Conjugated heat transfer in unsteady channel flows

Amilcare Pozzi, Renato Tognaccini
Dipartimento di Ingegneria Aerospaziale, Università di Napoli "Federico II", Italia
E-mail: pozzi@unina.it, rtogna@unina.it

Keywords: Thermo-fluid Dynamics, Conjugated heat transfer, analytical methods.

SUMMARY. The exact analytical solution of the unsteady impulsive Thermo-fluid dynamic field arising in a two-dimensional channel with thick solid walls is presented when the thermal field in the fluid is coupled with the thermal field in the solid. Two cases are considered depending on the boundary condition imposed on the unwetted sides of the channel walls: assigned temperature and adiabatic condition. The analytically computed temperature and heat flux at the solid-fluid interface are analysed as function of time and of the nondimensional parameters governing the problem.

1 INTRODUCTION

In Fluid dynamic problems both temperature and heat flux at a solid-fluid interface are in general *unknown* and should be determined by simultaneous and coupled solutions of the Thermo-fluid dynamic equations in the fluid and the energy equation in the solid. This problem is known in literature as *Conjugated heat transfer* [1]. Anyway conjugated phenomena are usually neglected in Fluid Dynamics because either the temperature or the heat flux are *assigned* as boundary conditions. These effects are, however, relevant in many applications such as aerospace and cooling technologies.

The lack of exact analytical solutions of the Thermo-fluid dynamic field and the difficulty in the solution of the coupled problem itself implies that conjugated problems are usually studied by approximate or numerical methods [2]. Therefore it is difficult to identify "a priori" when conjugated effects can be neglected and the standard boundary conditions can be used.This problem is further complicated by the lack of identification of the nondimensional parameters ruling the phenomenon. Moreover it is important to know how these parameters govern the field, difficult by numerical methods.

Just in the recent years exact analytical solutions describing conjugated effects have been published. Pozzi & Tognaccini[3] found the solution for the conjugated heat transfer in the case of an impulsively accelerated flow from rest to a constant speed over an infinite plate of finite thickness in the case of imposed temperature and of adiabatic condition on the unwetted side of the plate. Pozzi *et al.*[4] showed that, in the cases of plate of infinite thickness the solutions in both the fluid and in the solid are self-similar with very simple analytical expressions. These papers evidenced that the key role in conjugated thermal effects is played by the *thermal activity ratio* defined as the ratio between the thermal effusivities in the fluid and in the solid.

Pozzi & Tognaccini[5] have recently added another family to the short list of exact analytical solutions of the Navier-Stokes equations of practical relevance. Indeed they presented the solution of the completely developed unsteady flow in a two-dimensional channel when an arbitrary time varying pressure gradient is imposed (unsteady Poiseuille flow). The solution has been straightforward extended to the case of a moving wall with arbitrary time dependent velocity (unsteady Couette flow). In addition, the temperature field has also been analytically computed in the case of assigned time varying temperature at the walls for an arbitrary Prandtl number in the case of Eckert number equal to zero and for the relevant case of Prandtl number equal to one in the case of arbitrary Eckert number (whose effects are usually neglected in literature).

With the help of these results, we now present a further step in the analysis of conjugated heat transfer. Indeed these effects are analytically studied when the unsteady channel flow with assigned impulsive pressure gradient is developing between walls of finite thickness and the thermal boundary conditions (constant temperature or adiabatic wall) are assigned on the unwetted side of the channel. The physical problems in the fluid and in the solid are coupled enabling the continuity of the temperature and of the heat flux at the solid-fluid interface.

2 THE PHYSICAL PROBLEM

We consider an infinite two-dimensional channel with section length $2\ d$. Both solid walls of the channel have thickness b. We shall consider problems symmetrical with respect to the channel axis. At time $t = 0$ the fluid is impulsively accelerated by imposing a constant pressure gradient dp/dx. The boundary conditions for the thermal field are imposed on the unwetted side of the thick walls and are constant along the channel axis, therefore the solution only depends on time and on the spatial coordinate orthogonal to the wall.

We assume that an incompressible, laminar flow with constant properties (kinematic viscosity ν and thermal conductivity λ) arises in the channel. In this case the dynamic field is not coupled with the temperature one and the momentum equation is linear since the flow is parallel. An analytical solution of the problem has been proposed in [5].

The energy equations in the fluid and in the solid assume the following forms:

$$\frac{\partial \theta}{\partial \tau} - \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} = E\left(\frac{\partial u}{\partial \eta}\right)^2 , \qquad \frac{\partial \bar{\theta}}{\partial \tau} = t_{\mathrm{fs}}\frac{\partial^2 \bar{\theta}}{\partial Y^2} , \tag{1}$$

where η is the nondimensional spatial coordinate in the fluid (referenced to d) and origin placed on the channel axis, Y is the spatial coordinate in the solid lower wall (referenced to b) and origin placed on the unwetted side of the plate ($Y = 1$ corresponds to $\eta = 0$), τ is the non dimensional time referenced to d^2/ν, $\theta = (T - T_0)/T_0$ is the temperature in the fluid (T_0 is the initial temperature in the fluid), $\bar{\theta}$ is the temperature in the solid and u is the axial fluid velocity referenced to $V_{ref} = -(dp/dx)\ d^2\ /(2\mu)$ (μ is the dynamic viscosity). *Pr* is the Prandtl number; $E = V_{ref}^2/(c_p T_0)$ is the Eckert number (c_p is the specific heat at constant pressure in the fluid). $t_{\mathrm{fs}} = d\alpha_s/(V_{ref} b^2)$ is the ratio between the reference times in the fluid and in the solid (α_s is the thermal diffusivity in the solid and L a reference length for the fluid).

The coupling between the thermal field in the fluid and in the solid is obtained by imposing the continuity of the temperature and of the heat flux at the solid-fluid interface in this way:

$$\theta(\tau, -1) = \bar{\theta}(\tau, 1) = \theta_w(\tau), \qquad p\frac{\partial \theta}{\partial \eta}(\tau, -1) = \frac{\partial \bar{\theta}}{\partial Y}(\tau, 1) , \tag{2}$$

where $p = (b/d)(\lambda/\lambda_s)$ (λ_s is the thermal conductivity in the solid).

The initial conditions for the temperature fields are

$$\theta(0, \eta) = 0, \qquad \bar{\theta}(0, Y) = 0; \tag{3}$$

at the initial time the fluid and solid are in thermal equilibrium.

Two cases are studied here, depending on the condition imposed on the unwetted side of the plate:

(a) *isothermal case*, the temperature is kept at a constant value $\bar{\theta}(\tau, 0) = \theta_e$;

(b) *adiabatic case*, the heat flux $\partial\bar{\theta}/\partial Y(\tau, 0) = 0$.

3 THE TRANSFORMED SOLUTION

3.1 The solution in the fluid

We propose here the solution of the coupled equations (1) in the relevant case of $Pr = 1$.

Denoting with $\Theta(s,\eta) = \mathcal{L}_\tau[\theta(\tau,\eta)]$ the Laplace transform with respect time of the fluid temperature we have

$$\Theta(s,\eta) = \left[\Theta_w(s) + \frac{4E}{s^3}\right] \frac{\cosh(\sqrt{s}\,\eta)}{\cosh\sqrt{s}} + \Theta_p(s,\eta), \tag{4}$$

where $\Theta_w(s) = \mathcal{L}_\tau[\theta_w(\tau)]$ and $\Theta_p(s,\eta) = \mathcal{L}_\tau[\theta_p(\tau,\eta)]$, where θ_p is a particular integral of equation (1) in the fluid for $Pr = 1$:

$$\theta_p(\tau,\eta) = -\frac{E}{2}\left[u(\tau,\eta) - 2\tau\right]^2. \tag{5}$$

$u(\tau,\eta)$ has been proposed in [5] by an infinite series of Jacobi θ_2 functions. The calculation of the temperature field in physical variables only requires the knowledge of Θ_p at the solid-fluid interface where $u = 0$ and the Laplace transform of θ_p is trivial.

3.2 The solution in the solid

The solution in the solid depends on the assigned boundary condition imposed on the unwetted side of the channel wall.

Isothermal case

$$\bar{\Theta}(s,Y) = \Theta_w(s)\frac{\sinh(\sigma Y)}{\sinh\sigma} + \frac{\theta_e}{s}\,\frac{\sinh[\sigma(1-Y)]}{\sinh\sigma}, \tag{6}$$

where $\bar{\Theta}$ is the Laplace transform of the temperature in the solid and $\sigma = \sqrt{s/t_{\mathrm{fs}}}$.

Adiabatic case

$$\bar{\Theta}(s,Y) = \Theta_w(s)\frac{\cosh(\sigma Y)}{\cosh\sigma}. \tag{7}$$

Coupling condition

Both transformed solutions in the fluid and in the solid are unknown since depend on the unknown transformed interface temperature, which can be obtained by coupling the solutions in the fluid and in the solid. In the previous equations the continuity of the temperature across the solid-fluid interface has been already taken into account ($\bar{\Theta}_w = \Theta_w$). By imposing in transformed variables the continuity of the heat flux we obtain:

$$p\left(\frac{\partial\Theta}{\partial\eta}\right)_w(s) \quad = \quad \left(\frac{\partial\bar{\Theta}}{\partial Y}\right)_w(s)\,. \tag{8}$$

The temperature at the solid-fluid interface can be obtained substituting equations (4),(6) and (7) in equation (8). Specifying with $\Lambda = p\sqrt{t_{\mathrm{fs}}}$, we have:

Isothermal case

$$\Theta_w(s) = \frac{1}{\coth\sigma + \Lambda\tanh\sqrt{s}}\left[\frac{2E\Lambda}{s^2}\left(\frac{\tanh\sqrt{s}}{\sqrt{s}} - \frac{1}{\cosh^2\sqrt{s}}\right) + \frac{\theta_e}{\sqrt{s}\sinh\sigma}\right]; \tag{9}$$

Adiabatic case

$$\Theta_w(s) = \frac{1}{\tanh\sigma + \Lambda\tanh\sqrt{s}}\left[\frac{2E\Lambda}{s^{5/2}}\left(\frac{\tanh\sqrt{s}}{\sqrt{s}} - \frac{1}{\cosh^2\sqrt{s}}\right)\right]. \tag{10}$$

4 THE TEMPERATURE AT THE SOLID-FLUID INTERFACE

The physical temperatures at the solid-fluid interface are obtained by performing the inverse Laplace transforms of equations (9) and (10) which, however, are not straightforward.

4.1 The isothermal case

In this case equation (9) can be written as follows:

$$\Theta_w = 2E\Lambda\left(\frac{\Theta_{E1}}{s^3} - \frac{\Theta_{E2}}{s^{5/2}}\right) + \frac{\theta_e}{s}\Theta_e, \tag{11}$$

where

$$\Theta_{E1} = \frac{\tanh\sqrt{s}}{D_a}, \quad \Theta_{E2} = \frac{1}{\cosh^2\sqrt{s}}\frac{1}{D_a}, \quad \Theta_e = \frac{1}{\cosh^2\sigma}\frac{1}{D_a}, \tag{12}$$

with $D_a = \coth\sigma + \Lambda\tanh\sqrt{s}$.

Each term in equation (11) can be expanded in series by taking into account for the properties of the binomial series. In particular, specifying with $\beta = (1-\Lambda)/(1+\Lambda)$:

$$\begin{aligned}\Theta_{E1} &= \frac{1}{1+\Lambda}\{1-(1+\beta)\cdot \\ &\left.\sum_{h=0}^{\infty}(-1)^h\sum_{r=0}^{h}\beta^r\binom{h}{r}\sum_{j=0}^{r}\binom{r}{j}\left[e^{-\sqrt{s}(b_h+2u)} + e^{-\sqrt{s}(b_h+2)}\right]\right\}, \end{aligned} \tag{13}$$

$$\begin{aligned}\Theta_{E2} &= \frac{4}{1+\Lambda}\sum_{h=0}^{\infty}(-1)^h\sum_{i=0}^{h}\binom{h}{i}\sum_{j=0}^{i}\beta^{i-j}(1+\beta)^j\binom{i}{j}\cdot \\ &\sum_{s=0}^{i-j}\binom{i-j}{s}\sum_{r=0}^{j}\binom{j}{r}\left[e^{-\sqrt{s}(e_h+2)} - e^{-\sqrt{s}(e_h+2+2u)}\right], \end{aligned} \tag{14}$$

$$\Theta_e = \frac{2}{1+\Lambda}\sum_{h=0}^{\infty}(-1)^h\sum_{r=0}^{h}\beta^r\binom{h}{r}\sum_{j=0}^{r}\left[e^{-\sqrt{s}(b_h+u)} + e^{-\sqrt{s}(b_h+u+2)}\right], \tag{15}$$

where $b_h = 2[u(1-j)+h+j-r]$, $e_h = 2[u(h-i+s+j-r)+2h-j-2s]$ and $u = 1/\sqrt{t_{\mathrm{fs}}}$. The inverse Laplace transform of each term in these series can now be performed.

4.2 The adiabatic case

In the same way, equation (10) can be written as:

$$\Theta_w = 2E\Lambda\left(\frac{\Theta_{Eb1}}{s^3} - \frac{\Theta_{Eb2}}{s^{5/2}}\right), \tag{16}$$

where

$$\Theta_{Eb1} = \frac{\tanh\sqrt{s}}{D_b}, \qquad \Theta_{Eb2} = \frac{1}{\cosh^2\sqrt{s}}\frac{1}{D_b}, \tag{17}$$

with $D_b = \tanh\sigma + \Lambda\tanh\sqrt{s}$.

The series expansion of each term in equation (10) is

$$\begin{aligned}\Theta_{Eb1} &= \frac{1}{1+\Lambda}\left\{1 + (1+\beta)\sum_{h=0}^{\infty}\sum_{r=0}^{h}\beta^r\binom{h}{r}\cdot\right. \\ &\left.\sum_{j=0}^{r}(-1)^{2h-j}\binom{r}{j}\left[\mathrm{e}^{-\sqrt{s}(b_h+2u)} - \mathrm{e}^{-\sqrt{s}(b_h+2)}\right]\right\}, \end{aligned}\tag{18}$$

$$\begin{aligned}\Theta_{Eb2} &= \frac{4}{1+\Lambda}\sum_{h=0}^{\infty}\sum_{i=0}^{h}\binom{h}{i}\sum_{j=0}^{i}\left(\frac{2}{1+\Lambda}\right)^{i-j}\beta^j\binom{i}{j}\cdot \\ &\sum_{s=0}^{i-j}\binom{i-j}{s}\sum_{r=0}^{j}(-1)^{2h-i+s+r}\binom{j}{r}\left[\mathrm{e}^{-\sqrt{s}(c_h+2+2u)} + \mathrm{e}^{-\sqrt{s}(c_h+2)}\right], \end{aligned}\tag{19}$$

where $c_h = 2[u(r+s+h-i)+j-i+2h-2r]$. The inverse transform can be now easily found.

5 LOCAL AND ASYMPTOTIC ANALYSIS OF THE TEMPERATURE

The local ($\tau \to 0$) and asymptotic ($\tau \to \infty$) solutions can be obtained performing the analysis in transformed variables by the Abelian and Tauberian theorems.

5.1 Isothermal case

For $\tau \to 0$ we have

$$\Theta_w(s) \approx 2E\frac{\Lambda}{1+\Lambda}\frac{1}{s^3} \tag{20}$$

and

$$\theta_w(\tau) \approx E\frac{\Lambda}{1+\Lambda}\tau^2. \tag{21}$$

In this case of flow driven by an imposed impulsive pressure gradient, the temperature at the solid-fluid interface is continuous and grows as τ^2. On the contrary, the interface temperature was discontinuous at the initial time when the initial velocity had an impulsive jump, see [3].

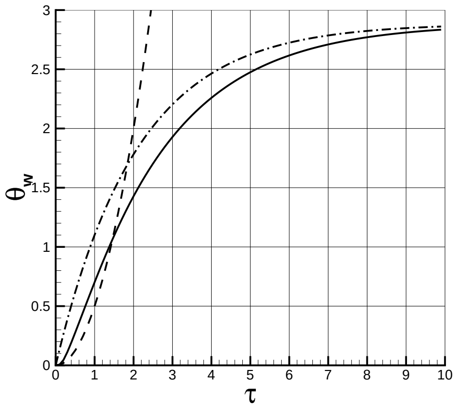
For $\tau \to \infty$ we have

$$\Theta_w(s) \approx \left(\frac{4E\Lambda}{3\sqrt{t_{\mathrm{fs}}}} + \theta_e\right)\left(\frac{1}{s} - \frac{a_1}{a_1 s + 1}\right) \tag{22}$$

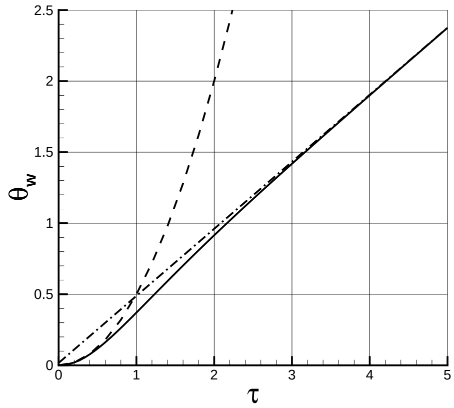
and

$$\theta_w(\tau) \approx \left(\frac{4E\Lambda}{3\sqrt{t_{\mathrm{fs}}}} + \theta_e\right)\left(1 - \mathrm{e}^{-\tau/a_1}\right), \tag{23}$$

where $a_1 = \Lambda/\sqrt{t_{\mathrm{fs}}} + 1/(3t_{\mathrm{fs}})$. Therefore, for large time values, the interface temperature exponentially tends towards a constant value $\theta_{wa\infty} = \frac{4E\Lambda}{3\sqrt{t_{\mathrm{fs}}}} + \theta_e$ which is always higher than the value imposed on the external wall.



(a)



(b)

Figure 1: Temperature at the solid fluid interface versus time. (a): isothermal case ($\theta_e = 1$). (b): adiabatic case. $E = 1$, $\Lambda = 1$, $t_{\rm fs} = 0.5$. —: exact solution; $---$: local solution; $-\cdot-$: asymptotic solution.

5.2 Adiabatic case

In this case, for $\tau \to 0$ we have the same behaviour of the interface temperature that we have obtained in the isothermal case, equations (20) and (21); i.e. for small time values the interface temperature does not depend on the condition imposed on the unwetted side of the plate. This result should not surprise, because in the solid, near the interface, as $\tau \to 0$ (small time and length scales), the unwetted side of the plate looks infinitely far.

For $\tau \to \infty$ we have

$$\Theta_w(s) \approx \frac{4}{3} E \frac{\Lambda}{\Lambda + 1/\sqrt{t_{\rm fs}}} \frac{1}{s^2} \tag{24}$$

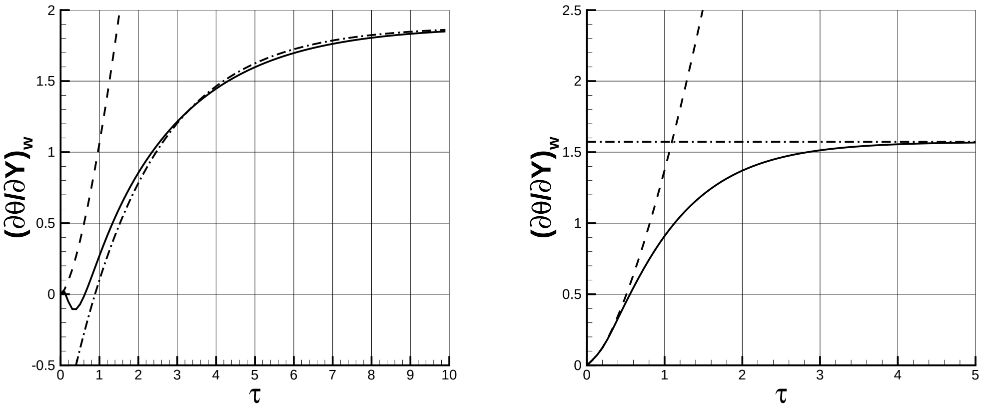
and

$$\theta_w(\tau) \approx \frac{4}{3} E \frac{\Lambda}{\Lambda + 1/\sqrt{t_{\rm fs}}} \tau. \tag{25}$$

In case of adiabatic external wall of the channel, due to the dissipation of kinetic energy, for large time values the temperature at the solid-fluid interface (and terefore in the fluid and in the solid too) indefinitely grows linearly.

6 THE HEAT FLUX

The transformed heat flux at the solid-fluid interface can be obtained differentiating equations (6) and (7).



(a) (b)

Figure 2: Heat flux at the solid fluid interface versus time. (a): isothermal case ($\theta_e = 1$). (b): adiabatic case. $E = 1$, $\Lambda = 1$, $t_{\rm fs} = 0.5$. —: exact solution; $- - -$: local solution; $- \cdot -$: asymptotic solution.

Isothermal case

$$\left(\frac{\partial\bar{\Theta}}{\partial Y}\right)_w (s) = \sigma\left[\Theta_w(s)\coth\sigma - \frac{1}{\sinh\sigma}\frac{\theta_e}{s}\right]. \tag{26}$$

Adiabatic case

$$\left(\frac{\partial\bar{\Theta}}{\partial Y}\right)_w (s) = \sigma\Theta_w(s)\tanh\sigma. \tag{27}$$

6.1 The isothermal case

In this case we have:

$$\left(\frac{\partial\bar{\Theta}}{\partial Y}\right)_w = \sigma\coth\sigma\left[2E\Lambda\left(\frac{\Theta_{E1}}{s^3} - \frac{\Theta_{E2}}{s^{5/2}}\right) + \frac{\theta_e}{s}\Theta_e\right] - \frac{u}{\sinh\sigma}\frac{\theta_e}{\sqrt{s}} \tag{28}$$

Taking again into account for the properties of the binomial series we have:

$$\coth\sigma = 1 - 2\sum_{p=0}^{\infty}(-1)^{2p+1}\mathrm{e}^{-2u\sqrt{s}(p+1)}. \tag{29}$$

Therefore a series expansion of the term within square brackets can be straightforward obtained. Once the series expansion has been obtained, the inverse transform of each term is straightforward as in the previous section.

The inverse transform of the last term in equation (28) can be obtained due to the property of the Jacobi θ_3 function [6] applied for $q = 1$ and $q = 1/2$:

$$\mathcal{L}_\tau^{-1}\left\{\frac{\cosh[(2q-1)\sqrt{s}}{\sqrt{s}\sinh\sqrt{s}}\right\} \quad = \quad \theta_3(q,\tau), \tag{30}$$

where

$$\theta_3(q,\tau) \quad = \quad \sum_{k=-\infty}^{+\infty} \mathrm{e}^{-(q+k)^2/\tau} \tag{31}$$

with $0 \le q \le 1$ and applied for $q = 1/2$.

As for the analysis of the temperature, a local and asymptotic analysis of the heat flux can be performed by the Abelian and Tauberian theorems. For $\tau \to 0$, the heat flux is given by

$$\left(\frac{\partial \bar{\theta}}{\partial Y}\right)_w = \frac{8E\Lambda}{3\sqrt{\pi}\sqrt{t_{\mathrm{fs}}}(1+\Lambda)}\tau^{3/2}. \tag{32}$$

For $\tau \to \infty$:

$$\left(\frac{\partial \bar{\theta}}{\partial Y}\right)_w = \left(\frac{4E\Lambda}{3\sqrt{t_{\mathrm{fs}}}} + \theta_e\right)\left(1 - \mathrm{e}^{-\tau/a_1}\right) - \theta_e. \tag{33}$$

For small and large time values, due to the dissipation of kinetic energy, the fluid is always heating the walls of the channel, even if $\theta_e > 0$. For large time values the heat flux become constant and is $\theta_{wa\infty} - \theta_e$.

6.2 The adiabatic case

The differentiation of equation (16) gives

$$\left(\frac{\partial \bar{\Theta}}{\partial Y}\right)_w = 2E\Lambda u\left(\tanh\sigma\frac{\Theta_{Eb1}}{s^{5/2}} - \tanh\sigma\frac{\Theta_{Eb2}}{s^2}\right) + \theta_0 u \tanh\sigma\frac{\Theta_{0b}}{s}. \tag{34}$$

Again, since

$$\tanh\sigma = 1 - 2\sum_{p=0}^{\infty}(-1)^p \mathrm{e}^{-2u\sqrt{s}(p+1)}, \tag{35}$$

we can obtain a series expression for each term of equation (34) and the inverse Laplace transform can be analytically obtained.

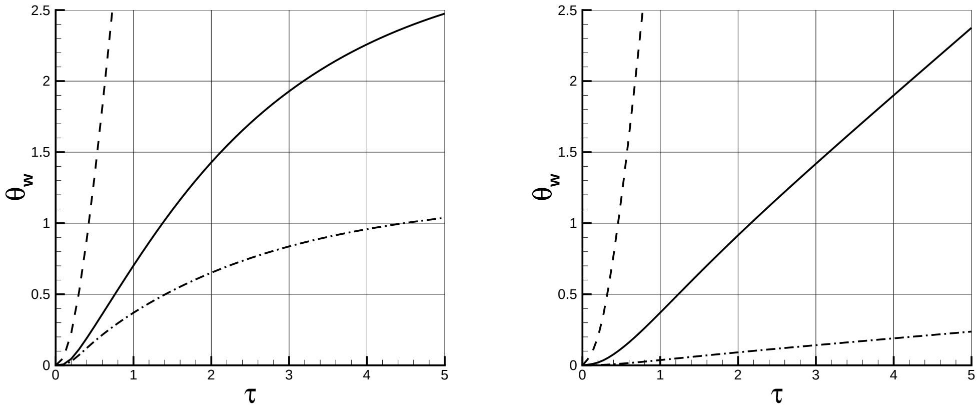
For $\tau \to 0$ the behavior of the heat flux is given by equation (32). For $\tau \to \infty$:

$$\left(\frac{\partial \bar{\theta}}{\partial Y}\right)_w = \frac{4}{3}E\frac{\Lambda}{t_{\mathrm{fs}}\Lambda + \sqrt{t_{\mathrm{fs}}}}. \tag{36}$$

7 ANALYSIS OF THE RESULTS

The temperature at the solid-fluid interface is plotted in figure 1(a) for the isothermal case and in figure 1(b) for the adiabatic case. The exact solutions are compared with the local and asymptotic ones. In the isothermal case (with $\theta_e > 0$), the temperature grows and reaches a constant asymptotic value which is higher than the imposed temperature θ_e (it depends on the Eckert number and on Λ). As in the case of Rayleigh flow over a thick plate [3], $\Lambda = p\sqrt{Pr}\sqrt{t_{\mathrm{fs}}}$ is the main additional parameter ruling the conjugated effects. Also in the present case Λ is related to the thermal activity ratio $E_R = \sqrt{\rho c_p \lambda}/\sqrt{\rho_s c_{p_s} \lambda_s}$:

$$\Lambda = \frac{E_R}{\sqrt{Re}}, \tag{37}$$



(a) (b)

Figure 3: Temperature at the solid fluid interface versus time for different values of the Eckert number. (a): isothermal case ($\theta_e = 1$). (b): adiabatic case. $\Lambda = 1$, $t_{\rm fs} = 0.5$. $-\cdot-$: $E = 0.1$; —: $E = 1$; $---$: $E = 10$.

where $Re = \rho V_{ref} d/\mu$ is the reference Reynolds number of the flow.

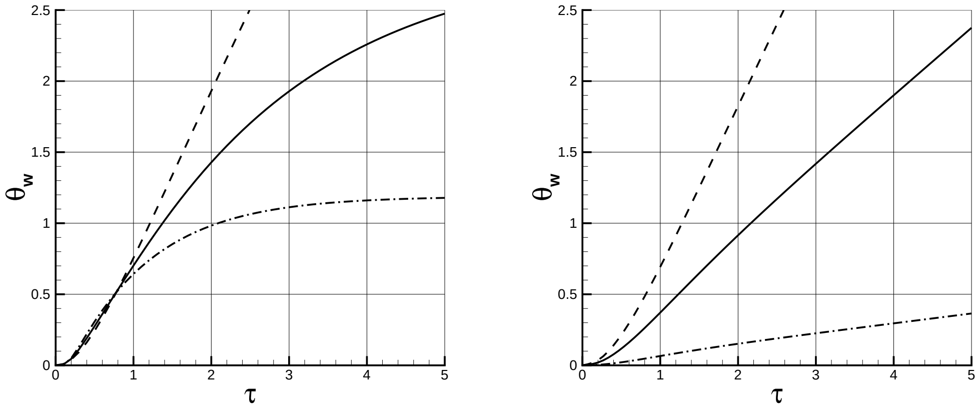
In the adiabatic case, if $E \neq 0$ the interface temperature, as already noted by the asymptotic analysis, indefinitely grows with linear law.

In figures 2(a) and 2(b) the heat fluxes at the interface versus time are plotted, respectively for the isothermal and adiabatic case. Again the exact solution is compared with the local and asymptotic ones. It is interesting to note that in the isothermal case, even if $\theta_e > 0$, at the initial time the fluid is heating the wall due to the dissipation of kinetic energy. There is a range of time in which the solid wall is heating the fluid, but in the asymptotic stage, the fluid will again heat the wall. For large values of E, the wall can heat the flow only for high values of θ_e.

Figures 3 and 4 show the interface temperature versus time respectively for different values of E and Λ. In each figure both the isothermal and adiabatic cases are proposed. It is interesting to note that a similar behavior of the temperature can be obtained by varying E or Λ.

8 CONCLUSIONS

In this paper we have presented an exact solution of the unsteady conjugated heat transfer problem of a flow arising in a 2D channel with *thick* walls. The flow is driven by a constant pressure gradient impulsively set to a constant value at the initial time. The solution of the thermal field has been obtained in the case of $Pr = 1$ taking into account for the effects of the dissipation of kinetic energy in the fluid (Eckert number different than 0). Two problems have been solved depending on



(a) (b)

Figure 4: Temperature at the solid fluid interface versus time for different values of the thermal activity ratio. (a): isothermal case ($\theta_e = 1$). (b): adiabatic case. $E = 1$, $t_{\rm fs} = 0.5$. $- \cdot -$: $\Lambda = 0.1$; —: $\Lambda = 1$; $- - -$: $\Lambda = 10$.

the thermal condition imposed on the unwetted side of the channel walls: isothermal and adiabatic case.

The time evolution of the temperature and heat flux at the solid-interface have been analysed and discussed in terms of the main parameters which are ruling the phenomena: the Eckert number E and the coupling parameter Λ which essentially is related with the thermal activity ratio between the fluid and the solid.

References

[1] L.T. Perelman, " On conjugated problems of heat transfer", Int. J. of Heat and Mass Transfer, **3**, 293-303, (1961).

[2] K.D. Cole, "Conjugate heat transfer from a small heated strip", Int. J. of Heat and Mass Transfer, **40**, 2709-2719, (1997).

[3] A. Pozzi and R. Tognaccini, "Time singularities in conjugated thermo-fluid dynamic phenomena", J. of Fluid Mech., **538**, 361-376, (2005).

[4] A. Pozzi, G. Quaranta and R. Tognaccini, "A self-similar unsteady flow with conjugated heat transfer", Int. J. Heat Mass Transfer, **51**, 1804-1809, (2008).

[5] A. Pozzi and R. Tognaccini, "Thermo-fluid dynamics of the unsteady channel flow", Europ. J. Mech./B Fluids, **28**, 299-308, (2009).

[6] A. Ghizzetti and A. Ossicini, *Trasformate di Laplace e calcolo simbolico*, (UTET, 1971).