as demonstrated by comparison with the theoretical $-5/3$ slope.

Due to the finite difference nature of the wavelet-based solver, and in order not to alter the energy content of the initial velocity field, the SCALES resolution is doubled in each direction with respect to SDNS. For this reason, SCALES is run using a maximum resolution corresponding to 512^3 wavelet collocation points. However, owing to the decaying nature of the turbulent flow considered, this maximum number of wavelets is only required during the initial period, with a gradual decrease of the maximum level of resolution as turbulence decays. In addition, the actual number of wavelets used in the simulation is very low with respect to the above maximum value, owing to the high compression property of the wavelet-transform-based method. The results are illustrated for a time interval corresponding to a few eddy-turnover times, namely, until the Taylor Reynolds-number becomes $Re_\lambda \cong 60$. A number of calculations are performed for different WTF levels, ranging from

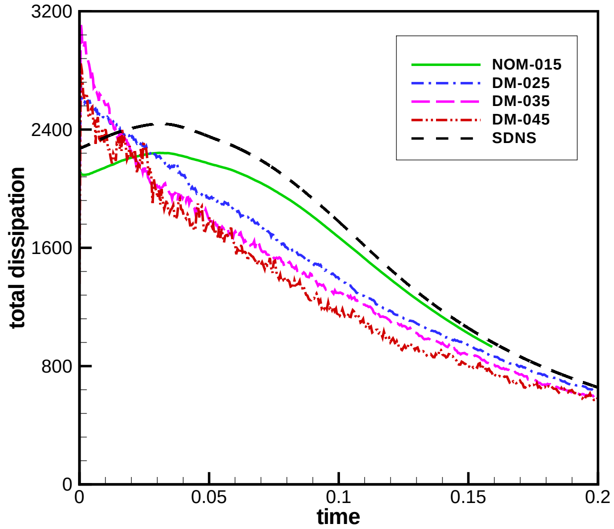

Figure 4: Total dissipation for different WTF levels, compared to SDNS.

$\epsilon = 0.45$ down to $\epsilon = 0.15$. The initial energy spectra for the different simulations are reported in Figure 2, where it is demonstrated how the small scale turbulence is partly represented by the filtered velocity fields.

The time evolution of resolved energy is illustrated in Figure 3 while the total energy dissipation is shown in Figure 4. The SGS contribution to total dissipation is depicted in Figure 5 and the fraction of SGS dissipation is shown in Figure 6. As expected, the SGS dissipation increases with the WTF level ϵ. Moreover, due to decaying turbulence, the fraction of dissipation provided by the model decreases during the simulation. Thus, one can think to use a time-variable thresholding strategy to make it possible to preserve a given level of SGS dissipation fraction. In Figure 7, the energy spectra at a given time instant ($t = 0.05$) are reported, demonstrating that the good performance of the method in representing small-scale turbulence are maintaned during the simulation. The high grid compression is demonstrated in Figure 8, where the percent of retained wavelets is reported. Yet in the most expensive case that is for $\epsilon = 0.15$, practically less than 2.5% of the wavelet collocation points are effectively used. Finally, the evolution of the Taylor Reynolds-number for different WTF levels is illustrated in Figure 9.

By looking at the presented results, one can see how the choice of a relatively high threshold – such as $\epsilon = 0.45$ – unavoidably leads to poor numerical resolution and the corresponding solution (DM-045) can not be considered as a good SCALES solution, as clearly demonstrated for example by inspection of energy spectra in Figure 7. For decreasing WTF level the real SCALES regime is however achieved. For instance, one can consider the simulations at $\epsilon = 0.35$ (DM-035) and, even better, $\epsilon = 0.25$ (DM-025) as providing typical SCALES solutions. When the WTF level is further decreased, the extra dissipation provided by the SGS model can be eventually neglected and the CVS regime is approached. In this study, the no-model solution at $\epsilon = 0.15$ (NOM-015) is interpreted

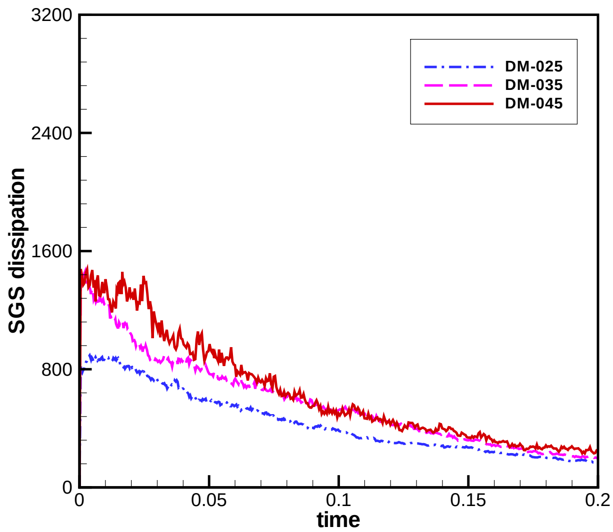

Figure 5: SGS dissipation for different WTF levels.

as CVS – though an even lower level should be considered – since the corresponding statistics well agree with reference SDNS.

In the ultimate case of very low threshold one can think about a WDNS solution. Here, owing to the high computational cost of the direct approach, a WDNS solution is obtained for decaying homogeneous turbulence at reduced Reynolds number that is $Re_\lambda = 72$, and maximum resolution corresponding to 256^3 wavelet collocation points. The WTF level $\epsilon = 0.05$ is assumed low enough to provide a WDNS solution. The corresponding energy spectra are illustrated – at different time instants during the decay – in Figure 10, compared with reference SDNS solution obtained with 128^3 Fourier modes. As one can see, the agreement is almost perfect.

4 CONCLUSIONS AND PERSPECTIVES

The systematic study of the effect of varying the wavelet thresholding level in wavelet-based numerical simulation of homogeneous decaying turbulence has been conducted. The present work should be considered as a preliminary step towards the development of a fully adaptive approach to the numerical simulation of unsteady turbulent flows. The basic idea behind SCALES can be taken one step further by applying a time-dependent wavelet thresholding strategy. Given the desired turbulence resolution, it can be ensured that a given fraction of the total dissipation is provided by the SGS model. With such a strategy the transition between the CVS and the SCALES regimes becomes natural as the SGS modeling procedure for SCALES can be automatically turned off, switching to the no-modeled CVS approach. That forms the basis for an even more sophisticated approach that leads to WDNS as the ultimate possibility. In practice, decreasing in time the value of the thresholding parameter will automatically result in increasing the grid resolution and, thus, reducing the SGS dissipation level. Alternatively, if the SGS dissipation falls below a certain given value,

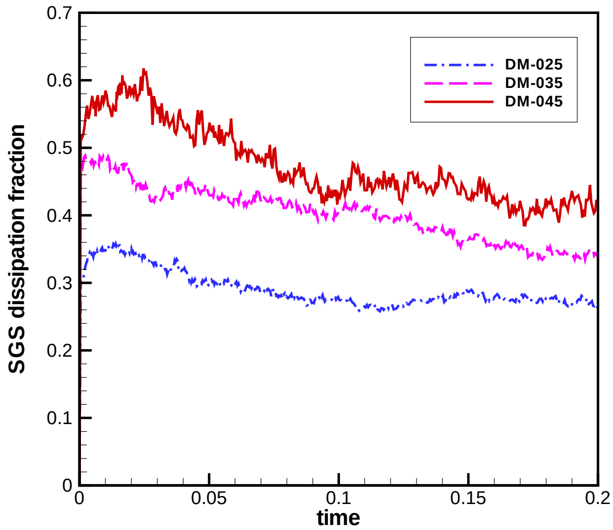

Figure 6: Fraction of SGS dissipation for different WTF levels.

the thresholding parameter is increased, resulting in the mesh coarsening that corresponds to higher SGS dissipation. This combined approach leads to a significant reduction in computational cost of calculations, thus allowing for the simulation of more complex unsteady turbulent flows for given computational resources and power. The time-variable thresholding strategy will be integrated with the dynamic eddy-viscosity SGS model. As a preliminary example of the application of such a strategy, in Figure 11, the WTF level and the SGS dissipation fraction for two different simulations with time-variable thresholding are reported. After some transient time, the two solutions that start with $\epsilon = 0.25$ and $\epsilon = 0.45$, respectively, approach the a-priori desired SGS dissipation fraction, which is prescribed to be 0.40. Then, the WTF level further varies following the decaying nature of the flow, while maintaining the given level of dissipation.

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References

[1] Moin, P., "Advances in large eddy simulation methodology of complex flows," *Int. J. Heat Fluid Flow*, **23**, 710-720 (2002).

[2] Pope, S.B., "Ten questions concerning the large-eddy simulation of turbulent flows," *New J. of Phys.*, **6**, 1-24 (2004).

[3] Farge, M., Schneider, K., Kevlahan, N.K.-R., "Non-Gaussianity and Coherent Vortex Simulation for two-dimensional turbulence using an adaptive orthogonal wavelet basis," *Phys. Fluids*, **11**, 2187-2201 (1999).

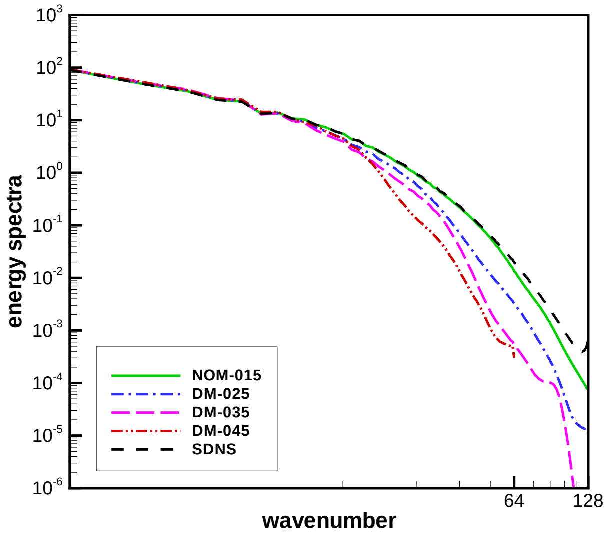

Figure 7: Energy spectra at t=0.05 for different WTF levels, compared to SDNS.

[4] Goldstein, D.E., Vasilyev, O.V., "Stochastic coherent adaptive large eddy simulation method," *Phys. Fluids*, **16**, 2497-2513 (2004).

[5] Vasilyev, O.V., Kevlahan, N.K.-R., "An adaptive multilevel wavelet collocation method for elliptic problems," *J. Comp. Phys.*, **206**, 412-431 (2005).

[6] De Stefano, G., Vasilyev, O.V., Goldstein, D.E., "Localized dynamic kinetic-energy-based models for stochastic coherent adaptive large eddy simulation," *Phys. Fluids*, **20**, 045102.1-045102.14 (2008).

[7] Vasilyev, O.V., De Stefano, G., Goldstein, D.E., Kevlahan, N.K.-R., "Lagrangian dynamic SGS model for stochastic coherent adaptive large eddy simulation," *J. of Turbulence*, **9**, 1-14 (2008).

[8] Goldstein, D.E., Vasilyev, O.V., Kevlahan, N.K.-R., "CVS and SCALES simulation of 3-D isotropic turbulence," *J. of Turbulence*, **6**, 1-20 (2005).

[9] Eswaran, V., Pope, S. B., "An examination of forcing in direct numerical simulations of turbulence," *Comput. Fluids*, **16**, 257-278 (1988)

[10] De Stefano, G., Goldstein, D.E., Vasilyev, O.V., "On the role of subgrid-scale coherent modes in large eddy simulation," *J. Fluid Mech.*, **525**, 263-274 (2005).

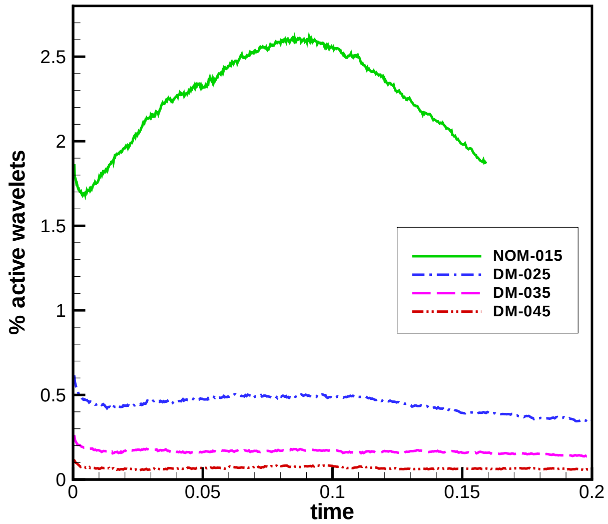

Figure 8: Percent of retained wavelets for different WTF levels.

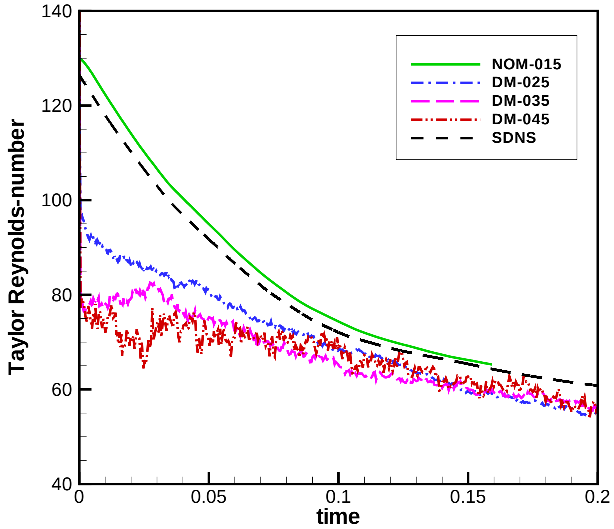

Figure 9: Taylor Reynolds-number for different WTF levels, compared to SDNS.

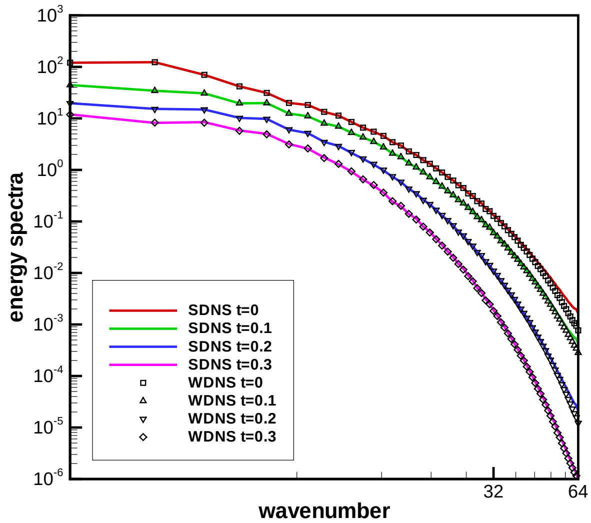

Figure 10: WDNS energy spectra at different times during the decay, compared to 128^3 SDNS solution.

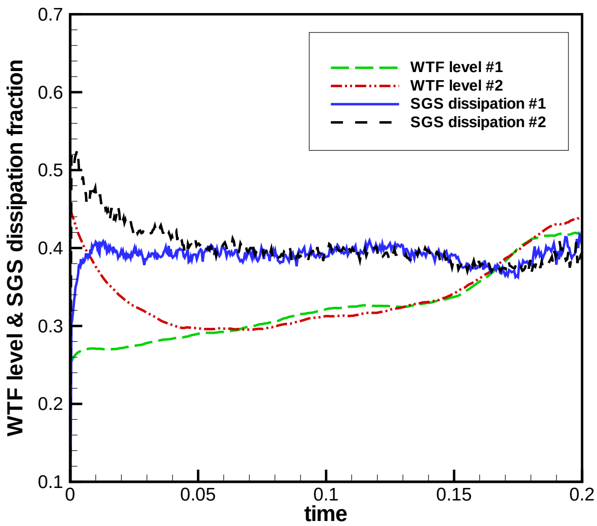

Figure 11: WTF level and SGS dissipation fraction for two different simulations with time-variable thresholding, starting with $\epsilon = 0.25$ (#1) and $\epsilon = 0.45$ (#2).